

Chapter 2: The Numerical Sequences

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1 The Numerical Sequences

1.1 Definition

We call a sequence element of E , any application defined as:

$$\begin{aligned}u : \mathbb{N} &\longrightarrow E \\ n &\longrightarrow u(n) = u_n\end{aligned}$$

such that $E = \mathbb{R}$ or $E = \mathbb{C}$.

- The image $u(n)$ is denoted u_n and called the term of index n or the general term of the sequence u , and u_0 is the initial term.
- The sequence is itself denoted $(u_n)_{n \in \mathbb{N}}$ or $(u_n)_{n \geq 0}$.
- The number of distinct terms is finite or countable.
- The terms of a sequence are not necessarily distinct.

Example 1.1. 1. $u : \mathbb{N}^* \longrightarrow \mathbb{R}, \quad n \longrightarrow u_n = 4n^2 + \frac{1}{4}$

2. $v : \mathbb{N} \longrightarrow \mathbb{R}, \quad n \longrightarrow v_n = \sum_{k=0}^n 2^k$

Remark 1.1. • If $E = \mathbb{R}$, the sequence (u_n) is said to be real.

- If $E = \mathbb{C}$, the sequence (u_n) is said to be complex.
- The data of a complex sequence $(z_n)_{n \geq 0}$ is equivalent to that of two real sequences $(u_n)_{n \geq 0}$ and $(v_n)_{n \geq 0}$ defined by:

$$\forall n \in \mathbb{N} : z_n = u_n + iv_n \quad \text{that is to say} \quad u_n = \operatorname{Re}(z_n) \quad \text{and} \quad v_n = \operatorname{Im}(z_n).$$

- The general term can be defined in two forms:
 - **Explicit form:** The expression of u_n is given as a function of n .
Ex: $u_n = \frac{1}{n + \frac{1}{n}}$ for $n \in \mathbb{N}^*$, $u_1 = 1$, $u_2 = \frac{2}{5}$, etc.
 - **Recurrent form:** A term u_{n+1} is calculated from the previous term (or the first k terms).
Ex:
$$\begin{cases} u_1 = 1 \\ \forall n \in \mathbb{N}^* : u_{n+1} = 1 + \frac{1}{u_n} \end{cases} ; \quad u_2 = 2, u_3 = \frac{3}{2}.$$

1.2 Operations on sequences

Let $(u_n)_{n \geq 0}$ and $(v_n)_{n \geq 0}$ be two numerical sequences:

- $(u_n)_{n \geq 0} = (v_n)_{n \geq 0} \iff u_n = v_n \quad \forall n \in \mathbb{N}$
- $(u_n)_{n \geq 0} + (v_n)_{n \geq 0} = (u_n + v_n)_{n \geq 0}$
- $(u_n)_{n \geq 0} \times (v_n)_{n \geq 0} = (u_n \times v_n)_{n \geq 0}$
- $\forall \lambda \in \mathbb{R} : \lambda(u_n)_{n \geq 0} = (\lambda u_n)_{n \geq 0}$
- $(u_n)_{n \geq 0} \leq (v_n)_{n \geq 0} \iff u_n \leq v_n \quad \forall n \in \mathbb{N}$

1.3 Periodic and stationary sequences

Definition 1.1 (Constant or stationary sequences). Let $(u_n)_{n \geq 0}$ be a numerical sequence. It is said to be **constant** if there exists $a \in \mathbb{R}$ such that $\forall n \in \mathbb{N} : u_n = a$.

It is said to be **stationary** if there exists $a \in \mathbb{R}$ and $n_0 \in \mathbb{N}$ such that $\forall n \geq n_0 : u_n = a$.

Definition 1.2 (Periodic sequences). Let $(u_n)_{n \geq 0}$ be a numerical sequence. It is said to be **periodic** if there exists a positive integer p (called the period) such that:

$$\forall n \in \mathbb{N} : u_{n+p} = u_n$$

If an integer p satisfies this property, all its multiples $p' = kp$ ($k \in \mathbb{N}^*$) also satisfy it. The period of the sequence is the smallest positive integer p satisfying this property; we then say that the sequence is p -periodic.

