

## Chapter 3: Stresses and Calculation Sections

### 3.1. Introduction

Structural members must satisfy both **ultimate** and **serviceability** requirements throughout their service life. They should provide adequate strength, stability, durability, and satisfactory performance under all expected loading conditions. Design codes ensure structural safety by defining **limit states**, which represent conditions beyond which a structure no longer meets the required performance. Each limit state must be checked separately using appropriate analysis methods. Ultimate limit state verification focuses on structural strength, while serviceability limit state verification considers deflection, cracking, creep, shrinkage, and prestress losses to ensure satisfactory long-term performance.

### 3.2. Limit States

#### 3.2.1. Ultimate Limit State (ULS)

The **Ultimate Limit State (ULS)** corresponds to structural failure or loss of stability. It ensures that the structure can safely resist the design loads without collapse. ULS verification mainly includes:

- **Strength failure:** insufficient load-carrying capacity.
- **Stability failure:** overturning or sliding.
- **Buckling:** instability of compression members.

Appropriate safety factors are applied to prevent these failure modes.

#### 3.2.2. Serviceability Limit State (SLS)

The **Serviceability Limit State (SLS)** ensures that the structure remains functional, durable, and comfortable during normal use. Typical serviceability criteria include:

- Limitation of deflections.
- Crack width control.
- Vibration control.
- Long-term durability against environmental effects.

SLS prevents excessive deformation or deterioration without compromising structural safety.

### 3.3. Actions

Actions are external loads or influences acting on a structure. They are classified as:

- **Permanent actions (G):** self-weight and permanent fixtures.
- **Variable actions (Q):** live loads, wind, snow, temperature, traffic, and other changing loads.
- **Accidental actions (A):** earthquakes, explosions, impacts, and other exceptional events.

Design codes specify combinations of these actions to verify structural safety under the most unfavorable loading conditions.

### 3.4. Internal Forces

Internal forces are the structural responses generated by external actions. They are used to evaluate the structural performance and design structural members.

#### 3.4.1. Types of Internal Forces

The principal internal forces are:

- **Axial force (N):** tension or compression.
- **Shear force (V):** force causing sliding between adjacent sections.
- **Bending moment (M):** force producing flexure.
- **Torsional moment (T):** force causing twisting.
- **Stress and strain:** internal stress distribution and resulting deformation within the material.

#### 3.4.2. Importance of Internal Forces

Internal forces are determined for each structural element to verify compliance with both ULS and SLS requirements. Their evaluation ensures that structural members possess sufficient strength, stability, and durability throughout their design life.

#### 3.4.3. Design Actions at the Ultimate Limit State (ULS)

##### A. Fundamental Load Combination

The design action at the ULS is generally expressed as:

$$\gamma_p + P_m + 1.35.G_{\max} + G_{\min} + \gamma_{Q1} Q_{1k} + \sum 1.3\psi_{0i} Q_{ik}. \quad (\text{Eq.3.1})$$

With :

$\gamma_p = 1$  in most cases.

$P_m$  : probable value of the prestressing action.

$G_{max}$  : set of permanent adverse actions.

$G_{min}$  : set of permanent favorable actions.

$\gamma_{Q1} = 1.5$  in the general case.

$Q_{1k}$  : the characteristic value of the basic action.

$\psi_{0i} Q_{ik}$  : the combination value of an accompanying action.

### **B. Accidental combinations:**

$$P_m + F_A + G_{max} + G_{min} + \psi_{11} Q_{1k} + \sum \psi_{2i} Q_{ik} \quad (\text{Eq.3.2})$$

With :

$F_A$  : the nominal value of the accidental share.

$\psi_{11} Q_{1k}$  : the frequent value of a variable action.

$\psi_{2i} Q_{ik}$  : the quasi-permanent value of another variable action.

### **3.4.4. Design stresses at the SLS**

#### **A. Rare combinations:**

$$P_d + G_{max} + G_{min} + Q_{1k} + \sum \psi_{0i} Q_{ik} \quad (\text{Eq.3.3})$$

With,  $P_d$ : the most unfavorable value of the prestressing action between  $P_1$  and  $P_2$ .

#### **B. Common combinations:**

$$P_d + G_{max} + G_{min} + \psi_{11} Q_{1k} + \sum \psi_{2i} Q_{ik} \quad (\text{Eq.3.4})$$

#### **C. Combinaisons quasi-permanentes :**

$$P_d + G_{max} + G_{min} + \sum \psi_{2i} Q_{ik} \quad (\text{Eq.3.5})$$

NB: The values of the coefficients  $\psi_0$ ,  $\psi_1$  and  $\psi_2$  vary according to the nature of the structure and are given by the BAEL 91 rules.

### **3.5. Design Sections:**

Design sections define the geometric properties required for structural analysis and member design. These properties determine the strength, stiffness, and stability of structural elements:

### .5.1. Cross-sectional Area (A)

The cross-sectional area is used to calculate normal stresses and axial resistance.

For a rectangular section

$$A = b \cdot h \quad (\text{Eq.3.6})$$

Or  $b$  = width and  $h$  = height.

