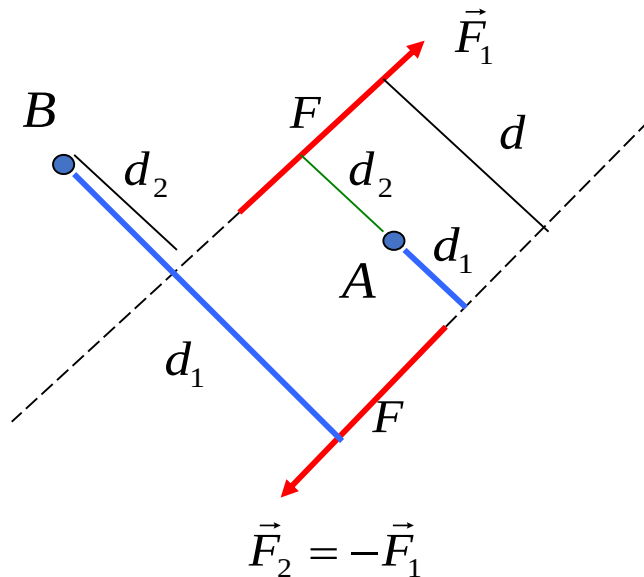


21. Moment of a Couple

The moment produced by two equal, opposite, and noncollinear forces is called a couple. Couples have certain unique properties and have important applications in mechanics.

Couple ----- a pair of forces with the same magnitude and opposite direction

Moments of a Couple:



$$M = \pm Fd$$

Distance between lines of action of the couple of force

Moment of the couple

Sign convention

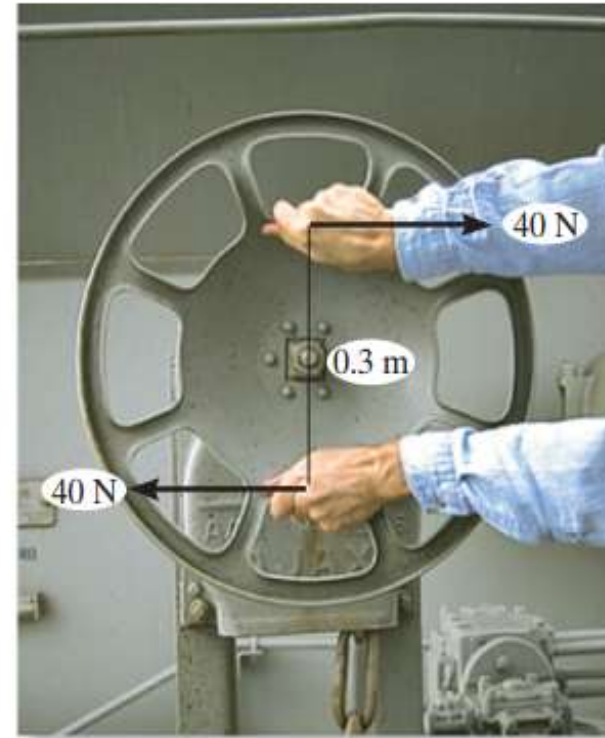
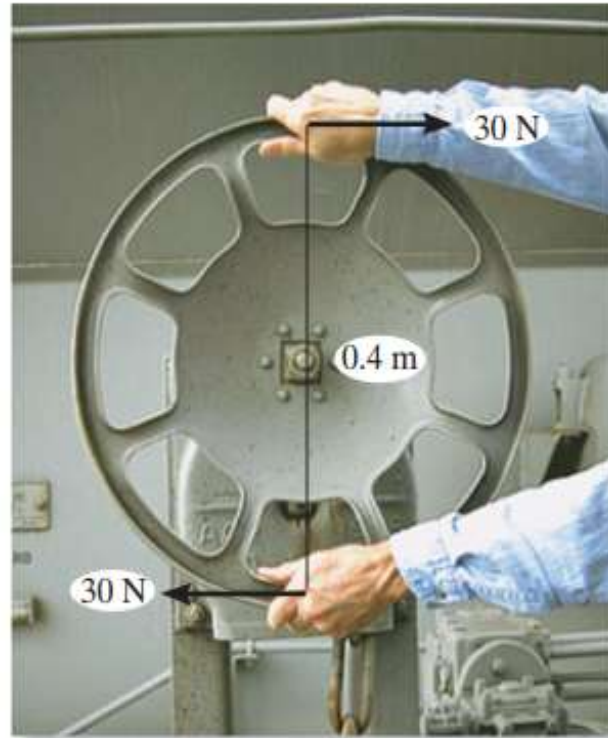
Magnitude of force

$$M^{(A)} = -Fd_1 - Fd_2 = -F(d_1 + d_2) = -Fd$$

$$M^{(B)} = -Fd_1 + Fd_2 = -F(d_1 - d_2) = -Fd$$

Moment of a couple about ANY point is the same!!

Equivalent Couples. If two couples produce a moment with the *same magnitude and direction*, then these two couples are *equivalent*. For example, the two couples shown in Fig. are *equivalent* because each couple moment has a magnitude of $M = 30\text{ N}(0.4\text{ m}) = 40\text{ N}(0.3\text{ m}) = 12\text{ N.m}$.