Remark 1.2. • Constant sequences are 1-periodic sequences.

- If the sequence $(u_n)_{n \geq 0}$ is p -periodic, then $(u_n)_{n \geq 0} = \{u_n\}_{0 \leq n \leq p-1}$.

1.4 Bounded sequence

Definition 1.3. Let $(u_n)_{n \geq 0}$ be a numerical sequence.

1. The sequence $(u_n)_{n \geq 0}$ is said to be **bounded above** if the set of numbers u_n has an upper bound:

$$\exists M \in \mathbb{R}, \quad \forall n \in \mathbb{N} : u_n \leq M$$

2. The sequence $(u_n)_{n \geq 0}$ is said to be **bounded below** if the set of numbers u_n has a lower bound:

$$\exists m \in \mathbb{R}, \quad \forall n \in \mathbb{N} : u_n \geq m$$

3. The sequence $(u_n)_{n \geq 0}$ is said to be **bounded** if it is both bounded above and bounded below:

$$\exists m, M \in \mathbb{R}, \quad \forall n \in \mathbb{N} : m \leq u_n \leq M$$

Example 1.2. 1. Let $(u_n)_{n \in \mathbb{N}^*}$ with $u_n = \frac{1}{n}$. We have $\forall n \in \mathbb{N}^* : 0 \leq \frac{1}{n} \leq 1$. The sequence $(u_n)_{n > 0}$ is bounded.

2. Let $(u_n)_{n \in \mathbb{N}}$ with $u_n = n$. We have $\forall n \in \mathbb{N} : u_n \geq 0$. It is bounded below by 0, but it is unbounded above.

3. Any constant sequence $u_n = c$ is bounded.

4. The sequence $u_n = (-1)^n \cdot n$ is neither bounded above nor bounded below.

Remark 1.3. To show that a sequence is bounded, proof by mathematical induction is often used.

Example 1.3. Consider the sequence defined by
$$\begin{cases} u_0 = 0 \\ u_{n+1} = \sqrt{u_n + 2}, \quad \forall n \in \mathbb{N} \end{cases}.$$

Prove that $\forall n \in \mathbb{N} : u_n \leq 2$:

1. For $n = 0$: $u_0 = 0 \leq 2$ (verified).
2. Assume $u_n \leq 2$ and show $u_{n+1} \leq 2$:

$$u_n \leq 2 \implies u_n + 2 \leq 4 \implies \sqrt{u_n + 2} \leq 2 \implies u_{n+1} \leq 2$$

Therefore, $\forall n \in \mathbb{N} : 0 \leq u_n \leq 2$, and the sequence $(u_n)_{n \in \mathbb{N}}$ is bounded.

1.5 Monotone sequences

Definition 1.4. Let $(u_n)_{n \in \mathbb{N}}$ be a numerical sequence.

1. The sequence $(u_n)_{n \in \mathbb{N}}$ is said to be **increasing** (resp. **strictly increasing**) if:

$$\forall n \in \mathbb{N}, \quad u_{n+1} \geq u_n \quad (\text{resp. } u_{n+1} > u_n)$$

2. The sequence $(u_n)_{n \in \mathbb{N}}$ is said to be **decreasing** (resp. **strictly decreasing**) if:

$$\forall n \in \mathbb{N}, \quad u_{n+1} \leq u_n \quad (\text{resp. } u_{n+1} < u_n)$$

3. The sequence $(u_n)_{n \in \mathbb{N}}$ is said to be **monotone** if it is increasing or decreasing.
4. The sequence $(u_n)_{n \in \mathbb{N}}$ is said to be **constant** if:

$$\forall n \in \mathbb{N}, \quad u_{n+1} = u_n = \cdots = u_0$$

Remark 1.4. 1. There are sequences that are neither increasing nor decreasing (e.g., $u_n = (-1)^n$).

