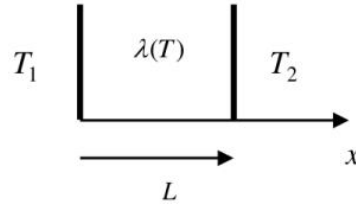


Exercise No. 1

A material fills the space between two parallel metal plates separated by a distance L . The thermal conductivity λ of this material is inversely proportional to the temperature according to the law

$$\lambda(T) = \frac{C}{T}$$



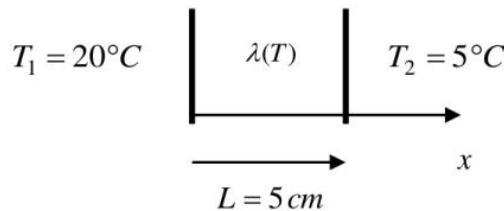
If the temperatures of the two plates are T_1 and T_2 .

- 1) Calculate the quantity of heat exchanged,
- 2) Deduce the temperature $T(x)$.

Exercise No. 2

A material fills the space between two parallel metal plates separated by a distance L . The thermal conductivity λ of this material is directly proportional to the temperature according to the law

$$\lambda(T) = 3T + 2$$



- a) Find the expression for the quantity of heat exchanged Q ,
- b) Calculate the numerical value of Q per unit area.

Exercise No. 3

Let a plane wall of thickness L be composed of a material with thermal conductivity λ varying with position x according to the law

$$\lambda(x) = \lambda_0 \sqrt{1 - \frac{x^2}{L^2}}$$

If the faces are at temperatures T_1 and T_2 , calculate the temperature $T(x)$.

Exercise No. 4

Calculate the temperature distribution $T(x)$ in a plane wall of thickness L and at thermal conductivity $\lambda = \lambda_0 e^{-\frac{x}{L}}$ where λ_0 is a positive constant. It will be assumed that the wall surfaces are at temperatures T_1 and T_2 .

Exercise No. 5

Consider a hollow sphere with inner radius R_i and outer radius R_o made of a material with thermal conductivity λ as a function of the radial distance r according to the law

$$\lambda(r) = \frac{\alpha}{r^3}$$

where α is a positive constant.

- Determine the units of the constant α . Given that the temperature of the inner surface of the sphere is T_i and the temperature of the outer surface of the sphere is T_o ($T_i > T_o$).
- Calculate the amount of heat Q that traverses the sphere,
- What is the thermal resistance R_{th} ,
- State the units of the thermal resistance R_{th} ,
- Determine the temperature distribution in the sphere $T(r)$.

Exercise No. 6

Consider a hollow cylinder with inner radius R_i and outer radius R_o made of a material with thermal conductivity λ as a function of temperature T according to the law

$$\lambda(T) = \frac{a}{T^2}$$

where a is a positive constant.

- Find the units of the constant a . Given that the temperature of the inner surface of the cylinder is T_i and the temperature of the outer surface of the cylinder is T_o ($T_i > T_o$).
- Calculate the amount of heat Q that passes through the cylinder,
- Determine the temperature distribution in the cylinder $T(r)$.

Exercise No. 7

The wall of a furnace consists of two layers: the first is made of refractory bricks with thickness $e_1 = 20 \text{ cm}$ and thermal conductivity $\lambda_1 = 1.38 \text{ W/(m}^\circ\text{C)}$, and the second layer is made of insulating bricks with thickness $e_2 = 10 \text{ cm}$ and thermal conductivity $\lambda_2 = 0.17 \text{ W/(m}^\circ\text{C)}$. The temperature inside the furnace is 1650°C and the convective heat exchange coefficient is $h_1 = 70 \text{ W/(m}^2\text{ }^\circ\text{C)}$. The ambient temperature is 25°C and the convective heat exchange coefficient is $h_2 = 10 \text{ W/(m}^2\text{ }^\circ\text{C)}$.

- Calculate the amount of heat lost through the furnace wall,
- The wall temperatures,
- The temperature at the interface of the two walls.

Exercise No. 8

The wall of a furnace is composed of three materials arranged in series:

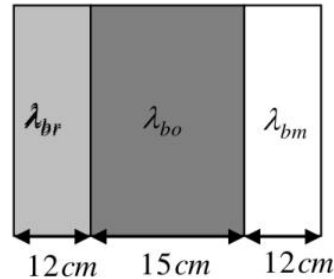
12 cm of refractory bricks, 15 cm of ordinary bricks, and 12 cm of magnesia bricks.