Example.3.

Replace the two couples acting on the pipe column in Fig. *a* by a resultant couple moment.

Solution

The couple moment developed by the forces at *A* and *B*, can easily be determined from **a scalar formulation**

$$M_1 = F \cdot d = 150 \text{ N} (0.4 \text{ m}) = 60 \text{ N} \cdot \text{m}$$

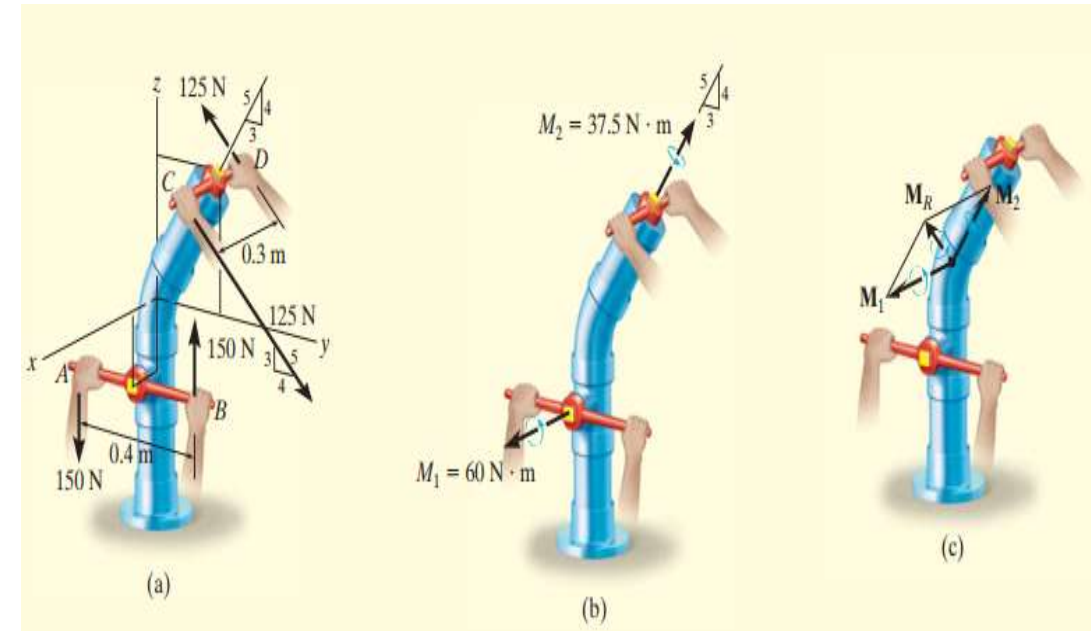
By the right-hand rule, acts in the direction $+\mathbf{i}$, Fig. *b*. Hence, $\mathbf{M}_1 = 60\mathbf{i} \text{ N} \cdot \text{m}$

Vector analysis will be used to determine $\mathbf{M}_2$, caused by forces at *C* and *D*. If moments are computed about point *D*, Fig. *b*,

$$\begin{aligned} \mathbf{M}_2 &= \mathbf{r}_{DC} \times \mathbf{F}_C \text{ then } \mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C = 0.3\mathbf{i} \times [125 (4/5) \mathbf{j} - 125 (3/5) \mathbf{k}] \\ &= 0.3\mathbf{i} \times [100\mathbf{j} - 75\mathbf{k}] = 30 (\mathbf{i} \times \mathbf{j}) - 22.5 (\mathbf{i} \times \mathbf{k}) \\ &= \{-22.5 \mathbf{j} + 30 \mathbf{k}\} \text{ N} \cdot \text{m} \end{aligned}$$

Since $\mathbf{M}_1$ and $\mathbf{M}_2$ are free vectors, they may be moved to some arbitrary point and added vectorially, Fig. *c*. The resultant couple moment becomes

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = 60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k} \text{ N} \cdot \text{m}$$



22. Moments in Three Dimensions

Cartesian Vector Formulation. If we establish x, y, z coordinate axes, then the position vector $\mathbf{r}$ and force $\mathbf{F}$ can be expressed as Cartesian vectors, Fig.7.a. we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where r_x, r_y, r_z represent the x, y, z components of the position vector drawn from point O to *any point* on the line of action of the force F_x, F_y, F_z represent the x, y, z components of the force vector. If the determinant is expanded, we have

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} - (r_x F_z - r_z F_x)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

The scalar magnitudes of the moments of these forces about the positive x -, y -, and z -axes through O can be obtained from the moment arm rule, and are

$$M_x = r_y F_z - r_z F_y \quad M_y = r_z F_x - r_x F_z \quad M_z = r_x F_y - r_y F_x$$

