

Exercise No. 1

Show that the general boundary condition

$$-\lambda \frac{\partial T}{\partial n} + q_a = h(T - T_\infty) + \varepsilon \sigma (T^4 - T_r^4)$$

For $\frac{T - T_r}{T_r} \ll 1$, it can be linearized and written in the form

$$-\lambda \frac{\partial T}{\partial n} + q_a = h(T - T_\infty) + h_r(T - T_r) \quad \text{où} \quad h_r = 4\varepsilon \sigma T_r^3 \text{ est le}$$

radiative transfer coefficient.

Solution

In the case where $\frac{|T - T_r|}{T_r} \ll 1$, it is possible to linearize the equation

$-\lambda \frac{\partial T}{\partial n} + q_a = h(T - T_\infty) + \varepsilon \sigma (T^4 - T_r^4)$, therefore let us assume that

$$\frac{T - T_r}{T_r} = \alpha \quad (\alpha \ll 1)$$

hence

$$T = (\alpha + 1)T_r$$

The term $T^4 - T_r^4$ can be written in the form

$$T^4 - T_r^4 = (T - T_r)(T^3 + TT_r^2 + T^2T_r + T_r^3)$$

Substituting T with its expression, we obtain

$$T^4 - T_r^4 = (T - T_r)(\alpha^3 + 4\alpha^2 + 6\alpha + 4)T_r^3$$

Since α is very small compared to unity, we can write this relation in the form

$$T^4 - T_r^4 = 4T_r^3(T - T_r)$$

Therefore, the equation becomes

$$-\lambda \frac{\partial T}{\partial n} + q_a = h(T - T_\infty) + h_r(T - T_r)$$

where h_r is the exchange coefficient for the given radiation, defined by the relation

$$h_r = 4\varepsilon \sigma T_r^3$$

Exercise No. 2

1) If the surface is black at temperature $T = 505 \text{ K}$, and if the reference temperature is $T_r = 500 \text{ K}$, calculate the heat exchange coefficient for radiation h_r .

2) Calculate the radiative heat flux q_{ray} with and without linearization.

Given: $\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)$

Solution

1) Calculation of the coefficient h_r

$$h_r = 4\varepsilon \sigma T_r^3 = 4 \times 1 \times 5.67 \times 10^{-8} \times 500^3 = 28,35 \text{ W/m}^2 \text{K}$$

2) Calculation of heat flux by radiation

a) With linearization

$$q_{ray} = h_r(T - T_r) = 28,35 \times (505 - 500) = 141,75 \text{ W / m}^2$$

b) Without linearization

$$q_{ray} = \varepsilon \sigma (T^4 - T_r^4) = 1 \times 5,67 \times 10^{-8} \times (505^4 - 500^4) = 143,89 \text{ W / m}^2$$

$$\frac{\Delta q(\text{sans} - \text{avec})}{q(\text{sans})} = \frac{143,89 - 141,75}{143,89} = 1,48\%$$

Exercise No. 3

Find the heat equation in Cartesian coordinates for an anisotropic medium.

Exercise No. 4

The thermal conductivity tensor of an anisotropic solid is given by the matrix

$$\lambda = \begin{pmatrix} \alpha_1 & \beta & 0 \\ \beta & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}$$

a) Write the expressions for the heat fluxes q_x , q_y , and q_z along the directions (Ox, Oy, Oz) ,

b) Derive the heat equation.

Solution

a) In an anisotropic solid, the components of the heat flux vector are given by

$$\begin{pmatrix} -q_x \\ -q_y \\ -q_z \end{pmatrix} = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} \begin{pmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{pmatrix} = \begin{pmatrix} \alpha_1 & \beta & 0 \\ \beta & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix} \begin{pmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{pmatrix}$$

$$q_x(\vec{r}, t) = -\alpha_1 \frac{\partial T(\vec{r}, t)}{\partial x} - \beta \frac{\partial T(\vec{r}, t)}{\partial y}$$

$$q_y(\vec{r}, t) = -\beta \frac{\partial T(\vec{r}, t)}{\partial x} - \alpha_2 \frac{\partial T(\vec{r}, t)}{\partial y}$$

$$q_z(\vec{r}, t) = -\alpha_3 \frac{\partial T(\vec{r}, t)}{\partial z}$$

$$\rho C_P \frac{\partial T(\vec{r}, t)}{\partial t} = -\vec{\nabla} \cdot \vec{q} + g(\vec{r}, t)$$

let

$$\rho C_P \frac{\partial T(\vec{r}, t)}{\partial t} = -\frac{\partial q_x(\vec{r}, t)}{\partial x} - \frac{\partial q_y(\vec{r}, t)}{\partial y} - \frac{\partial q_z(\vec{r}, t)}{\partial z} + g(\vec{r}, t)$$

