

Chapter 1: Design of Flat Slabs and Flat Plates

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1. Introduction

Floor systems are horizontal load-bearing elements that separate the stories of a building. Their primary function is to receive and support vertical loads (dead loads, live loads, and climatic loads) and transfer them to supporting elements such as beams, ribs, columns, or shear walls [1]. In addition to their structural role, floors must provide thermal and acoustic insulation between stories in residential buildings; indeed, slab thicknesses in such buildings are often determined by insulation requirements, resulting in thicknesses that exceed those dictated by structural strength criteria alone.

This chapter outlines the sizing, reinforcement design, and construction details for certain types of floor systems commonly used in reinforced concrete construction.

2. Définition:

Reinforced concrete floor systems come in various structural configurations to meet requirements regarding span, loading, and construction method. The following floor types can be distinguished (based on their load-bearing elements) [1]:

1-Flat slab (Fig. 1);

2-One-way beam-and-slab floor (Fig. 2);

3-Two-way beam (grid/waffle) floor (Fig. 3);

4- Ribbed floor (joists perpendicular to beams, with the slab spanning perpendicular to the ribs) (Fig. 4).

Reinforced concrete floors can be either prefabricated or cast-in-place. In this course, we will examine only the latter case.

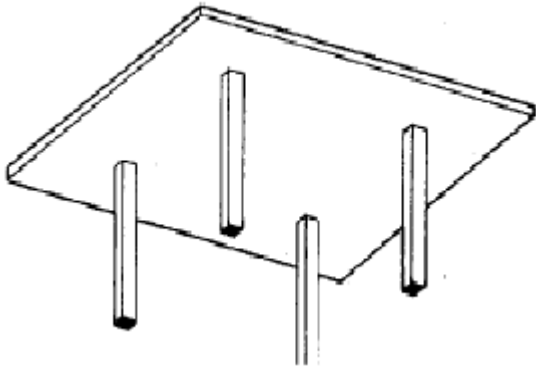


Fig. 1: Slab floor

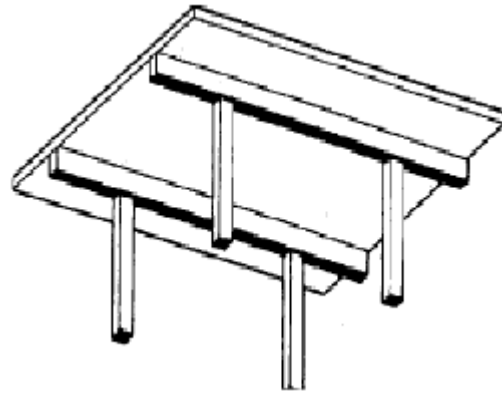


Fig. 2: One-way beam floor system



Fig. 3: Ribbed floor.

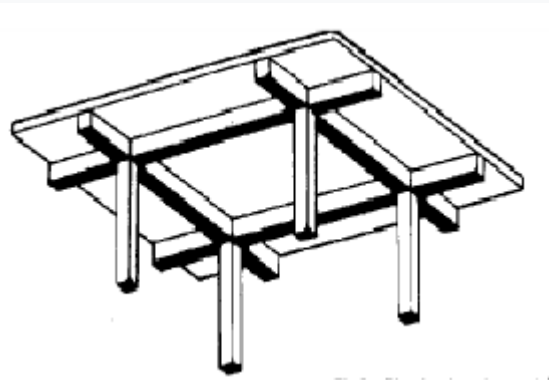


Fig. 4: Grid floor system.

3. Flat slabs and mushroom slabs:

A flat slab is a floor system with a flat underside, devoid of beams, resting directly on columns. This type of floor may feature column capitals at the top (an enlargement of the column head); in such cases, they are referred to as mushroom slabs (Fig. 5). The function of the mushroom slab capital is to reduce the slab span, increase rigidity, and prevent punching shear failure [2]. It is also possible to provide a local increase in thickness at the column location to enhance resistance to bending and shear (Fig. 5).

Mushroom slabs are, in principle, reserved for industrial floors subject to heavy superimposed loads.

- A general grid arrangement with a bottom layer of bars (in the span) that may, in certain cases, be terminated before reaching the support centerlines.
- A grid arrangement with a top layer of bars (hogging reinforcement/starter bars) that may be supplemented by additional top bars outside the zones adjacent to the supports.