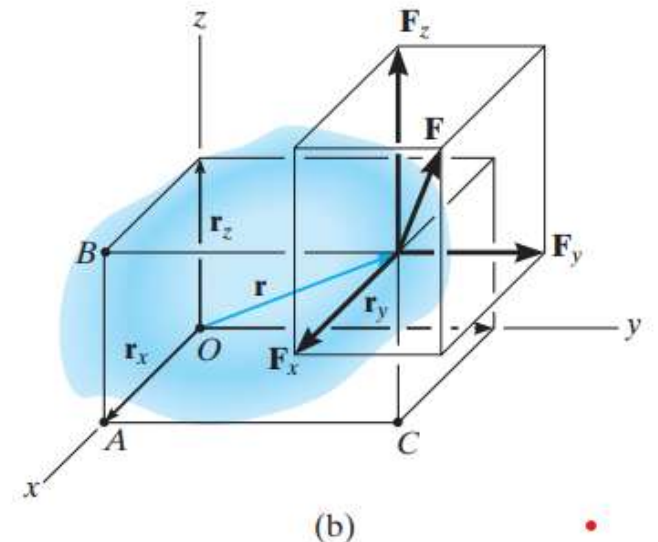
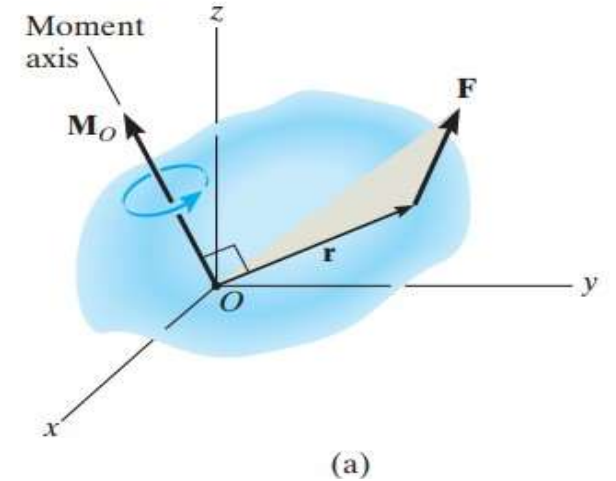


Fig.7. The moment in three dimensions.

23. Equilibrium of a Particle

A particle is in equilibrium when its acceleration is zero.

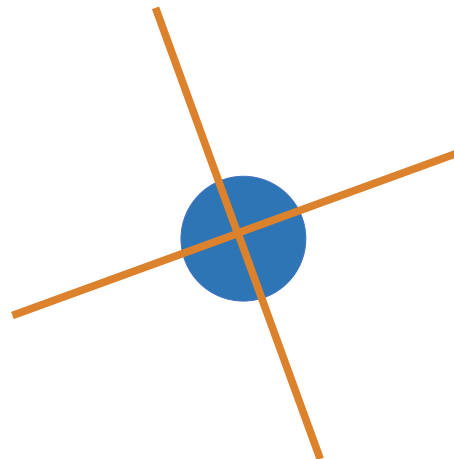
Therefore, the vector sum of all forces must vanish.

In 2D, two independent equilibrium equations are available.

In 3D, three force-equilibrium equations are available.

$$\Sigma \mathbf{F} = 0$$

$$\Sigma F_x = 0 \quad \Sigma F_y = 0$$



24. Equilibrium Equations for a Rigid Body

For a planar rigid body in equilibrium:

Force equilibrium prevents translation.

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

Moment equilibrium prevents rotation.

$$\Sigma M_O = 0 \text{ about any convenient point } O.$$

These equations are used to determine support reactions and unknown forces.

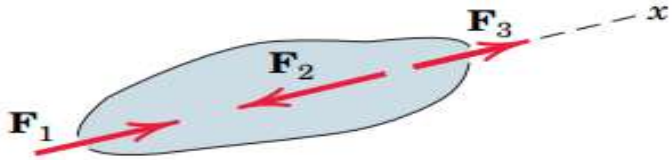
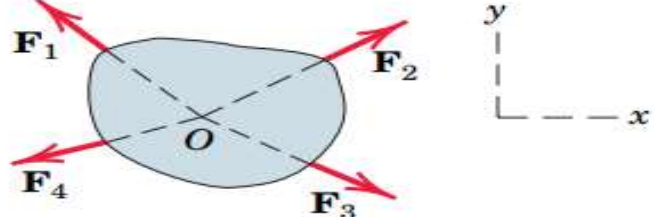
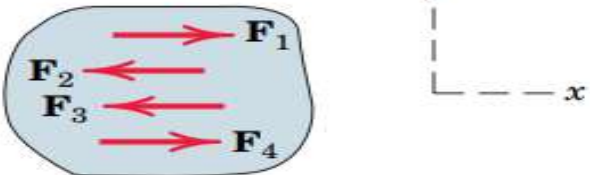
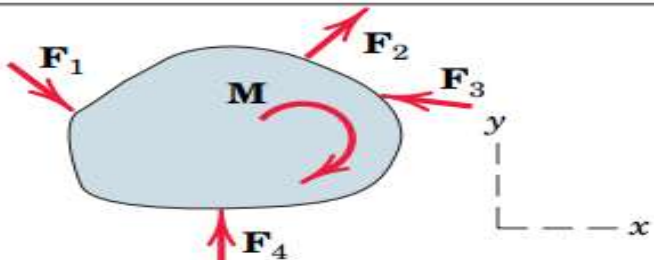
$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M_O = 0$$

Categories of Equilibrium

The categories of force systems acting on **bodies in two-dimensional equilibrium** are summarized in a table 1.