We obtain the heat equation in Cartesian coordinates in an anisotropic medium in the form

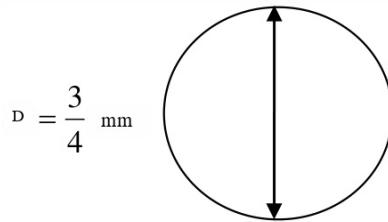
$$\rho C_p \frac{\partial T(\vec{r}, t)}{\partial t} = \alpha_1 \frac{\partial^2 T(\vec{r}, t)}{\partial x^2} + \alpha_2 \frac{\partial^2 T(\vec{r}, t)}{\partial y^2} + \alpha_3 \frac{\partial^2 T(\vec{r}, t)}{\partial z^2} + 2\beta \frac{\partial^2 T(\vec{r}, t)}{\partial x \partial y} + g(\vec{r}, t)$$

Exercise No. 5

The temperature of a gas stream must be measured using a thermocouple. The thermocouple junction can be modeled as a sphere with diameter $D = \frac{3}{4}$ mm, of thermal conductivity $\lambda = 30 \text{ W/m}^\circ\text{C}$, volumetric mass density $\rho = 8400 \text{ kg/m}^3$, and specific heat capacity $C_p = 0.4 \text{ kJ/kg}^\circ\text{C}$. If the heat exchange coefficient between the gas flow and the thermocouple is $h = 600 \text{ W/m}^2^\circ\text{C}$, how long will it take for the thermocouple to record 99% of the temperature difference between the gas temperature and the initial temperature of the thermocouple.

Solution

Assuming that the thermocouple head is a sphere



The characteristic length of the system is

$$L_c = \frac{V}{S} = \frac{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}{4\pi\left(\frac{D}{2}\right)^2} = \frac{D}{6} = \frac{\left(\frac{3}{4} \times 10^{-3}\right)}{6} = 0,00075 \text{ m}$$

Applying the formula

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hS}{\rho CV}} = e^{-\frac{h}{\rho CL_c}}$$

Let

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hS}{\rho CV}t} = e^{-\frac{h}{\rho CL_c}t}$$

The term

$$\frac{h}{\rho C_p L_c} = \frac{600}{8400 \times 400 \times 0,00075} = 0,158 \text{ s}^{-1}$$

To read 0.99 of the initial temperature, we must have

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0,01$$

$$0,01 = e^{-0,158t}$$

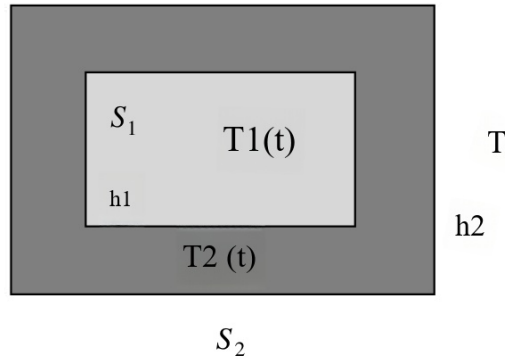
hence

$$t = \frac{\ln(0,01)}{-0,158} = 29,14 \text{ s}$$

Thus, it will take 29.14 seconds for the thermocouple to record 99% of the temperature difference between the gas temperature and the initial temperature of the thermocouple.

Exercise No. 6

A container with an inner surface S_1 and an outer surface S_2 contains a fluid at temperature T_0 . The ambient air temperature is constant and equal to T_∞ . If one assumes that the internal and external convective heat transfer coefficients are h_1 and h_2 , calculate the temperatures of the container and the fluid as a function of time, $T_1(t)$ and $T_2(t)$.



Solution

For the fluid

$$(-\rho CV)_1 \frac{dT_1(t)}{dt} = h_1 S_1 (T_1(t) - T_2(t))$$

For the container

$$(-\rho CV)_2 \frac{dT_2(t)}{dt} = h_2 S_2 (T_2(t) - T_\infty) - h_1 S_1 (T_1(t) - T_2(t))$$

$$t = 0, T_1(0) = T_2(0) = T_0$$

$$t = 0, \frac{dT_1(0)}{dt} = \frac{dT_2(0)}{dt} = 0$$

$$K_1 = \frac{h_1 S_1}{\rho_1 C_1 V_1}, K_2 = \frac{h_1 S_1}{\rho_2 C_2 V_2}, K_1 = \frac{h_2 S_2}{\rho_2 C_2 V_2}$$

Avec l'opérateur $D = \frac{d}{dt}$, $D^2 = \frac{d^2}{dt^2}$, nous obtenons

$$\begin{aligned}(D + K_1)T_1(t) - K_1T_2(t) &= 0 \\ -K_2T_1(t) + (D + K_2 + K_3)T_2(t) &= K_3T_\infty \\ (D^2 + (K_1 + K_2 + K_3)D + K_1K_3)T_1(t) &= K_1K_3T_\infty\end{aligned}$$

