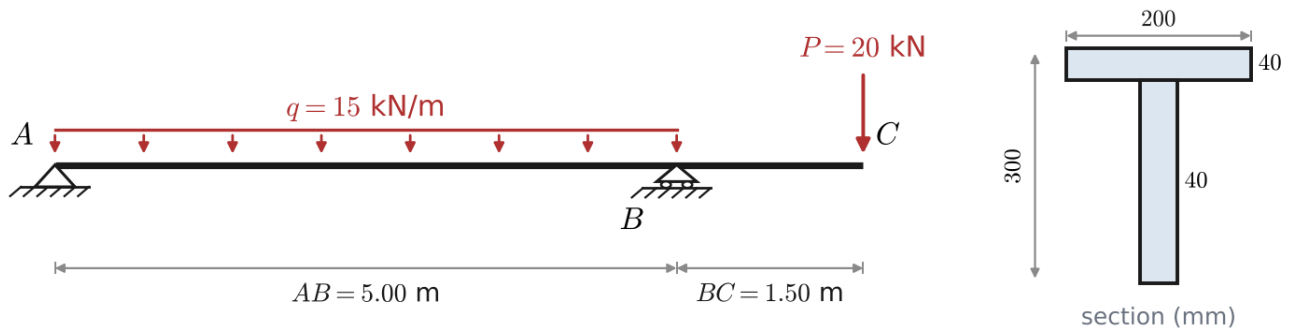


Strength of Materials II — Chapter 1 · Tutorial sheet TD 1

Internal forces and stresses



The data, common to all three exercises. The beam ABC is simply supported at A and B with $AB = 5.00 \text{ m}$, and overhangs by $BC = 1.50 \text{ m}$. It carries a uniform load $q = 15 \text{ kN/m}$ on AB only, and a point load $P = 20 \text{ kN}$ at the free end C . Its cross-section is the T drawn above: flange $200 \times 40 \text{ mm}$ on top, web $40 \times 260 \text{ mm}$ below, total depth 300 mm . Conventions: M_z is positive when it sags, V is positive when it tends to rotate the isolated element clockwise, σ is positive in tension. Solve each exercise **by the method named in its title**. Give every answer with its sign, its unit and three significant figures, and say where in the beam and where in the section it occurs.

Exercise 1: Shear force and bending moment diagrams. Find the two reactions. Write $V(x)$ and $M_z(x)$ on each segment and draw both diagrams to scale. Give the largest sagging moment and its position, the moment at B , and the point of contraflexure where M_z changes sign. Finally, compare the largest sagging moment with the value $qL^2/8$ that the span alone would give, and explain the difference in one sentence. *Watch out for:* the overhang carries no distributed load, so V is constant along BC ; and at B the shear force jumps by the whole of the reaction while the bending moment does not jump at all — only its slope changes.

Exercise 2: Normal stresses (Navier). Locate the neutral axis of the T-section. Compute the second moment of area I_z and the two section moduli W_{sup} and W_{inf} . Find the extreme normal stresses under the sagging moment obtained in Exercise 1, then under the hogging moment at B . State which fibre, in which section of the beam, carries the largest tensile stress and which carries the largest compressive stress. If the material were cast iron — strong in compression, weak in tension — would you keep this orientation of the T? *Watch out for:* a T is unsymmetric, so the neutral axis is **not** at mid-depth and there are **two** different section moduli; both extreme fibres must be checked, under both signs of the moment.

Exercise 3: Shear stresses (Jouravski). The largest shear force in the beam is $V = 43.5 \text{ kN}$, just to the left of B . Find the shear stress at the neutral axis and on each side of the flange-web junction, and the shear flow at that junction in kN per metre of length. Sketch the distribution of τ over the whole depth. Compare τ_{max} with the largest normal stress of Exercise 2, and with the crude estimate V/A_{web} ; comment on both comparisons. *Watch out for:* τ jumps where the width b jumps, because S_z^* is continuous but b is not; the shear flow $q = \tau b$ is continuous, and it is the flow, not the stress, that a weld or a row of nails has to carry.

Reminders

$$\begin{aligned} dV/dx &= -q, \quad dM_z/dx = V \\ \sigma_x &= M_z y/I_z, \quad W = I_z/y_{ext} \end{aligned}$$

$$\begin{aligned} \bar{y} &= \sum A_i y_i / \sum A_i \\ \tau &= V S_z^* / (I_z b) \end{aligned}$$

$$\begin{aligned} I_z &= \sum (b_i h_i^3 / 12 + A_i d_i^2) \\ q_{flow} &= V S_z^* / I_z \end{aligned}$$