2. A non-increasing sequence is not necessarily a decreasing sequence in general.

Example 1.4. 1. Let $u_n = \frac{1}{n}$ for $n \in \mathbb{N}^*$.

$$u_{n+1} - u_n = \frac{1}{n+1} - \frac{1}{n} = \frac{-1}{n(n+1)} < 0 \implies u_{n+1} < u_n$$

Thus, $(u_n)_{n \in \mathbb{N}^*}$ is strictly decreasing.

2. Let $u_n = n^2$ for $n \in \mathbb{N}$.

$$u_{n+1} - u_n = (n+1)^2 - n^2 = 2n+1 > 0 \implies u_{n+1} > u_n$$

Thus, $(u_n)_{n \in \mathbb{N}}$ is strictly increasing.

3. Let $u_n = (-1)^n$ for $n \in \mathbb{N}$.

$$u_{n+1} - u_n = (-1)^{n+1} - (-1)^n = (-1)^n(-2)$$

The sign changes depending on parity, so the sequence is neither increasing nor decreasing.

1.6 Convergent sequences

Definition 1.5. Let $(u_n)_{n \in \mathbb{N}}$ be a numerical sequence. We say that the sequence $(u_n)_{n \in \mathbb{N}}$ is **convergent** and has limit $l \in \mathbb{R}$ if:

$$\forall \epsilon > 0, \quad \exists N = N(\epsilon) \in \mathbb{N}, \quad \forall n > N \implies |u_n - l| < \epsilon$$

We write $u_n \longrightarrow l$ as $n \longrightarrow +\infty$, or $\lim_{n \rightarrow +\infty} u_n = l$.

Example 1.5. Let $u_n = \frac{2n}{3n+1}$. Show that $(u_n)_{n \in \mathbb{N}}$ converges to $\frac{2}{3}$.

Let $\epsilon > 0$. We seek N such that for $n > N$, $|u_n - \frac{2}{3}| < \epsilon$:

$$\left| \frac{2n}{3n+1} - \frac{2}{3} \right| = \left| \frac{6n - (6n+2)}{3(3n+1)} \right| = \frac{2}{3(3n+1)} < \epsilon$$

This inequality yields:

$$3(3n+1)\epsilon > 2 \implies 3n > \frac{2}{3\epsilon} - 1 \implies n > \frac{2}{9\epsilon} - \frac{1}{3}$$

It is sufficient to take $N = E\left(\frac{2}{9\epsilon} - \frac{1}{3}\right) + 1$. Thus, $\lim_{n \rightarrow +\infty} u_n = \frac{2}{3}$.

Theorem 1.1. *If a sequence $(u_n)_{n \in \mathbb{N}}$ converges, its limit l is unique.*

Proof. Suppose that $\lim_{n \rightarrow +\infty} u_n = l$ and $\lim_{n \rightarrow +\infty} u_n = l'$ with $l \neq l'$.

Let $\epsilon = \frac{1}{4}|l - l'| > 0$.

Since $u_n \rightarrow l$, $\exists N_1 \in \mathbb{N}$ such that $\forall n > N_1, |u_n - l| < \epsilon$.

Since $u_n \rightarrow l'$, $\exists N_2 \in \mathbb{N}$ such that $\forall n > N_2, |u_n - l'| < \epsilon$.

Let $N_0 = \max(N_1, N_2)$. For $n > N_0$, we have:

$$|l - l'| = |l - u_n + u_n - l'| \leq |l - u_n| + |u_n - l'| < \epsilon + \epsilon = 2\epsilon = \frac{1}{2}|l - l'|$$

which is impossible. Therefore $l = l'$. □

Remark 1.5. To prove that a sequence diverges, it suffices to show that it tends to two different values as $n \rightarrow +\infty$.

Example 1.6. For $u_n = (-1)^n$:

- For $n = 2k$ (even), $u_n = 1 \rightarrow 1$.
- For $n = 2k + 1$ (odd), $u_n = -1 \rightarrow -1$.

Since $1 \neq -1$, the sequence (u_n) is divergent.