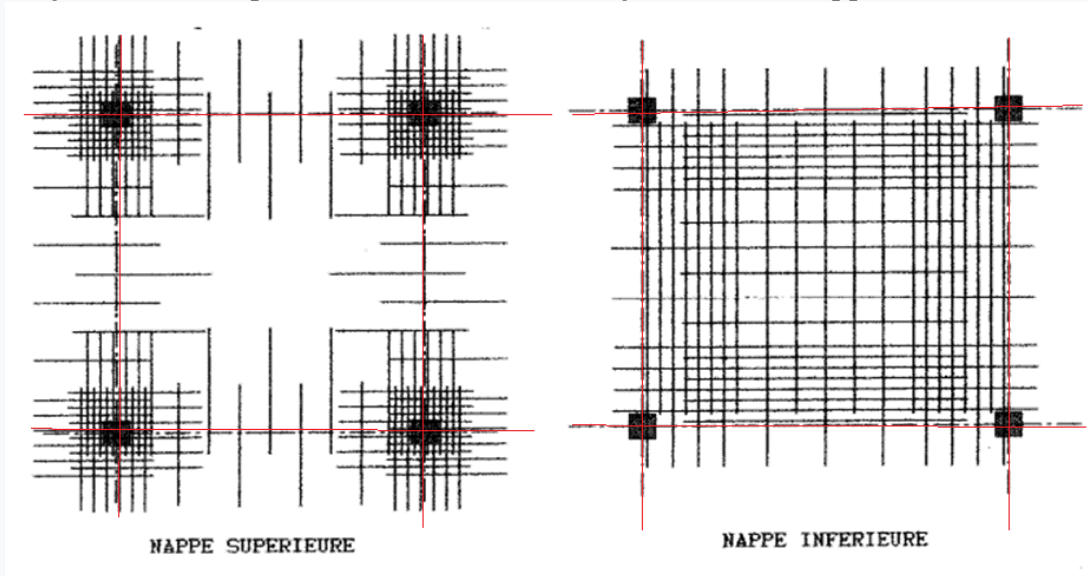


Fig. 7: Reinforcement layout for floor slabs [4]

In the case of flat slabs with column capitals, inclined construction bars are generally installed to ensure structural continuity and maintain the shape of the capitals (Fig. 8). It should be noted that the use of so-called "flat slabs" or "flat slabs with column capitals" is prohibited in seismic zones under RPA2024 [5].

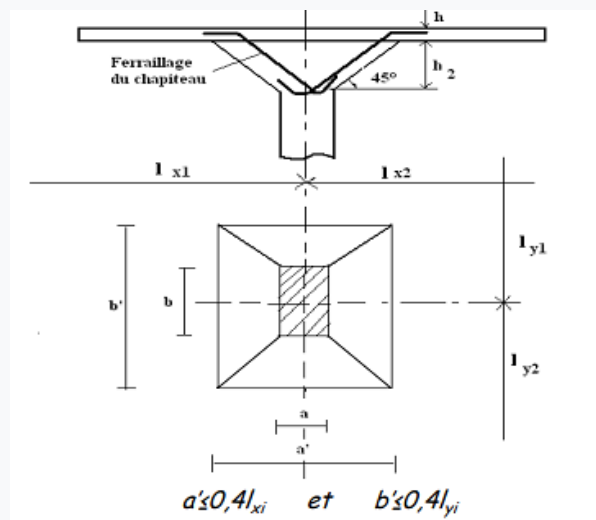


Fig. 8: Reinforcement layout for the column capitals.

4. Hollow-core floors:

This type of floor is frequently used in the construction of single-family homes [2].

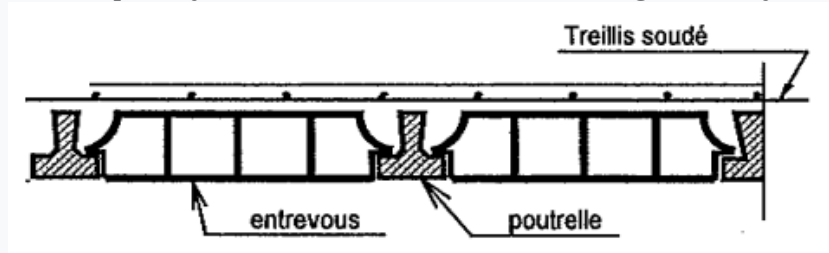


Fig. 9: Hollow-block floor [6].

This type of floor system generally consists of:

- Ribs (also known as joists) that support the hollow blocks.
- Hollow blocks (also known as *hourdis* or infill blocks) that serve as formwork between the joists.
- A compression slab that ties the entire floor assembly together.

*Hollow blocks—also known as *entrevous*—are prefabricated infill elements made of fine-aggregate concrete, terracotta, or polystyrene; they generally act as formwork for the joists and the concrete slab that covers them.

These hollow blocks come in various sizes (thickness = 12, 15, 16, 18, 20, or 25 cm; width = 30, 56, or 65 cm).

