

Tutorial Sheet 1 – Introduction to Dynamical Systems

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Exercise 1

For each equation, determine whether it is linear or nonlinear, and indicate its order.

1.

$$t^2 \frac{d^2 y}{dt^2} + 3t \frac{dy}{dt} + y = 0,$$

2.

$$y'' + ty' + e^t y = 0,$$

3.

$$y' + \cos y = 0,$$

4.

$$y'' + (y')^2 + y = 0,$$

5.

$$ty' + y = t^3,$$

6.

$$y' = y \ln(y).$$

Exercise 2

Solve the following differential equations.

1.

$$\frac{dx}{dt} = te^{-x},$$

2.

$$\frac{dx}{dt} = tx,$$

3.

$$\frac{dy}{dt} = t(1 + y^2),$$

4.

$$\frac{dy}{dt} = e^{t-y},$$

5.

$$\frac{dy}{dt} = (t^2 + 1)y,$$

6.

$$\frac{dy}{dt} = t^2 \sin(y),$$

7.

$$\frac{dy}{dt} = \frac{t}{1+y},$$

8.

$$\frac{dy}{dt} = t^2 \sin(y),$$

9.

$$\frac{dy}{dt} = y \cos(t).$$

Exercise 3

Consider the following initial value problem:

$$\begin{cases} y'(t) = ty(t), \\ y(0) = 1. \end{cases}$$

1. Show that the solution exists globally.
2. Determine the explicit solution and study its behavior as $t \rightarrow \infty$.

Exercise 4

Show the equivalence between the differential form and the integral form:

$$y(t) = y_0 + \int_{t_0}^t f(s, y(s)) ds.$$

Exercise 5

1. Let $f(x) = x^2$. Is f globally Lipschitz continuous on $\mathbb{R}$?
2. Let $f(x) = \sin(x)$. Show that f is Lipschitz continuous on $\mathbb{R}$.

Exercise 6

Write the following differential system:

$$\begin{cases} u'' = au' + bu + cv' + dv, \\ v'' = ev' + fv + gu' + hu \end{cases}$$

in the form of a linear system:

$$x' = Ax,$$

where x is a vector of a suitable dimension n and A is a square matrix of size $n \times n$.

Next, write the following nonautonomous differential system:

$$\begin{cases} u'' = au' + bu + cv' + dv + \alpha(t), \\ v'' = ev' + fv + gu' + hu + \beta(t) \end{cases}$$

in the form of an autonomous system:

$$x' = f(x),$$

where x is a vector of a suitable dimension n .

Finally, write the following differential system:

$$\begin{cases} u' = uv - \sin u \cos v = h(u, v), \\ v'' = uv' + 2uv + u^3v' + ev = k(u, v, v') \end{cases}$$

in the form of a system:

$$x' = g(x),$$

where x is a vector of a suitable dimension.

Exercise 7

Show that the following initial value problem:

$$x' = |x|^{1/2}$$

has four solutions passing through the point $(0, 0)$, and plot these solutions in the (t, x) -plane.

Exercise 8

Consider the differential equation:

$$x' = x|\sin t|.$$

1. Write the equation in the form of an autonomous system.
2. Study the existence and uniqueness of solutions on $\mathbb{R}$.

3. Let $x(t)$ be the maximal solution satisfying the initial condition $x(0) = x_0$. Show that if $x_0 < 0$, then for every $t \in \mathbb{R}$, $x(t) > 0$.
4. Calculate the solutions explicitly.

Exercise 9

Show that if

$$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix},$$

then

$$e^A = e^a \begin{pmatrix} \cos b & -\sin b \\ \sin b & \cos b \end{pmatrix}.$$

Exercise 10

Calculate e^{tA} for

$$A = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix},$$

and deduce the solution of the system

$$\dot{X} = AX.$$

Exercise 11

Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Calculate e^{tA} .

Exercise 12

Solve the system:

$$\begin{cases} x' = 2x + y, \\ y' = -x + y, \end{cases} \quad (x(0), y(0)) = (1, 0).$$