Theorem 1.2. *Every convergent sequence is bounded.*

Proof. Let $u_n \rightarrow l$. For $\epsilon > 0$ (e.g., $\epsilon = 1$), $\exists N \in \mathbb{N}$ such that

$$\forall n > N, |u_n - l| < 1 \implies |u_n| < |l| + 1.$$

Let $k = \max(\{|u_0|, |u_1|, \dots, |u_N|, |l| + 1\})$. Then $\forall n \in \mathbb{N}, |u_n| \leq k$. □

1.7 Subsequences

Definition 1.6. Let (u_n) be a numerical sequence, and let $(n_k)_{k \in \mathbb{N}}$ be a strictly increasing sequence of natural numbers. The sequence $(u_{n_k})_{k \in \mathbb{N}}$ is called a **subsequence** (or extracted sequence) of (u_n) .

Any subsequence of a convergent sequence converges to the same limit. The converse is generally false.

Theorem 1.3 (Bolzano-Weierstrass Theorem). *Every bounded sequence of real numbers has a convergent subsequence.*

1.8 Theorem of convergence of monotone sequences

Theorem 1.4. *A sequence (u_n) that is increasing and bounded above is convergent to $l = \sup(u_n)$.*

Proof. Let (u_n) be an increasing and bounded sequence. Therefore, the set of numbers (u_n) has an upper bound $l = \sup(u_n)$.

For any $\epsilon > 0$, by definition of supremum, $\exists N \in \mathbb{N}$ such that $l - \epsilon < u_N \leq l$.

Since (u_n) is increasing, for all $n > N$, $l - \epsilon < u_N \leq u_n \leq l < l + \epsilon$, which implies $|u_n - l| < \epsilon$.

Thus $\lim_{n \rightarrow +\infty} u_n = l = \sup(u_n)$. $\square$

Theorem 1.5. *A sequence (u_n) that is decreasing and bounded below is convergent to $l = \inf(u_n)$.*

Proof. An analogous proof to the previous one. $\square$

1.9 Theorems on convergent sequences

Theorem 1.6. *Let $\lim u_n = l$ and $\lim v_n = l'$.*

1. $\lim(u_n + v_n) = l + l'$
2. $\lim(u_n \times v_n) = l \times l'$
3. $\lim(\lambda u_n) = \lambda l \quad (\lambda \in \mathbb{R})$
4. $\lim |u_n| = |l|$
5. *If $u_n \neq 0$ and $l \neq 0$, $\lim \frac{1}{u_n} = \frac{1}{l}$*
6. *If $u_n \geq 0$, then $l \geq 0$.*
7. *If $u_n \leq 0$, then $l \leq 0$.*
8. *If $u_n \leq v_n$ for all $n > N$, then $l \leq l'$.*

Proposition 1.1 (Squeeze Theorem). *If $u_n \leq w_n \leq v_n$ for all $n \geq N$, and $\lim u_n = \lim v_n = l$, then $\lim w_n = l$.*

Example 1.7. Additional Example for Squeeze Theorem:

Let $w_n = \frac{\sin(n^2)}{n}$ for $n \geq 1$. We know that for all $n \geq 1$:

$$-1 \leq \sin(n^2) \leq 1 \implies -\frac{1}{n} \leq \frac{\sin(n^2)}{n} \leq \frac{1}{n}$$

Let $u_n = -\frac{1}{n}$ and $v_n = \frac{1}{n}$. Since $\lim_{n \rightarrow +\infty} u_n = 0$ and $\lim_{n \rightarrow +\infty} v_n = 0$, by the Squeeze Theorem, we conclude that $\lim_{n \rightarrow +\infty} w_n = 0$.

Theorem 1.7. *If $\lim u_n = 0$ and (v_n) is bounded, then $\lim(u_n \cdot v_n) = 0$.*

1.10 Extension to infinite limits

Definition 1.7. 1. $\lim_{n \rightarrow +\infty} u_n = +\infty \iff \forall A > 0, \exists N \in \mathbb{N}, \forall n > N \implies u_n > A$.