*Joists: these are the floor components that may rest at their ends on main beams or load-bearing walls. The compression slab is a reinforced concrete slab, typically with a thickness of 4 cm or more, and is generally reinforced with welded wire mesh. The slab is cast in situ over the entire surface of the floor assembly formed by the joists and hollow blocks. The mesh size of the welded wire mesh must not exceed 200 x 300 mm (Fig. 10). The reinforcement sections for the compression slab must comply with the requirements of BAEL 91/99 (B.6.8.42):

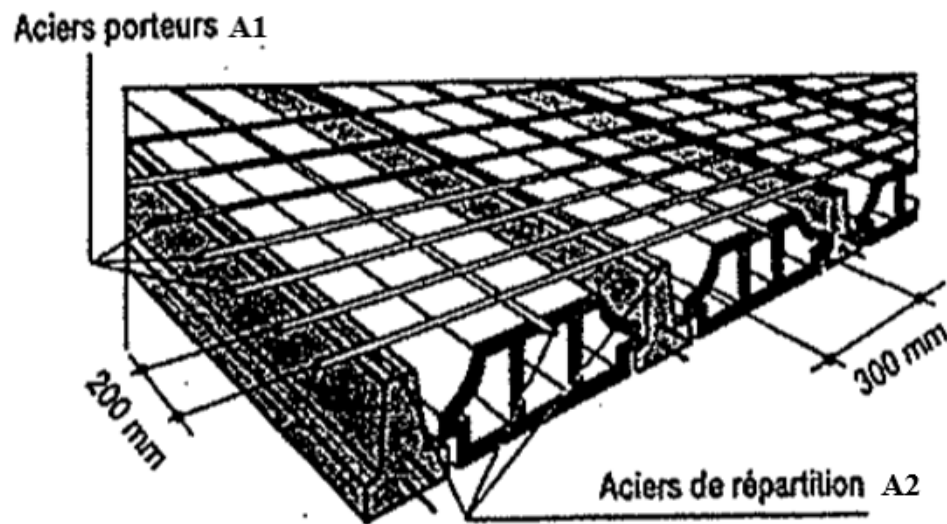


Fig. 10: Reinforcement of a hollow-core slab [6]

4.1. Preliminary sizing of joists:

The floor thickness is determined based on the following condition [7]:

$$L/25 \leq h_t \leq L/20 \text{ or } h_t \geq L/22.5$$

h_t : Total slab thickness = sum of the compression slab and the hollow blocks.

L : Maximum clear span between joist supports.

The most common depth is (16+4) cm for a standard floor and (20+5) cm for joists with long spans (subject to very heavy superimposed loads).

Together with the compression slab, the joists form a T-shaped section (Fig. 11).

Width b_0 : $0.3 \times h$

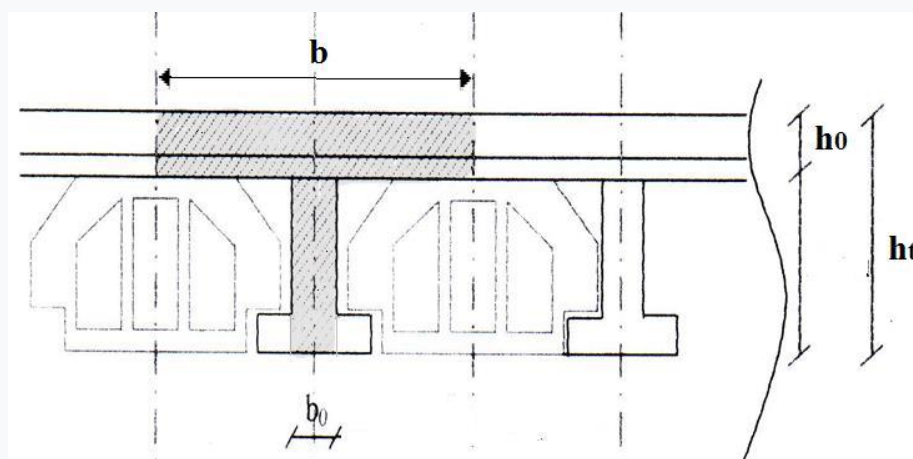


Fig. 11: Calculation diagram for the joists

4.2. Reinforcement of joists:

The joists are designed as continuous beams subjected to simple bending. Depending on the specific case, these continuous beams are analyzed using either exact methods from the strength of materials (RDM) or standard reinforced concrete design methods, such as the simplified method (*méthode forfaitaire*) or Caquot's method. To resist bending moments, main reinforcement is placed in the bottom tension zones (within the span), in the top tension zones over intermediate supports, and at end supports in cases of fixed-end conditions (Fig. 12). Additional reinforcement is required as strengthening reinforcement at the edges of floor openings (Fig. 13).

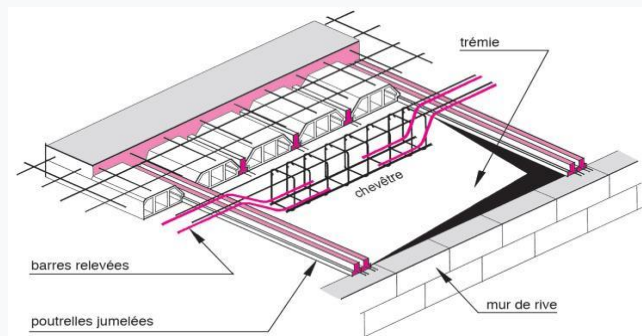
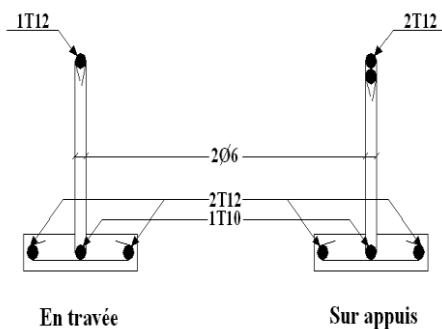