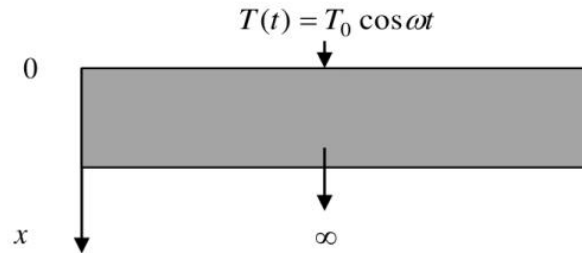
$$\begin{aligned}T_1(t) &= T_\infty + Ae^{\eta t} + Be^{\eta_2 t} \\ r_1 &= \frac{-(K_1 + K_2 + K_3) + \sqrt{(K_1 + K_2 + K_3)^2 - 4K_1K_3}}{2} \\ r_2 &= \frac{-(K_1 + K_2 + K_3) - \sqrt{(K_1 + K_2 + K_3)^2 - 4K_1K_3}}{2}\end{aligned}$$

By calculating the constants A and B , we find

$$\frac{T_1(t) - T_\infty}{T_0 - T_\infty} = \frac{r_2}{r_2 - r_1} e^{\eta t} - \frac{r_1}{r_2 - r_1} e^{\eta_2 t}$$

Exercise No. 7

Given that the daily variation of temperature at the soil surface is $T(t) = T_0 \cos(\omega t)$,



and we propose to calculate the ground temperature $T(x, t)$; for this, we must solve the Fourier equation

$$\frac{1}{a} \frac{\partial T(x, t)}{\partial t} = \frac{\partial^2 T(x, t)}{\partial x^2}$$

a) Write the appropriate boundary conditions,

b) Show that $(i\omega)^{\frac{1}{2}} = \sqrt{\frac{\omega}{2}} (1 + i)$,

c) Seek the solution in the form: $T(x, t) = X(x)e^{i\omega t}$,

d) By taking the real part of the solution, calculate the integration constant.

e) What conclusions can be drawn from this solution.

Solution

a) Boundary conditions

$$\begin{aligned}x = 0; T(0; t) &= T_0 \cos \omega t \\ x = \infty; T(x; t) &= \text{Finie}\end{aligned}$$

b) Write $(i\omega)^{\frac{1}{2}}$ in the form

$$(i\omega)^{\frac{1}{2}} = a + ib$$

Let

$$i\omega = a^2 - b^2 + 2iab$$

Hence

$$a = b = \sqrt{\frac{\omega}{2}}$$

Therefore $(i\omega)^{\frac{1}{2}} = \sqrt{\frac{\omega}{2}} + i\sqrt{\frac{\omega}{2}}$

c) By substituting the form of the solution $T(x, t) = X(x)e^{i\omega t}$ into the Fourier equation, we obtain

$$\frac{i\omega}{a} X(x)e^{i\omega t} = X''(x)e^{i\omega t}$$

Let

$$X''(x) - \frac{i\omega}{a} X(x) = 0$$

The solution to this ordinary differential equation is

$$X(x) = Ae^{x\sqrt{\frac{i\omega}{a}}} + Be^{-x\sqrt{\frac{i\omega}{a}}}$$

$$T(x, t) = \left(Ae^{x\sqrt{\frac{i\omega}{a}}} + Be^{-x\sqrt{\frac{i\omega}{a}}} \right) e^{i\omega t}$$

However, since the temperature must be finite for $x = \infty$, the constant A must be zero, hence

$$T(x, t) = Be^{-x\sqrt{\frac{i\omega}{a}}} e^{i\omega t} = Be^{-x\sqrt{\frac{\omega}{2a}}} e^{-xi\sqrt{\frac{\omega}{2a}}} e^{i\omega t} = Be^{-x\sqrt{\frac{\omega}{2a}}} e^{i\left(\omega t - x\sqrt{\frac{\omega}{2a}}\right)}$$

d) The real part of this solution is

$$T(x, t) = Be^{-x\sqrt{\frac{\omega}{2a}}} \cos\left(\omega t - x\sqrt{\frac{\omega}{2a}}\right)$$