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Equilibrium of Concurrent Forces in Space

- For a concurrent force system in space, expressing the resultant in terms of orthogonal components of individual forces, the equilibrium condition can be stated as

$$\vec{R} = (\sum F_x)\vec{i} + (\sum F_y)\vec{j} + (\sum F_z)\vec{k} = \vec{0}$$

- If all the forces acting on the body are non-concurrent in space, the necessary and sufficient conditions for equilibrium will be same as that for coplanar non-concurrent forces with the addition of the Z-component for , and X and Y components for .

$\Sigma \mathbf{F} = \mathbf{0}$	or	$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$
$\Sigma \mathbf{M} = \mathbf{0}$	or	$\begin{cases} \Sigma M_x = 0 \\ \Sigma M_y = 0 \\ \Sigma M_z = 0 \end{cases}$

Free-Body Diagrams (FBD)

A free body diagram is a sketch of a body, a portion of a body, or two or more bodies completely isolated or free from all other bodies, showing **the forces exerted** by all other bodies on the one being considered. Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various **types of reactions that occur at supports** and points of contact between bodies subjected to coplanar force systems. As a general rule.

• Example.1

- Draw free body diagrams for the

Following cases:

- (i) A block or a ball resting on a smooth horizontal plane

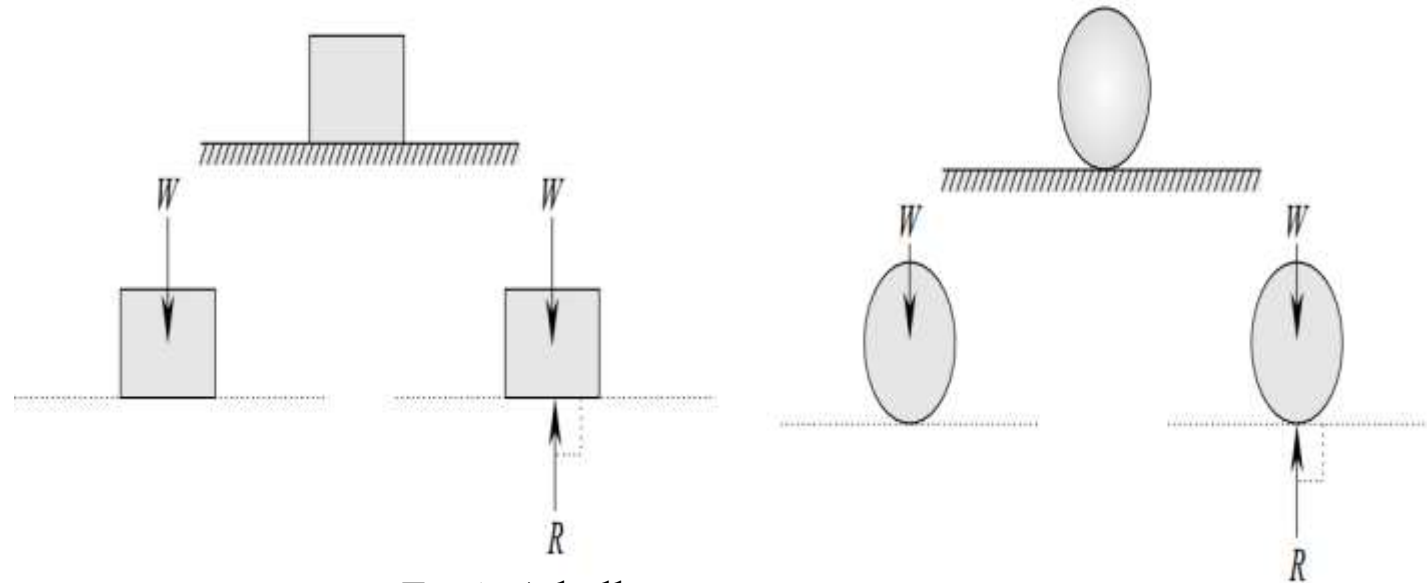


Fig.1. A ball resting.

(ii) A block on a smooth inclined plane is restrained from moving downwards by a string attached to