Fig. 12 Example of beam reinforcement

Fig. 13 Reinforcement detail of an opening [8]

5. Solid slab floors:

A solid slab can be considered as a wide, flat beam, where the smallest dimension is at least five times its total thickness. It may be supported by beams, ribs, walls, or structural panels. These supports influence the way the slab functions—and, consequently, the arrangement of the reinforcement—leading to the following cases [6]:

*a slab resting on simple supports, such as a slab forming a terrace (Fig. 14);

*a slab with fixed (encastred) ends (Fig. 15);

*a slab supported on three or four sides, or alternatively on only two sides (Fig. 16)

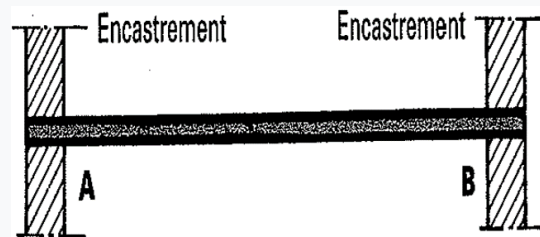
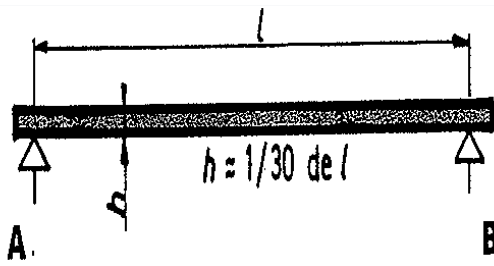


Fig. 14: Slab simply supported at A and at B [6]. Fig. 15: Slab with fixed ends [6].

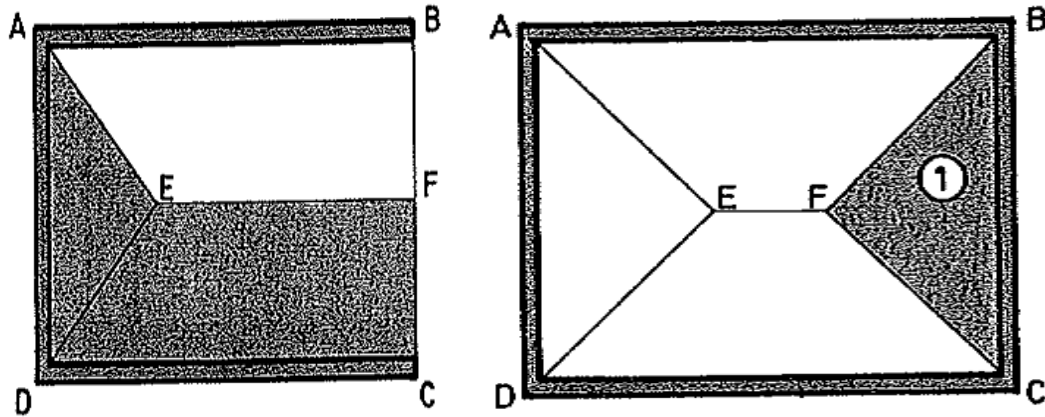


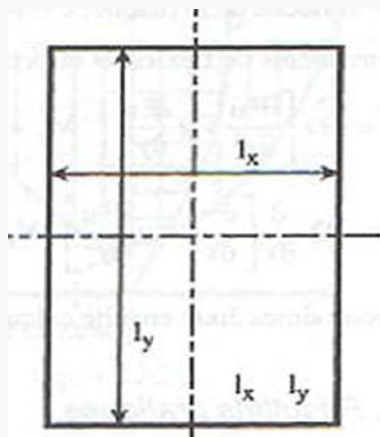
Fig. 16: Slab supported on three or four supports [6]

Depending on their supports, slabs subjected to uniformly distributed loads can be classified as follows [7,9]:

- **One-way spanning (beam-slab):** for a slab panel supported (or fixed) on two sides, or supported on four sides, that meets the following condition:

The ratio $L_x / L_y < 0.4$, where L_x is the minimum span and L_y is the maximum span;

- **Two-way spanning (slab):** for a slab panel supported on four sides that meets the following condition: $0.4 \leq L_x / L_y \leq 1$.