2. $\lim_{n \rightarrow +\infty} u_n = -\infty \iff \forall A > 0, \exists N \in \mathbb{N}, \forall n > N \implies u_n < -A$.

1.11 Adjacent sequences

Definition 1.8. Two sequences (u_n) and (v_n) are said to be **adjacent** if:

1. (u_n) is increasing,
2. (v_n) is decreasing,
3. $\lim_{n \rightarrow +\infty} (v_n - u_n) = 0$.

Theorem 1.8. *If (u_n) and (v_n) are adjacent, they converge to the same limit.*

Proof. Let (u_n) and (v_n) be two adjacent sequences. We suppose that (u_n) is increasing and (v_n) is decreasing.

Let the sequence (w_n) be defined by $w_n = v_n - u_n$. We have $w_{n+1} - w_n = (v_{n+1} - v_n) - (u_{n+1} - u_n) \leq 0$. Thus, (w_n) is decreasing and bounded below by its limit $l = 0$.

Assuming $w_n \geq 0 \implies v_n \geq u_n$, we obtain:

$$u_0 \leq u_1 \leq \dots \leq u_n \leq u_{n+1} \leq v_{n+1} \leq v_n \leq \dots \leq v_1 \leq v_0$$

The sequence (u_n) is increasing and bounded above by v_0 , thus (u_n) converges to l with $l \leq v_0$. Similarly, (v_n) is decreasing and bounded below by u_0 , thus (v_n) converges to l' with $l' \geq u_0$. Since $\lim_{n \rightarrow +\infty} (v_n - u_n) = l' - l = 0$, it follows that $l' = l$. Therefore, both sequences converge to the same limit. $\square$

1.12 Recurrent sequences

Definition 1.9. A sequence defined by an initial term u_0 and a relation $u_{n+1} = f(u_n)$ for a continuous function f is called a **recurrent sequence**.

Theorem 1.9. *If a recurrent sequence $u_{n+1} = f(u_n)$ converges to l and f is continuous, then $f(l) = l$ (l is a fixed point of f).*

Proof. Since $\lim_{n \rightarrow +\infty} u_n = l$ and f is continuous, we have $\lim_{n \rightarrow +\infty} f(u_n) = f(\lim_{n \rightarrow +\infty} u_n) = f(l)$. Since $\lim_{n \rightarrow +\infty} f(u_n) = \lim_{n \rightarrow +\infty} u_{n+1} = l$, it follows that $f(l) = l$. $\square$

1.12.1 Monotonicity of recurrent sequences

- **If f is increasing:** (u_n) is monotone.
 - Increasing if $f(u_0) \geq u_0$.
 - Decreasing if $f(u_0) \leq u_0$.
- **If f is decreasing:** $f \circ f$ is increasing. The subsequences (u_{2n}) and (u_{2n+1}) are monotonic, and (u_n) converges if and only if both subsequences converge to the same limit.

1.13 Cauchy sequences

Definition 1.10. A sequence (u_n) is a **Cauchy sequence** if:

$$\forall \epsilon > 0, \quad \exists N \in \mathbb{N}, \quad \forall p, q > N \implies |u_p - u_q| < \epsilon$$

Theorem 1.10 (Cauchy Criterion). *A sequence of real numbers is convergent if and only if it is a Cauchy sequence.*

Proof. 1. ($\implies$) **Every convergent sequence is a Cauchy sequence:**

Let (u_n) converge to $l \in \mathbb{R}$. For every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n > N$, $|u_n - l| < \frac{\epsilon}{2}$. Thus, for all $p, q > N$, we have:

$$|u_p - u_q| = |u_p - l - (u_q - l)| \leq |u_p - l| + |u_q - l| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

Therefore, (u_n) is a Cauchy sequence.