From the first boundary condition, we have

$$T(0, t) = B \cos \omega t = T_0 \cos \omega t$$

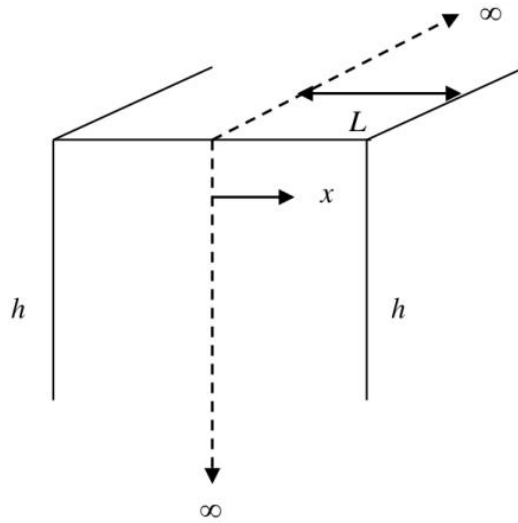
$$B = T_0$$

Ainsi

$$T(x, t) = T_0 e^{-x\sqrt{\frac{\omega}{2a}}} \cos\left(\omega t - x\sqrt{\frac{\omega}{2a}}\right)$$

e) In conclusion, the ground temperature has an amplitude that is damped by the term $e^{-x\sqrt{\frac{\omega}{2a}}}$ and is phase-shifted relative to the temperature $T(t) = T_0 \cos(\omega t)$ by an angle equal to $x\sqrt{\frac{\omega}{2a}}$

Exercise No. 8



Exercise No. 9

Given that the spectral emissive power curve $E_{\lambda,b}(\lambda, T)$ exhibits a maximum, find the equation that allows one to deduce Wien's displacement law.

Solution

We note that the spectral emissive power distribution $E_{\lambda,b}(\lambda, T)$ of a black body exhibits a maximum at $\lambda_{\max}$ which depends on the temperature T . To find this maximum, let us calculate

$$\frac{dE_{\lambda,b}(\lambda, T)}{d\lambda} = 0$$

by differentiating relation VI. 148 with respect to the wavelength λ , we obtain

$$\frac{dE_{\lambda,b}(\lambda, T)}{d\lambda} = \frac{-5C_1\lambda^4 \left(e^{\frac{C_2}{\lambda T}} - 1 \right) + C_1C_2 \frac{\lambda^3}{T} e^{\frac{C_2}{\lambda T}}}{\left(\lambda^5 \left(e^{\frac{C_2}{\lambda T}} - 1 \right) \right)^2} = 0$$

from which

$$-5C_1\lambda^4\left(e^{\frac{C_2}{\lambda T}}-1\right)+C_1C_2\frac{\lambda^3}{T}e^{\frac{C_2}{\lambda T}}=0$$

by simplifying and rearranging, we obtain

$$\lambda T\left(e^{\frac{C_2}{\lambda T}}-1\right)=\frac{C_2}{5}e^{\frac{C_2}{\lambda T}}$$

If we denote by $x=\frac{C_2}{\lambda T}$, this relation can then be written in the form

$$\frac{e^x-1}{e^x}=\frac{x}{5}$$

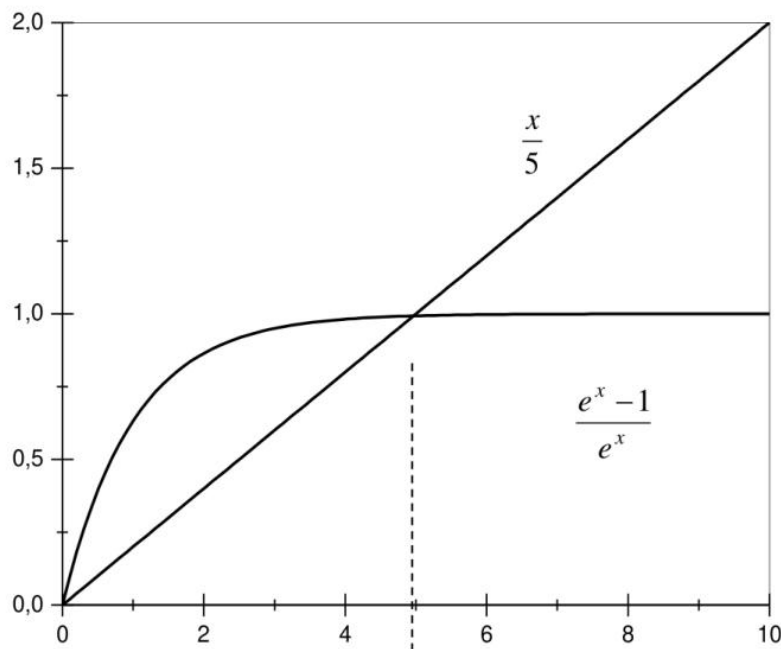


Figure VI. 14. Fonctions $\frac{e^x-1}{e^x}$ et $\frac{x}{5}$.