5.1. Preliminary sizing of the solid slab:

The thickness *h* of the chosen solid slab must satisfy the following conditions:

*Flexural resistance [6]:

1/30 to 1/35 of the span for a slab supported in two directions;

1/40 to 1/50 of the span for a slab supported in 3 or 4 directions;

The literature suggests other ratios to consider when selecting the thickness of floor slabs [7, 10]:

Table 1: Common slab thicknesses [7, 10]

	$\frac{L_x}{L_y} < 0.4$	$\frac{L_x}{L_y} > 0.4$
Isolated panel	$\frac{h}{l_x} \geq \frac{1}{20}$	$\frac{h}{l_x} \geq \frac{1}{30}$
Continuous slab	$\frac{h}{l_x} \geq \frac{1}{25}$	$\frac{h}{l_x} \geq \frac{1}{40}$

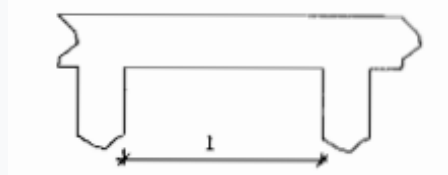
*Acoustic insulation requires a minimum thickness of 15 cm.

*Fire safety (7 cm for 1-hour fire resistance; 11 cm for 2-hour fire resistance) [6].

5.2. Calculation of internal forces in a slab:

The calculation of bending moments in slabs is performed along one or two directions, depending on the slab's structural behavior. In general, the calculation of bending moments in one direction is determined by treating the slab as a flat beam with a width $b = 1 \{ \text{m} \}$ and a height h [6].

It should be noted that, in general, the span used for the calculation of reinforced concrete elements is the clear span between the inner faces of the supports (except in the exceptional case of a beam



equipped with specific bearing devices) [9].

A. One-way spanning slab (Simplified/Empirical method):

Slabs that span in one direction are calculated as continuous beams to determine the primary reinforcement (load-bearing steel) placed in the direction of the shorter span (L_x). In the other

direction (L_y), distribution reinforcement is required, with an area at least equal to one-quarter ($1/4$) of the primary reinforcement (per unit length) [6, 9].

The simplified (empirical) method is applicable only to flexural elements such as beams, joists, and one-way spanning slabs.

- **Scope of validity [9]:**

- Moderate imposed loads (Q): Q is at max, equal to 2 times the permanent load or 5kN/m .
- The ratio between two adjacent spans is between 0.8 and 1.25.
- The moments of inertia (I) of the cross-sections are the same across the different spans.
- Cracking is considered to be of little consequence (non-critical). The empirical method (or "fixed-coefficient method") consists of determining the maximum bending moments—both within the span and at the supports—by applying pre-established, fixed ratios to the moment M_0 of a simply supported beam (the isostatic moment) that has the same span and is subjected to the same loads as the span under consideration [7,9]:

- Maximum moment in an end span: $M_t \geq \left[\frac{1.2+0.3\alpha}{2} \right] M_0$, where: $\alpha = \frac{Q}{G+Q}$
- Moment in an intermediate span: $M_t \geq \left[\frac{1+0.3\alpha}{2} \right] M_0$,

M_0 is the maximum moment in a simply supported (statically determinate) span; for a span of length L subjected to a uniformly distributed load q : $M_0 = qL^2/8$

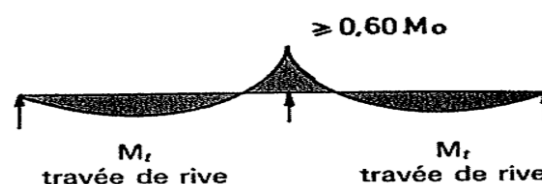
-The negative moments at the intermediate supports (top reinforcement/hogging reinforcement) are summarized in the figure (Fig. 17).

-At the end supports, the top reinforcement is sized to balance a moment $\geq 0.15 M_0$.

For each span, the following inequality must essentially be verified:

$$M_t + \frac{M_w + M_e}{2} \geq \text{Max} [1.05M_0 ; (1+0.3\alpha).M_0]$$

Where : M_w Bending moment at the left support and M_e moment at the right support



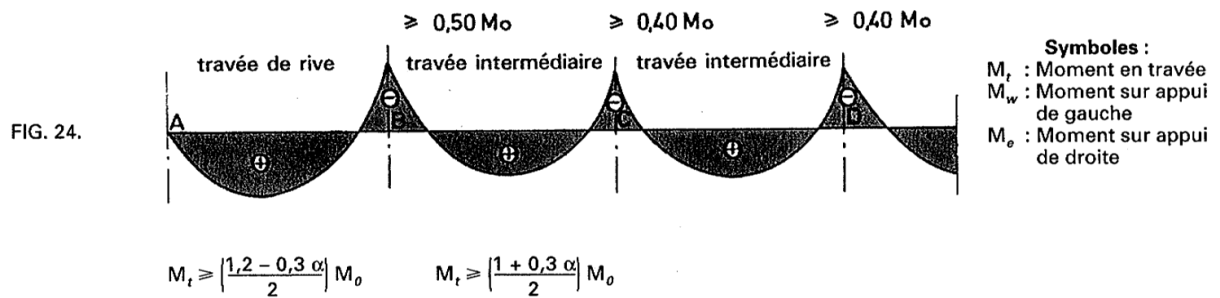


Fig. 17: Standardized moments for spans and supports [6].