2. ($\impliedby$) Every Cauchy sequence is convergent:

Let (u_n) be a Cauchy sequence. By Proposition 1.13.1, (u_n) is bounded. By the Bolzano-Weierstrass Theorem, we can extract a convergent subsequence with limit l . Since (u_n) is a Cauchy sequence, for a given $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $|u_n - u_p| < \frac{\epsilon}{2}$ for all $n, p > N$. Choosing $p > N$ such that the term u_p belongs to the subsequence converging to l (so $|u_p - l| < \frac{\epsilon}{2}$), we get for all $n \geq N$:

$$|u_n - l| \leq |u_n - u_p| + |u_p - l| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

Therefore, (u_n) converges to l . $\square$

Remark 1.6. (u_n) is not a Cauchy sequence $\implies (u_n)$ is divergent.

Remark 1.7. The Cauchy criterion allows us to prove the convergence of a sequence without knowing the limits..

Example 1.8. Simple Example for a Cauchy Sequence:

Consider the sequence $u_n = \frac{1}{n}$ for $n \in \mathbb{N}^*$. Let us show that (u_n) is a Cauchy sequence. Let $\epsilon > 0$ be given. We want to find an integer N such that for all integers $p, q > N$:

$$|u_p - u_q| < \epsilon$$

Let us estimate the distance between two arbitrary terms u_p and u_q (assuming without loss of generality that $p \geq q \geq 1$):

$$|u_p - u_q| = \left| \frac{1}{p} - \frac{1}{q} \right| = \frac{1}{q} - \frac{1}{p} < \frac{1}{q}$$

To ensure that $\frac{1}{q} < \epsilon$, it suffices to choose $q > \frac{1}{\epsilon}$. Thus, by choosing an integer $N > \frac{1}{\epsilon}$, then for any $p, q > N$, we automatically have:

$$|u_p - u_q| < \frac{1}{q} < \frac{1}{N} < \epsilon$$

Therefore, by definition, $(u_n) = \left(\frac{1}{n}\right)$ is a Cauchy sequence (which aligns with the Cauchy criterion since we already know it converges to 0).

1.14 Arithmetic and geometric sequences

1.14.1 Arithmetic sequence

Defined by $u_{n+1} = u_n + r$ ($r \in \mathbb{R}$ common difference):

1. $u_n = u_0 + nr$
2. $u_n = u_p + (n - p)r$
3. $\sum_{k=p}^n u_k = (u_p + u_n) \frac{n-p+1}{2}$

Example 1.9. Additional Example for Arithmetic Sequence:

Let (u_n) be an arithmetic sequence with initial term $u_0 = 3$ and common difference $r = 4$.

1. The general explicit term is given by $u_n = u_0 + nr = 3 + 4n$.
2. The term of index 10 is $u_{10} = 3 + 4(10) = 43$.
3. The sum of the first 11 terms (from $n = 0$ to $n = 10$) is:

$$\sum_{k=0}^{10} u_k = (u_0 + u_{10}) \frac{10 - 0 + 1}{2} = (3 + 43) \frac{11}{2} = 46 \times \frac{11}{2} = 23 \times 11 = 253.$$

1.14.2 Geometric sequence

Defined by $u_{n+1} = q \cdot u_n$ ($q \in \mathbb{R}$ common ratio):

1. $u_n = u_0 \cdot q^n$
2. $u_n = u_p \cdot q^{n-p}$
3. $\sum_{k=p}^n u_k = u_p \left(\frac{1-q^{n-p+1}}{1-q} \right) \quad (q \neq 1)$

Example 1.10. Additional Example for Geometric Sequence:

Let (v_n) be a geometric sequence with initial term $v_0 = 2$ and common ratio $q = 3$.

1. The general explicit term is given by $v_n = v_0 \cdot q^n = 2 \cdot 3^n$.
2. The term of index 4 is $v_4 = 2 \cdot 3^4 = 2 \cdot 81 = 162$.
3. The sum of the first 5 terms (from $n = 0$ to $n = 4$) is:

$$\sum_{k=0}^4 v_k = v_0 \left(\frac{1 - q^{4-0+1}}{1 - q} \right) = 2 \left(\frac{1 - 3^5}{1 - 3} \right) = 2 \left(\frac{1 - 243}{-2} \right) = 2 \left(\frac{-242}{-2} \right) = 242.$$

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