In Figure VI. 14, we have plotted the functions $\frac{e^x - 1}{e^x}$ and $\frac{x}{5}$ at their point of intersection is the solution to the equation $\frac{e^x - 1}{e^x} = \frac{x}{5}$. The value x that corresponds to the solution is $x = 4,965$. Knowing that $x = \frac{C_2}{\lambda T}$, we can then write

$$\frac{C_2}{\lambda_{\max} T} = 4,965$$

and by substituting the numerical value of the constant C_2 , we obtain

$$\lambda_{\max} T = 2898 \mu m K$$

This relation is known as Wien's displacement law. This law shows that the wavelength corresponding to the maximum value of the spectral emissive power is inversely proportional to the absolute temperature of the black body.

Exercise No. 10

Given that the maximum spectral emissive power corresponds to $\lambda_m T = C_3$, and is $E_b(\lambda_{\max}, T) = C_4 T^5$, calculate the numerical value of the constant C_4 .

Solution

The maximum spectral emissive power $E_{\lambda_{\max}}(T)$, which corresponds to the maximum wavelength $\lambda_{\max}$, is obtained by substituting the relation $\lambda_{\max} T = C_3$ into the equation

$$E_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5 \left(e^{\frac{C_2}{\lambda T}} - 1 \right)}$$

let

$$E_b(\lambda_{\max}, T) = C_4 T^5$$

This relation shows that the maximum spectral emissive power $E_{\lambda_{\max}}(T)$ depends on the fifth power of the absolute temperature of the black body.

Exercise No. 11

According to the electromagnetic theory of light, the radiation pressure in a cavity of volume V and temperature T is $P = \frac{u}{3}$, where u is the energy density per unit volume. Derive the Stefan-Boltzmann law from the first and second principles of thermodynamics.

Solution

Consider a cavity of volume V at temperature T containing electromagnetic waves whose energy density is u . Based on the electromagnetic theory of light, the radiation pressure P in the cavity is

$$P = \frac{u}{3}$$

if U is the internal energy of the cavity, we can then write that

$$U = uV$$

From the first principle of thermodynamics, we have

$$TdS = dU + PdV$$

if we substitute the expressions for the internal energy U and the pressure P , it follows that

$$TdS = d(uV) + \frac{u}{3} dV = Vdu + udV + \frac{u}{3} dV$$

from which the expression for entropy dS

$$dS = \frac{V}{T} du + \frac{4}{3T} udV$$

Since the energy density u of electromagnetic waves in the cavity depends only on the temperature T , we can write that

$$du = \frac{du}{dT} dT$$

then in this case, equation VI. 180 becomes

$$dS = \frac{V}{T} \frac{du}{dT} dT + \frac{4}{3T} udV$$

Since the entropy S is a function of the temperature T and the volume V , we can write

$$\begin{aligned} dS &= \left(\frac{\partial S}{\partial T} \right)_V dT + \left(\frac{\partial S}{\partial V} \right)_T dV \\ \left(\frac{\partial S}{\partial T} \right)_V &= \frac{V}{T} \frac{du}{dT} \\ \left(\frac{\partial S}{\partial V} \right)_T &= \frac{4}{3T} u \end{aligned}$$

since the entropy S is a state function, it must necessarily satisfy the condition

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$$

calculate $\frac{\partial^2 S}{\partial T \partial V}$ et $\frac{\partial^2 S}{\partial V \partial T}$, d'où

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial}{\partial V} \left(\frac{\partial S}{\partial T} \right) = \frac{\partial}{\partial V} \left(\frac{V}{T} \frac{du}{dT} \right) = \frac{1}{T} \frac{du}{dT}$$

$$\frac{\partial^2 S}{\partial V \partial T} = \frac{\partial}{\partial T} \left(\frac{\partial S}{\partial V} \right) = \frac{\partial}{\partial T} \left(\frac{4}{3T} u \right) = \frac{4}{3} \left(\frac{1}{T} \frac{du}{dT} - \frac{u}{T^2} \right)$$

if we substitute these expressions into equation VI. 186, we obtain

$$\frac{1}{T} \frac{du}{dT} = \frac{4}{3} \left(\frac{1}{T} \frac{du}{dT} - \frac{u}{T^2} \right)$$

equation, which after simplification becomes

$$\frac{4u}{T} = \frac{du}{dT}$$

let

$$\frac{du}{u} = 4 \frac{dT}{T}$$

the solution to this separable differential equation is

$$u(T) = CT^4$$

Exercise No. 12

Calculate the quantity of heat emitted by a black surface at 4000°C and deduce the fractions of energy emitted in the following spectral bands:

$$[0 - 0.38], [0.38 - 0.78], [0.78 - \infty] \mu m$$

Exercise No. 13

The temperature of the filament in an incandescent lamp is 2500 K. If we assume that the filament is a black body, determine:

- The fraction of energy radiated in the visible band,
- The wavelength corresponding to the maximum emission.