B. Two-way spanning slab

For this type of slab, moments act in both directions, meaning that the reinforcement bars function as load-bearing elements in both directions. First, the maximum moments at the center of the slab are determined by considering it to be simply supported along its edges and subjected to a uniformly distributed load [9]:

$$M_x \text{ dans le sens } L_x \text{ de la petite portée} = \mu_x p l_x^2$$

$$M_y \text{ dans le sens } L_y = \mu_y M_x$$

P : la charge uniformément répartie par unité de surface.

μ_x and μ_y are reduction coefficients provided in Appendix E3 of the BAEL [9] as a function of the ratio L_x/L_y .

Fig. 17: Standardized (empirical) moments for spans and supports [6].

(Table 2), using the following values for the Poisson's ratio ν of concrete, in accordance with the BAEL: 0 for the ultimate limit state (ULS) and 0.2 for the serviceability limit state (SLS).

Table 2: Values of the coefficients μ_x and μ_y

DALLES RECTANGULAIRES UNIFORMÉMENT CHARGÉES ARTICULÉES SUR LEUR CONTOUR

Calcul des
moments au centre :

$$0,4 \leq \alpha = \frac{l_x}{l_y} \leq 1$$

$$M_x = \mu_x \cdot p \cdot l_x^2$$

$$M_y = \mu_y \cdot M_x$$

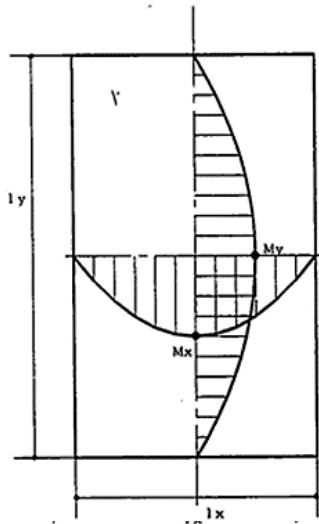
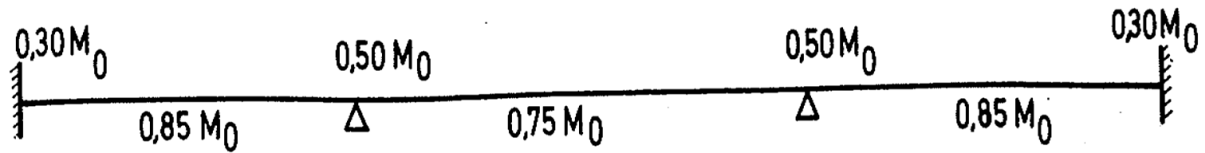


Fig. 2. — Dalle uniformément chargée : moments au centre.

$\alpha = \frac{l_x}{l_y}$	ELU $v = 0$		ELS $v = 0,2$	
	μ_x	μ_y	μ_x	μ_y
0,40	0,1101	0,2500	0,1121	0,2854
0,41	0,1088	0,2500	0,1110	0,2924
0,42	0,1075	0,2500	0,1098	0,3000
0,43	0,1062	0,2500	0,1087	0,3077
0,44	0,1049	0,2500	0,1075	0,3155
0,45	0,1036	0,2500	0,1063	0,3234
0,46	0,1022	0,2500	0,1051	0,3319
0,47	0,1008	0,2500	0,1038	0,3402
0,48	0,0994	0,2500	0,1026	0,3491
0,49	0,0980	0,2500	0,1013	0,3580
0,50	0,0966	0,2500	0,1000	0,3671
0,51	0,0951	0,2500	0,0987	0,3758
0,52	0,0937	0,2500	0,0974	0,3853
0,53	0,0922	0,2500	0,0961	0,3949
0,54	0,0908	0,2500	0,0948	0,4050
0,55	0,0894	0,2500	0,0936	0,4150
0,56	0,0880	0,2500	0,0923	0,4254
0,57	0,0865	0,2582	0,0910	0,4357
0,58	0,0851	0,2703	0,0897	0,4462
0,59	0,0836	0,2822	0,0884	0,4565
0,60	0,0822	0,2948	0,0870	0,4672
0,61	0,0808	0,3075	0,0857	0,4781
0,62	0,0794	0,3205	0,0844	0,4892
0,63	0,0779	0,3338	0,0831	0,5004
0,64	0,0765	0,3472	0,0819	0,5117
0,65	0,0751	0,3613	0,0805	0,5235
0,66	0,0737	0,3753	0,0792	0,5351
0,67	0,0723	0,3895	0,0780	0,5469
0,68	0,0710	0,4034	0,0767	0,5584
0,69	0,0697	0,4181	0,0755	0,5704
0,70	0,0684	0,4320	0,0743	0,5817
0,71	0,0671	0,4471	0,0731	0,5940
0,72	0,0658	0,4624	0,0719	0,6063
0,73	0,0646	0,4780	0,0708	0,6188
0,74	0,0633	0,4938	0,0696	0,6315
0,75	0,0621	0,5105	0,0684	0,6447
0,76	0,0608	0,5274	0,0672	0,6580
0,77	0,0596	0,5440	0,0661	0,6710
0,78	0,0584	0,5608	0,0650	0,6841
0,79	0,0573	0,5786	0,0639	0,6978
0,80	0,0561	0,5959	0,0628	0,7111
0,81	0,0550	0,6135	0,0617	0,7246
0,82	0,0539	0,6313	0,0607	0,7381
0,83	0,0528	0,6494	0,0596	0,7518
0,84	0,0517	0,6678	0,0586	0,7655
0,85	0,0506	0,6864	0,0576	0,7794
0,86	0,0496	0,7052	0,0566	0,7933
0,87	0,0486	0,7244	0,0556	0,8074
0,88	0,0476	0,7438	0,0546	0,8216
0,89	0,0466	0,7635	0,0537	0,8358
0,90	0,0456	0,7834	0,0528	0,8502
0,91	0,0447	0,8036	0,0518	0,8646
0,92	0,0437	0,8251	0,0509	0,8799
0,93	0,0428	0,8450	0,0500	0,8939
0,94	0,0419	0,8661	0,0491	0,9087
0,95	0,0410	0,8875	0,0483	0,9236
0,96	0,0401	0,9092	0,0474	0,9385
0,97	0,0392	0,9322	0,0465	0,9543
0,98	0,0384	0,9545	0,0457	0,9694
0,99	0,0376	0,9771	0,0449	0,9847
1,00	0,0368	1,0000	0,0441	1,0000