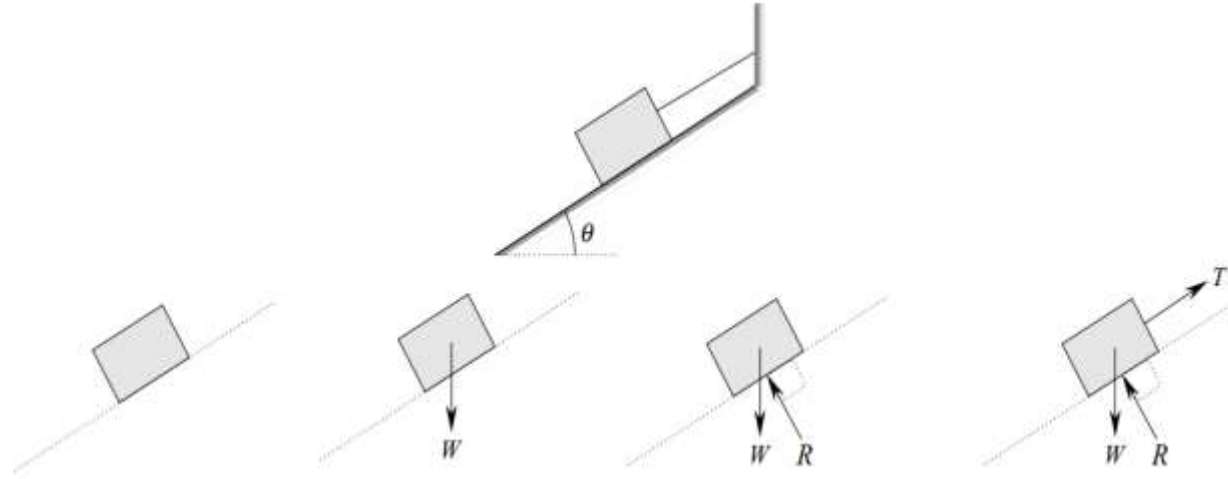


Fig.2. A block on a smooth inclined plane.

(iii) A sphere on a smooth inclined plane is restrained from moving downwards by a vertical plane

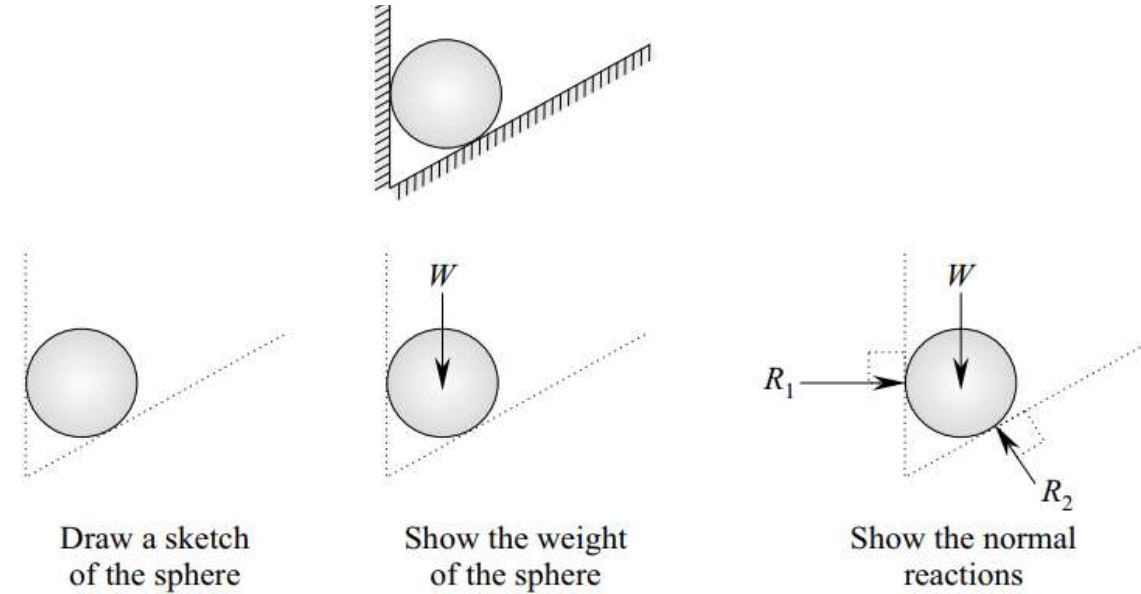


Fig.3. A sphere on a smooth inclined plane.

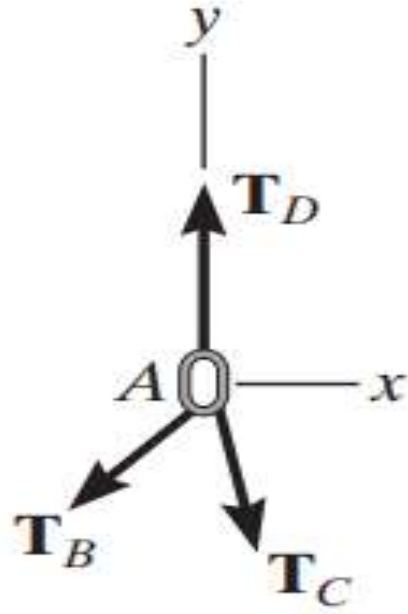
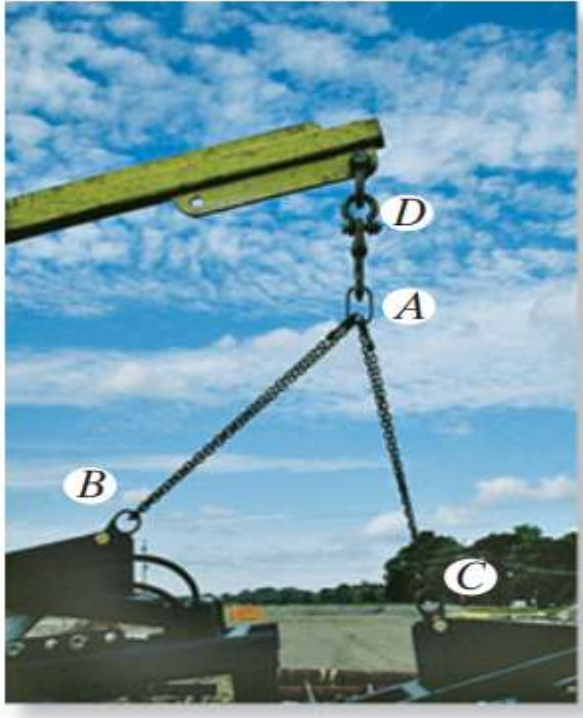