Data $\lambda_1 = 0.4 \mu m, \lambda_2 = 0.76 \mu m$

Exercise No. 14

Consider a sphere of diameter 20 cm at 800 K suspended in air. If we assume that the sphere is a black body, determine:

- The total emissive power,
- The amount of radiation emitted during five minutes,
- The emissive power at a wavelength of 3 μm .

Exercise No. 15

- Find the expression for the surface element in spherical coordinates,
- Derive the expression for the solid angle in spherical coordinates on a sphere of radius r ,
- Calculate the solid angle associated with a sphere and with a hemisphere.

Exercise No. 16

The energy density contained within the volume of a cavity containing electromagnetic radiation is

$$u_\nu = \frac{8\pi\nu^2}{c^3} \bar{E}$$

From statistical thermodynamics, we know that the probability of finding a system N_i in the energy state E_i is given by what we call the Boltzmann distribution.

$$P = Ae^{-\frac{E_i}{kT}}$$

where k is the Boltzmann constant and T is the absolute temperature of the system. Consequently, the average energy $\bar{E}$ of such a system is

$$\bar{E}_i = \frac{\int_0^\infty E_i e^{-\frac{E_i}{kT}} dE_i}{\int_0^\infty e^{-\frac{E_i}{kT}} dE_i}$$

Planck's hypothesis is that the energy exchanged between the oscillators on the wall of a black body cavity and the standing electromagnetic waves present within it occurs in discrete energy packets called quanta, given by the formula

$$E_n = nh\nu$$

where n is an integer.

- a) Calculate the average energy $\bar{E}$ of the oscillators,
- b) Calculate the energy density u_ν contained within the volume of the cavity containing,
- c) Given that $E_{\nu,b} = \frac{c}{4} u_\nu$, find the expression for the spectral emissive power E_ν .

Solution

Planck's hypothesis is that the energy exchanged between the oscillators on the wall of a black body cavity and the stationary electromagnetic waves present within it occurs in discrete packets of energy called quanta, given by the formula

$$E_n = nh\nu$$

where n is an integer.

Let us now calculate the average energy $\bar{E}$ of the oscillators located on the wall of the cavity constituting the black body. This average energy $\bar{E}$ is given by the formula where we have replaced the integral with a summation since the energies E_n are exchanged in discrete packets $E_n = nh\nu$

$$\bar{E} = \frac{\sum_{n=0}^{\infty} nh\nu e^{-\frac{nh\nu}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{kT}}}$$

If we let $x = \frac{h\nu}{kT}$, the previous formula can be written as

$$\bar{E} = kT \frac{\sum_{n=0}^{\infty} nx e^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = kT \frac{N(x)}{D(x)}$$

where the functions $N(x)$ and $D(x)$ are defined by the following expressions

$$N(x) = \sum_{n=0}^{\infty} nx e^{-nx}$$

$$D(x) = \sum_{n=0}^{\infty} e^{-nx}$$

If we explicitly define the function $D(x)$

$$D(x) = 1 + e^{-x} + e^{-2x} + e^{-3x} + \dots + e^{-nx}$$

we observe that it represents a geometric sequence with ratio e^{-x} ; consequently, we can express the function $D(x)$ in the form

$$D(x) = \frac{1}{1 - e^{-x}}$$

Explicitly, the function $N(x)$ is written as

$$N(x) = xe^{-x} + 2xe^{-2x} + 3xe^{-3x} + \dots + nxe^{-nx}$$

$$N(x) = x(e^{-x} + 2e^{-2x} + 3e^{-3x} + \dots + ne^{-nx})$$

this function $N(x)$ can also be written in the form

$$N(x) = -x \frac{d}{dx} (1 + e^{-x} + e^{-2x} + e^{-3x} + \dots + e^{-nx}) = -x \frac{d}{dx} \left(\frac{1}{1 - e^{-x}} \right)$$

let

$$N(x) = \frac{xe^{-x}}{(1 - e^{-x})^2}$$

By substituting the expressions for the functions $N(x)$ and $D(x)$ into Equation VI. 126, we obtain the following relation for the mean energy:

$$\bar{E} = kT \frac{x e^{-x} (1 - e^{-x})}{(1 - e^{-x})^2} = kT \frac{h\nu}{kT} \frac{e^{-x}}{(1 - e^{-x})} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Therefore, the energy density u_ν contained in the volume of a cavity containing electromagnetic radiation is

$$u_\nu = \frac{8\pi h}{c_0^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

Let us now find the relationship between the energy density u_ν and the spectral emissive power E_ν .