For slabs that are fully or partially restrained at their edges, the evaluation of the span moments M_{tx} and M_{ty} and the support moments M_{ax} and M_{ay} is performed by reducing the previously obtained moment values, M_x and M_y [9]. The fractions to be used for calculating the span and support moments are shown in the figure (Fig. 18).



Cas d'une dalle continue uniformément chargée avec la condition : $M_t + \frac{M_w + M_e}{2} \geq 1,25 M_0$

M_w et M_e étant les valeurs absolues prises en compte pour les moments sur appuis de gauche et de droite.

Fig. 18: Standardized (empirical) moments at mid-span and over supports.

C. Slab subjected to concentrated loads:

The concentrated load P is considered to be distributed over certain widths a and b (Fig. 19) [10]:

$$a = a_0 + h + 2\xi h_1 \text{ and } b = b_0 + h + 2\xi h_1$$

a_0 and a are the dimensions parallel to the L_x direction.

b_0 and b are the dimensions parallel to the L_y direction.

$\xi = 1$ for reinforced concrete coverings.

$\xi = 0.75$ for less resistant coverings.

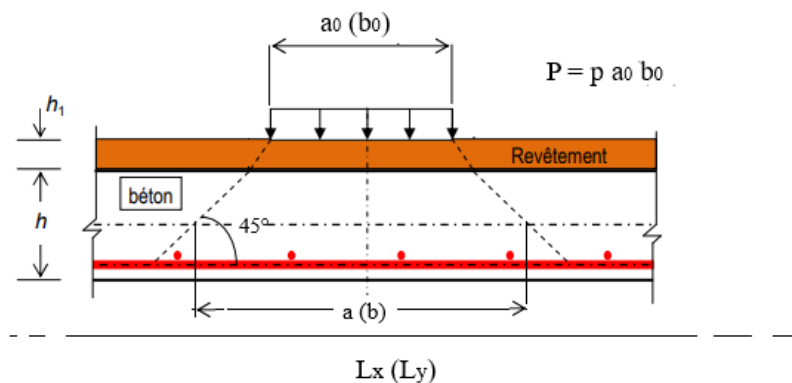


Fig. 19: Concentrated force [10].

- Bending moments for a centered load**

The bending moments M_1 and M_2 resulting from a load centered on the slab can be evaluated using Pigeaud's charts (Fig. 20). These calculations apply to unit-width strips at the center of the slab panel and are based on the following parameters [10]: $\alpha = L_x/L_y$, a/L_x , and b/L_y .

$$M_{0x} = M_1.P \text{ (in the } L_x \text{ direction)}$$

$$M_{0y} = M_2.P \text{ (in the } L_y \text{ direction)}$$

P: total load applied over the rectangular contact area(a.b).

- **Bending moments in the case of an off-center load**

When the applied load is not centered on the slab, the calculation can be reduced to that of a centered load by superimposing load cases corresponding to centered rectangles, all subjected to the same load intensity (Fig. 21) [10].