Fig.4. The process of drawing a FBD is illustrated by the sequence shown.



The chains exert three forces on the ring at **A**, as shown on its **free-body diagram**. The ring will not move, or will move with constant velocity, provided the summation of these forces along the x and along the y axis is zero. If one of the three forces is known, the magnitudes of the other two forces can be obtained from the two equations of equilibrium.

Types of Support

A structure beam may have the following types of supports.

1. **Roller Support** In this case, end of the beam rests on some sliding surface, rollers or any flat surface like a smooth masonry wall. It is free to roll or move in the horizontal direction.
2. **Hinged Support** It resists Vertical and horizontal displacements, but allows rotation. It has two reactions- vertical (V) and (H), as shows in Fig.4.
3. **Fixed Support** A fixed support is built monolithically with the wall, this connection is so stiff that it does not allow any kind of displacement (neither translation nor rotation).

This support has three reactions, a vertical reaction (V), a horizontal reaction (H) and moments (M). A fixed support is also known as *a built-in support*.

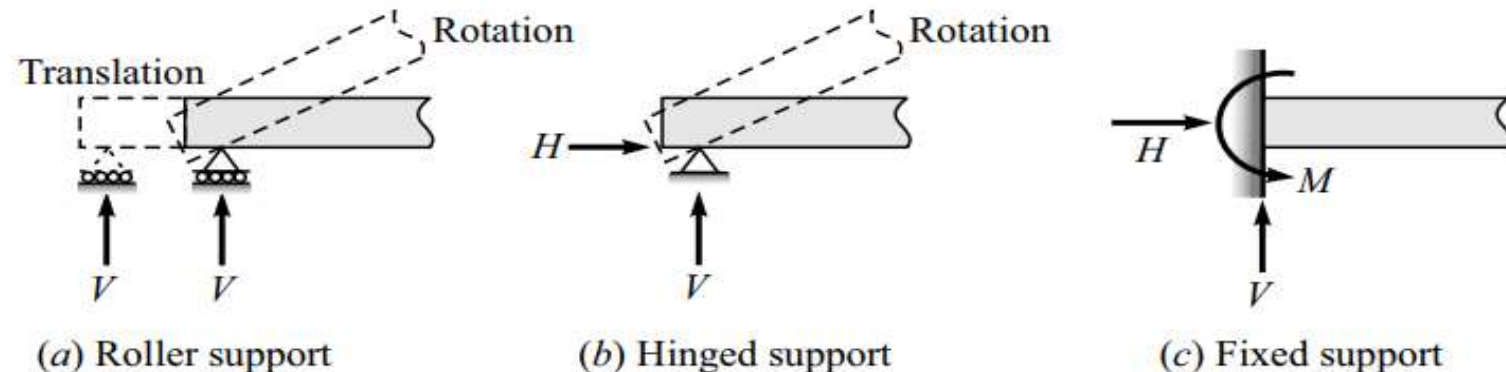
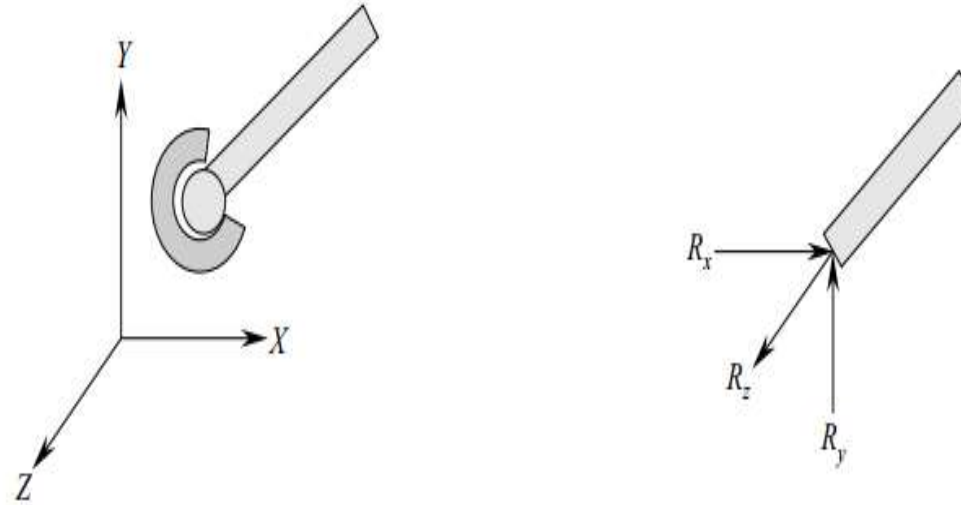


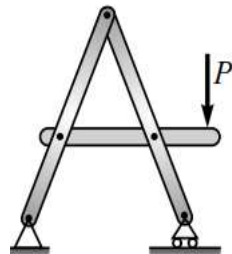
Fig.4. Types of supports.