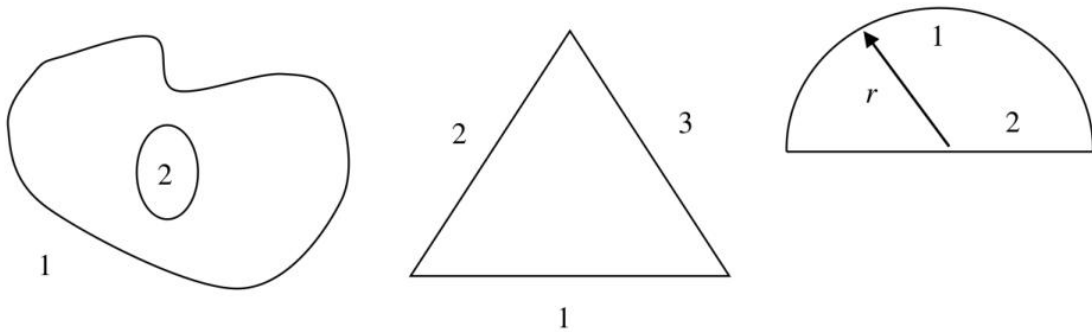
Exercise No. 17

Assuming the Sun radiates as a black body, calculate its temperature from the following data:

$R_S = 7.0 \times 10^8 \text{ m}$: radius of the sun, $R_{ST} = 15 \times 10^{10} \text{ m}$: sun-earth distance, $S = 1380 \text{ W/m}^2$: solar constant (average radiated energy flux incident on Earth's atmosphere).

Exercise No. 18

Calculate the shape factors for the following configurations:



Exercise No. 19

Calculate the shape factor and the net radiated energy between two infinite parallel surfaces of respective temperatures T_1 and T_2 . The two surfaces are assumed to be opaque and gray with respective emissivities ϵ_1 and ϵ_2 .

Exercise No. 20

Calculate the shape factor and the net radiated energy between two infinite concentric cylinders with respective temperatures T_1 and T_2 . The two surfaces are assumed to be opaque and gray with respective emissivities ϵ_1 and ϵ_2 .

Exercise No. 21

Consider a cylindrical furnace such that $R = H = 1$ m, the upper surface (S_1) and the base (S_2) have emissivities $\epsilon_1 = 0.8$ and $\epsilon_2 = 0.4$, respectively, and are maintained at temperatures $T_1 = 700$ K and $T_2 = 500$ K. The lateral surface is considered black and is maintained at temperature $T_3 = 300$ K.

- a) Calculate the energy radiated by each surface,
- b) Explain how to maintain these surfaces at the specified temperatures during a steady-state regime.

Exercise No. 1

If ρ_α is the volumetric mass of species α , the total volumetric mass of the mixture will be

$$\rho = \sum_{\alpha=1}^{\alpha=N} \rho_\alpha$$

where N is the total number of species constituting the mixture. From this definition, we can define the following quantities:

- 1) The mass fraction of species α

$$\omega_\alpha = \frac{\rho_\alpha}{\rho}$$

- 2) The molar concentration of the species α

$$c_\alpha = \frac{\rho_\alpha}{M_\alpha}$$

where M_α is the molar mass of species α .

- 3) The molar fraction of species α

$$x_\alpha = \frac{c_\alpha}{c}$$

With these definitions, show the following relations:

$$\text{a) } \sum_{\alpha=1}^{\alpha=N} x_\alpha = 1$$

$$\text{b) } \sum_{\alpha=1}^{\alpha=N} x_\alpha M_\alpha = M$$

$$\text{c) } x_\alpha = \frac{\frac{\omega_\alpha}{M_\alpha}}{\sum_{\beta=1}^{\beta=N} \frac{\omega_\beta}{M_\beta}}$$

$$\text{d) } \sum_{\alpha=1}^{\alpha=N} \omega_\alpha = 1$$

$$\text{e) } x_\alpha = \omega_\alpha \frac{M}{M_\alpha}$$

$$\text{f) } \sum_{\alpha=1}^{\alpha=N} \frac{\omega_\alpha}{M_\alpha} = \frac{1}{M}$$

$$\text{g) } \omega_\alpha = \frac{x_\alpha M_\alpha}{\sum_{\beta=1}^{\beta=N} x_\beta M_\beta}$$

Exercise No. 2

For a binary mixture, we have for species **A**

$$x_A = \frac{\frac{\omega_A}{M_A}}{\frac{\omega_A}{M_A} + \frac{\omega_B}{M_B}}$$

to show that

$$\vec{\nabla} x_A = \frac{\frac{\vec{\nabla} \omega_A}{M_A M_B}}{\left(\frac{\omega_A}{M_A} + \frac{\omega_B}{M_B} \right)^2}$$

and for species **B**

$$\vec{\nabla} x_B = \frac{\frac{\vec{\nabla} \omega_B}{M_A M_B}}{\left(\frac{\omega_A}{M_A} + \frac{\omega_B}{M_B} \right)^2}$$