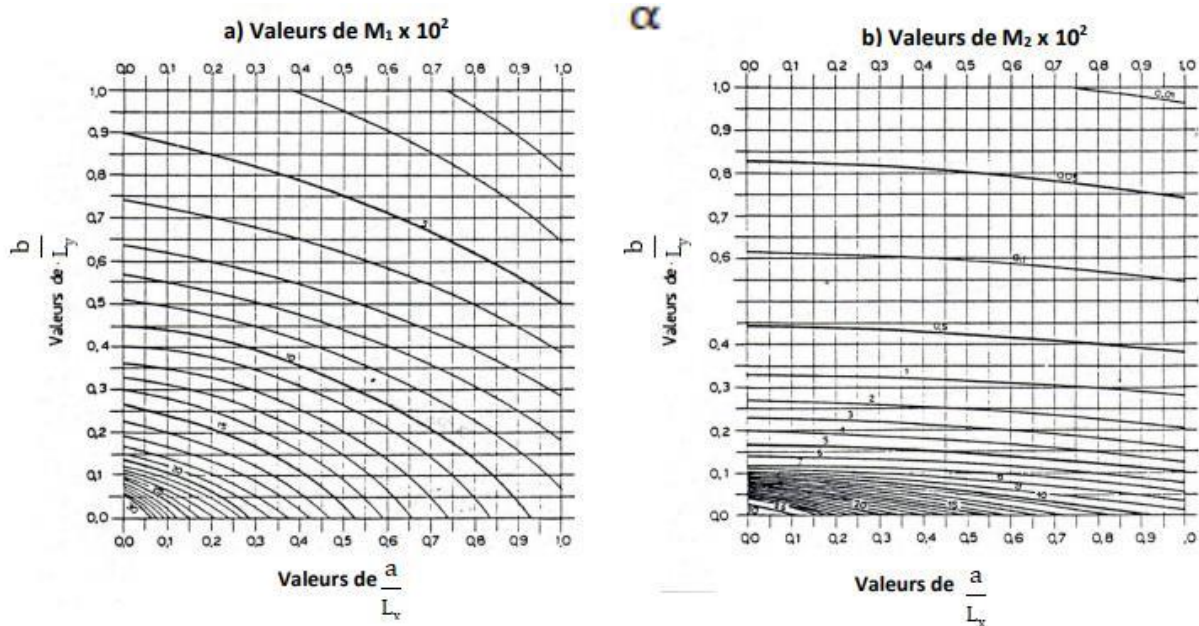


Fig. 20: Pigeaud's charts

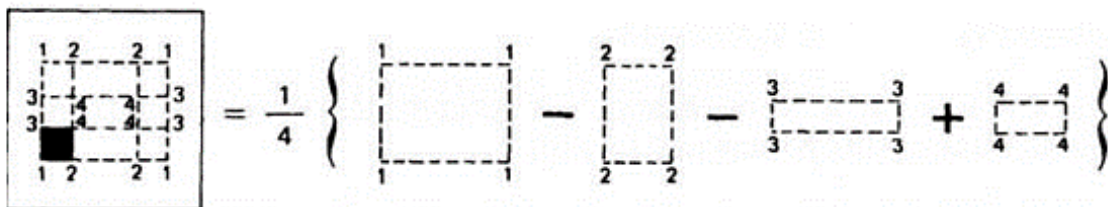


Fig. 21: Example of the decomposition of a concentrated load

5.3. Structural detailing [6-10]

→The reinforcement for a slab spanning in two directions acts as a load-bearing system in both directions. The grid thus formed typically consists of welded wire mesh. The positioning and lengths of the reinforcement panels must compensate for the concrete's inability to resist tension in the tension zones.

→The maximum diameter of the steel bars must be: $\emptyset \leq 1/10$ of the slab thickness.

→The spacing between the bars must be equal to the smaller of the two values presented in Table 3 (where h is the thickness of the slab).

Table 3: Spacing between the slab bars

Direction	Distributed loads only	Concentrated loads
For load-bearing steels (Lx)	3 h et 33cm	2 h et 25cm
For distribution bars (Ly)	4 h et 45cm	3 h et 33cm

→ Verification against punching shear is necessary when applied forces are localized.

→ The "non-brittleness" condition requires a minimum percentage of reinforcement to balance the concrete's ultimate (failure) moment. For slabs with a thickness between 12 and 30 cm, the following formulas should be considered:

Direction Ly (where h is expressed in meters):

$$A_{y \min} (\text{cm}^2/\text{m}) = \begin{cases} 12h : \text{ronds lisses} \\ 8h : \text{FeE400} \\ 6h : \text{FeE500 ou TS} \end{cases}$$

12h: plain round bars

8h: FeE400 steel

6h: FeE500 steel or welded wire mesh

Direction Lx : $A_{x \min} (\text{cm}^2/\text{m}) = A_{y \min} (\text{cm}^2/\text{m}) \cdot 3 - \alpha / 2$

→ The reinforcement located closest to the tension face of the slab must correspond to the direction subject to the greatest stress (parallel to Lx).

→ The primary steel reinforcement located in the central zone of the slab must be extended to the supports:

- Entirely, if the slab is subjected to concentrated and moving loads;
- Every other bar, in all other cases.

→ For standard buildings, a load with an intensity of less than one-quarter of the total variable load applied to the slab panel is not considered a concentrated load. In such cases, the rules for distributed loads apply (for example, in the case of walls).

5.4. Calculation of internal forces in a slab (Yield-line method)[1,11,12]