4. **Ball-and-socket joint** This type of support is same as the hinge support in three dimensions. Here, the body is restrained from moving in all three directions, but is free to rotate in any direction. Hence, reactions R_x , R_y and R_z act on the free-body diagram of the body



Frames

Frames are the structures made of interconnected inextensible members. They are designed to support applied loads and moments. A simple A frame is shown in Fig.5.



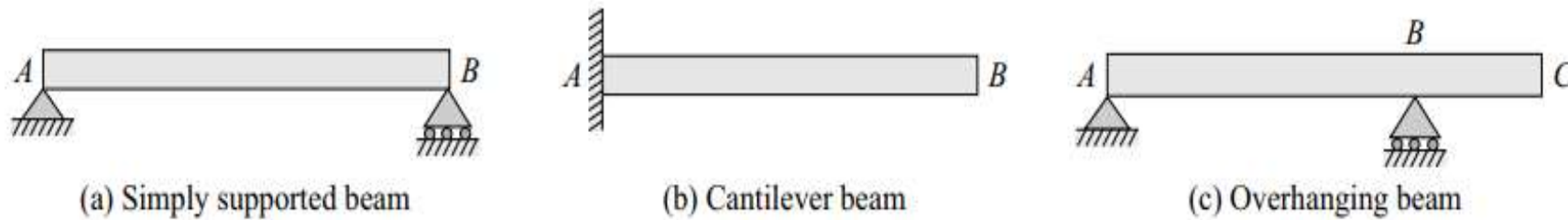
(a) A-Frame



(b) Multi-force-member of frame

Beams

Beams and Types of Beams One of the structural member that we come across in this sections is a beam. It is a horizontal structural member that is designed to resist forces transverse to its axis, It is held in position by various supports. Depending upon the nature of the supports, beams can be classified as follows.



Simply supported Beam Simply supported beam that is supported by a hinge one end and a roller at the other end.

Cantilever beam is a beam that is supported by a built-in or fixed support at one end and the other end free.

Overhanging beam The beam is simply supported at A and B but it also projects beyond the support to the point C, which is a free end.

Modeling the Action of Forces

Figure.5. shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted *on* the body to be *isolated*, by the body to be *removed*.