### 3.5.2. First Moment of Area (S)

The first moment of area is used to locate the centroid of a section)

- **Unit** :  $\text{m}^3$ ,  $\text{cm}^3$ ,  $\text{mm}^3$ .
- **Usefulness** : Used to determine the center of gravity (or geometric center) of the section.
- **Example** For a rectangular section:  
$$S_x = A \cdot y$$
  
(Eq.3.7)

where  $y$  = distance from the center of gravity to the reference axis.

### 3.5.3. Center of gravity (G)

The centroid is the geometric center of the section and is required for determining the neutral axis and section properties.

### 3.5.4. Second Moment of Area (I)

The second moment of area measures the flexural stiffness of a section.

- **Unit** :  $\text{m}^4$ ,  $\text{cm}^4$ ,  $\text{mm}^4$ .
- Rectangular section:

$$I_x = \frac{bh^3}{12} \quad (\text{Eq.3.8})$$

- Circular section:

$$I_x = \frac{\pi d^4}{64} \quad (\text{Eq.3.9})$$

or d = diameter

**Noticed :**

- $I_x$  : Moment of inertia with respect to the horizontal axis (x).
- $I_y$  : Moment of inertia with respect to the vertical axis (y).

### 3.5.5. Section Modulus $W_x$ and $W_y$ (cm<sup>3</sup>):

The resistance modulus is equal to the quotient of the axial moment of inertia by the distance from the axis to the furthest fiber.

$$W_x = \frac{I_x}{y}, W_y = \frac{I_y}{x} \quad (\text{Eq.3.10})$$

### 3.5.6. Flexural modulus (W)

- **Definition** : Ratio between the moment of inertia and the maximum distance to the neutral axis.
- **Unit** : m<sup>3</sup>, cm<sup>3</sup>, mm<sup>3</sup>.
- **Purpose** : Used to calculate maximum stresses due to bending.
- **Formula** :

$$W = \frac{I}{v} \quad (\text{Eq.3.11})$$

Or v = the distance between the neutral axis and the furthest fiber.

Example: for a rectangular section:  $W_x = \frac{bh^2}{6}$

### 3.5.7. Radius of gyration (i)

- **Definition** : A measure of the distribution of area around an axis.
- **Unit** : m, cm, mm.
- **Usefulness** : Used to evaluate buckling resistance.

- **Formula :**

$$i = \sqrt{\frac{I}{A}} \quad (\text{Eq.3.12})$$

### 3.5.8. Polar moment of inertia (J)

- **Definition :** A measure of resistance to torsion.
- **Unit :**  $\text{m}^4$  ,  $\text{cm}^4$  ,  $\text{mm}^4$  .
- **Usefulness :** Used to calculate torsional stresses.
- **Formula :** For a circular section

$$J = \frac{\pi d^4}{32} \quad (\text{Eq.3.13})$$

### 3.5.9. Section Efficiency:

The efficiency of a section is given by:

$$\rho = \frac{I}{BV_i V_s} \quad (\text{Eq. 3.14})$$

It characterizes the economic aspect of a section with respect to its bending behavior.

### 3. 5.10. Types of Sections:

The concepts of **gross** and **net section** are essential in civil engineering for the dimensioning and analysis of structural elements. They allow us to describe the effective surface area of a cross-section, taking into account or not material reductions (holes, perforations, etc.).

#### A. Gross section

The **gross section** represents the total surface area of the cross-section of a structural element **without taking into account any openings, perforations or material reductions** . It includes the entire surface of the section, whether it is made of concrete, steel, wood or any other material.

For a rectangular section:  $\text{Gross } A = b \cdot h$  , where  $b$  = width and  $h$  = height.

#### B. Net section

The **net section** represents the effective surface area of the cross-section **after deducting openings, perforations or material reductions**. It takes into account voids that reduce the load-bearing capacity of the element.

$$Net A = Gross A - \text{surface area of the holes}.$$

### C. Homogeneous section:

This is the cross-section obtained by adding the cross-section of the bonded longitudinal reinforcement multiplied by a suitable equivalence coefficient "n" to the net cross-section, provided that the reinforcement is bonded to the concrete (which is the case in pre-tensioning, and in post-tensioning after the injection of appropriate grout). The homogeneous cross-section is used to calculate the stresses due to the loads applied after the injection of the conduits, which generally correspond to the variable loads.

$$A_h = A_n + (\text{section of bonded longitudinal reinforcement}) \times (\text{Equivalence coefficient } n).$$

n = 5: case of short-term actions.

n = 15: case of long-term actions.

### D. Cracked transformed section:

Obtained by neglecting the concrete in tension. It includes only the compression concrete, passive reinforcement (n=15), and prestressing steel using equivalent coefficients depending on the prestressing system:

- 15 in the case of pre-tension.
- 0 for non-adherent reinforcements in post-tensioning.
- 15/2 for bonded reinforcements in post-tension.

$$A_{hr} = (\text{Section of the concrete under compression alone}) + (\text{section of passive reinforcement}) \times (n) + (\text{section of prestressing reinforcement}) \times (n').$$

The cracked transformed section is commonly used for stress verification of Class III prestressed concrete members..