For a binary mixture, for species **A**, we have the relation

$$\omega_A = \frac{x_A M_A}{x_A M_A + x_B M_B}$$

to show that

$$\vec{\nabla} \omega_A = \frac{M_A M_B \vec{\nabla} x_A}{(x_A M_A + x_B M_B)^2}$$

and deduce for species **B** the relation

$$\vec{\nabla} \omega_B = \frac{M_A M_B \vec{\nabla} x_B}{(x_A M_A + x_B M_B)^2}$$

Exercise No. 3

For a binary mixture composed of two species **A** and **B**, the mass fluxes $\vec{j}_A$ and $\vec{j}_B$ of species **A** and **B** are defined by

$$\vec{j}_A = \rho \omega_A (\vec{V}_A - \vec{V})$$

$$\vec{j}_B = \rho \omega_B (\vec{V}_B - \vec{V})$$

where $\vec{V} = \omega_A \vec{V}_A + \omega_B \vec{V}_B$ is the mean mass velocity. Show that

a) $\dot{\mathbf{j}}_A + \dot{\mathbf{j}}_B = \mathbf{0}$

b) $D_{AB} = D_{BA}$

Exercise No. 4

If $\vec{V}_\alpha$, $\vec{V}$, and $\vec{V}^*$ are respectively the velocities of species α with respect to a stationary coordinate system, the mean mass velocity, and the mean molar velocity, the mass flux of species α is given by one of the formulas

$$\vec{n}_\alpha = \rho_\alpha \vec{V}_\alpha$$

$$\vec{j}_\alpha = \rho_\alpha (\vec{V}_\alpha - \vec{V})$$

$$\vec{j}_\alpha^* = \rho_\alpha (\vec{V}_\alpha - \vec{V}^*)$$

as for the molar flux of species α , it is given by one of the formulas

$$\vec{N}_\alpha = c_\alpha \vec{V}_\alpha$$

$$\vec{J}_\alpha = c_\alpha (\vec{V}_\alpha - \vec{V})$$

$$\vec{J}_\alpha^* = c_\alpha (\vec{V}_\alpha - \vec{V}^*)$$

Show that:

a)

$$\sum_{\alpha=1}^{\alpha=N} \vec{n}_\alpha = \rho \vec{V}$$

b)

$$\sum_{\alpha=1}^{\alpha=N} \vec{N}_\alpha = c \vec{V}^*$$

c)

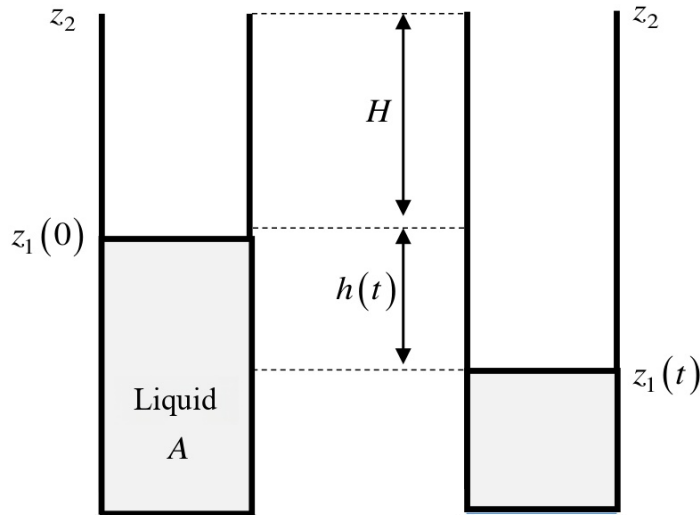
$$\sum_{\alpha=1}^{\alpha=N} \vec{j}_\alpha = \vec{0}$$

d)

$$\sum_{\alpha=1}^{\alpha=N} \vec{J}_\alpha^* = \vec{0}$$

Exercise No. 5

Let us now study the variation in the height $z_1(t)$ of liquid A in a mixture of composition x_{A2} .



Exercise No. 6

Consider diffusion in a stagnant gaseous film surrounding a liquid droplet. The molar fraction $x_A(r)$ is obtained from the differential equation

$$\frac{d}{dr} \left(\frac{r^{\frac{3n+4}{2}}}{(1-x_A(r))} \frac{dx_A(r)}{dr} \right) = 0$$

- Define the appropriate boundary condition.
- Calculate the total molar flux $N_{Ar}(r_1)$.

$$N_{Ar}(r_1) = \left[- \frac{cD_{AB,l} r^{\frac{3n}{2}}}{(1-x_A(r)) r_1^{\frac{3n}{2}}} \frac{dx_A(r)}{dr} \right]_{r=r_1}$$

Exercise No. 7

Consider the two-dimensional laminar runoff of a liquid **B** of constant thickness δ on a vertical surface composed of a species **A** slightly soluble in this liquid.