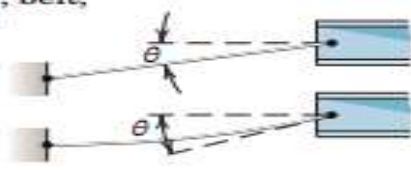
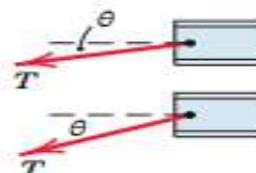
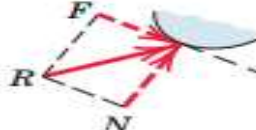
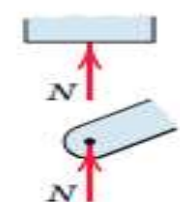
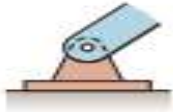
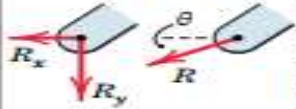
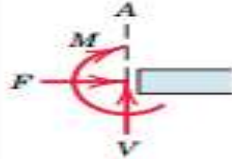
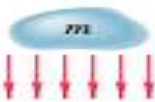
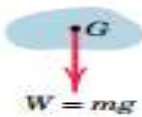
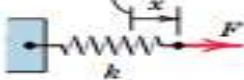
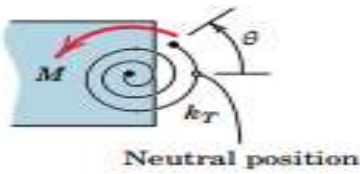
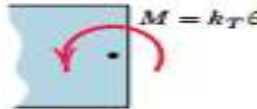
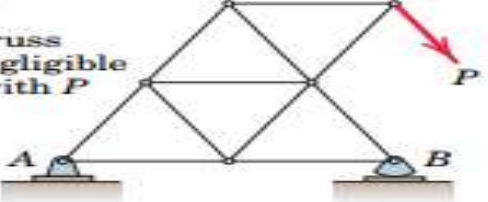
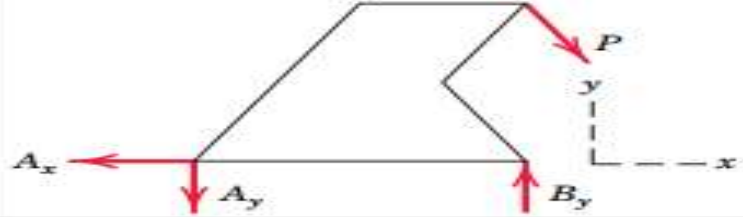
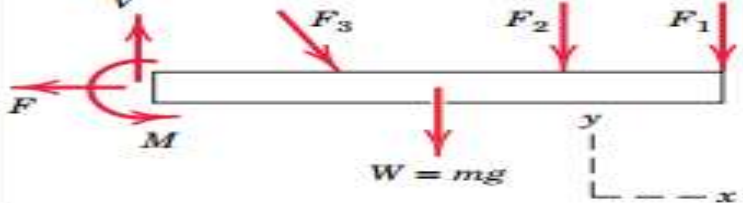
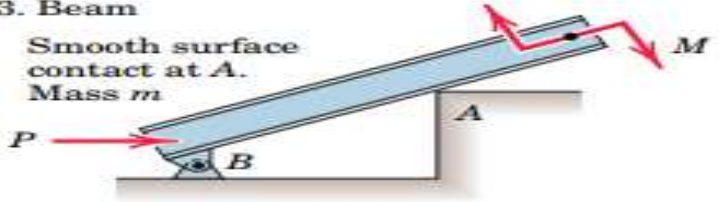
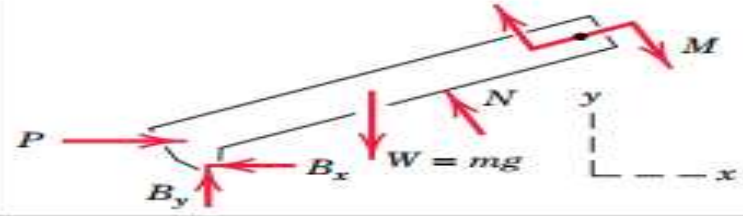
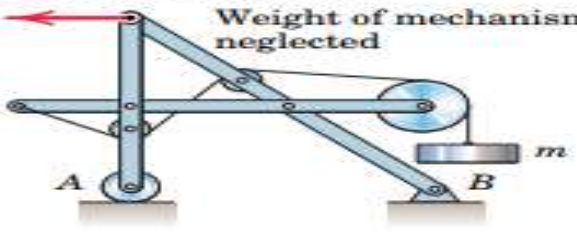
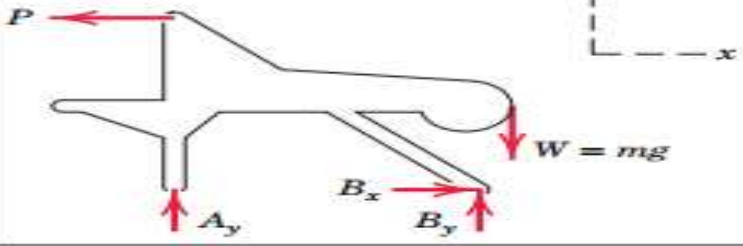
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

Figure.6. shows the common types of force application on mechanical systems for analysis in two dimensions.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)		
Type of Contact and Force Origin	Action on Body to Be Isolated	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.
9. Spring action Neutral position  Linear  Nonlinear 		Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.
10. Torsional spring action 		For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

Examples of Free-Body Diagrams

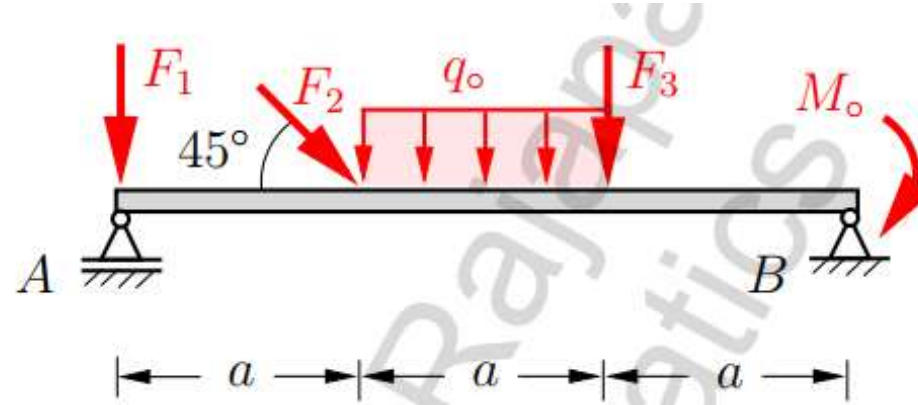
Figure .7. gives four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes are omitted for clarity. In each case we treat the entire system as a single body, so that the internal forces are not shown

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam  Mass m	 $W = mg$
3. Beam Smooth surface contact at A. Mass m 	 $W = mg$
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	 $W = mg$

Example 1

The simply supported beam (length $a = 1$ m) shown in Fig. 1. is subjected to the three concentrated forces $F_1 = 4$ kN, $F_2 = 2$ kN, $F_3 = 3$ kN, the line load $q_0 = 5$ kN /m and the moment $M_0 = 4$ kNm.

Calculate the support reactions



Solution

We free the beam from the supports and make the reactions A_H and B_V visible in the free-body diagram. The line load is replaced by its resultant $q_0 \cdot a$