If the inner surface is at 827°C and the outer surface is at 112°C , calculate the quantity of heat lost through the wall per unit area. Given:

$$\lambda_{br} = 0,177 \text{ kcal/hm}^\circ\text{C}$$

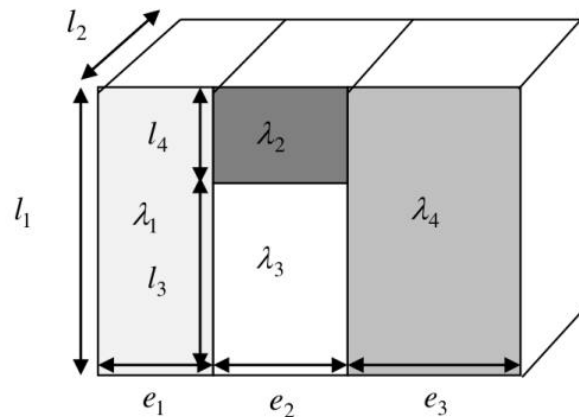
$$\lambda_{bo} = 0.223 \frac{\text{kcal}}{\text{hm} \cdot ^\circ\text{C}}$$

$$\lambda_{bm} = 3.08 \frac{\text{kcal}}{\text{hm} \cdot ^\circ\text{C}}$$



Exercise No. 9

- Calculate the thermal resistance of each layer.
- What is the equivalent resistance.
- Calculate the amount of heat passing through the structure.



Exercise No. 10

Calculate the thermal losses through a unit-area window in the following two cases:

- single glass with a thickness of four millimeters.
- Double glazing, composed of two glass panes each four millimeters thick and an intermediate air layer six millimeters thick. Convection in the air layer is neglected.

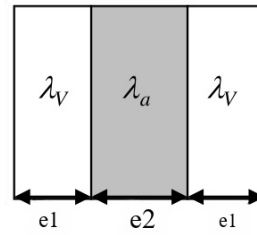
Given:

$$\lambda_v = 1.2 \frac{\text{W}}{\text{m} \cdot ^\circ\text{C}}$$

$$\lambda_a = 0.024 \frac{\text{W}}{\text{m} \cdot ^\circ\text{C}}$$

$$h = 12 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}}$$

$$T_{\infty 1} = 20^\circ\text{C}, T_{\infty 2} = 0^\circ\text{C}$$



Exercise No. 11

Consider a wall of thickness e and thermal conductivity λ . The surfaces of the wall are at temperatures T_1, T_2 and the surrounding air is at temperatures T_∞, T_∞' .

- Calculate the quantities of heat exchanged,
- Calculate the equivalent thermal resistance,
- If one defines $Q = US\Delta T$ where US is the global conductance coefficient, calculate this coefficient.
- Provide the equivalent electrical circuit.

Exercise No. 12

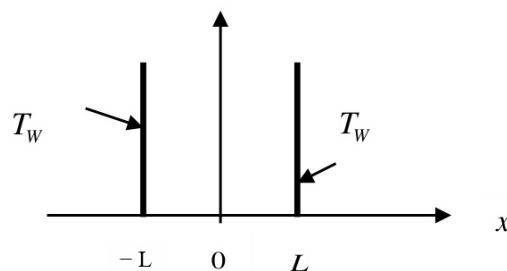
A cylinder with inner radius R_i and outer radius R_e is made of a material with thermal conductivity $\lambda = a + bT$. If the temperatures of the two walls are T_i and T_e , calculate the amount of heat exchanged.

Exercise No. 13

Find the temperature distribution in a cylinder with a circular cross-section undergoing constant heat generation q_s using the energy balance method.

Exercise No. 14

A plane wall of thickness $2L$ is subject to a volumetric internal heat generation of the type $q_s = q_0 \cos(ax)$, where a is a positive constant. If the two sides of the wall are maintained at a constant temperature T_w , calculate the quantity of heat lost by the wall.



Exercise No. 15

A planar wall of thickness L having one insulated face and the other face at temperature T_w . Calculate the temperature $T(x)$ if the internal heat source is:

$$q_s(x) = q_0 \left(1 - \frac{x}{L}\right)$$

Exercise No. 16

Find the temperature distribution in a plane wall subject to heat generation $q_s = q_0(1 - \beta(T - T_w))$, if the two faces are at temperature T_w and separated by a distance $2L$.

Exercise No. 17

A long solid cylinder of length L and diameter r_0 made of a material with constant thermal conductivity λ is subject to a constant internal heat generation Q_0 . The outer surface of the cylinder is maintained at a constant temperature T_0 . Given that the heat equation in this case is

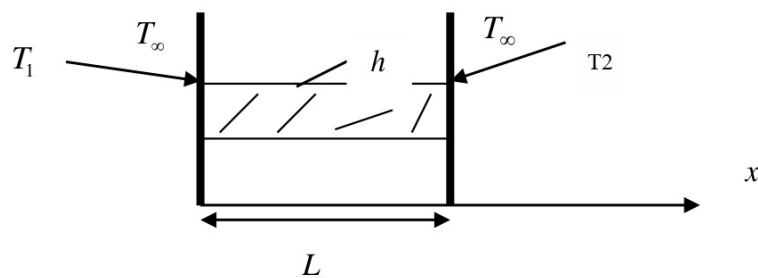
$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{Q_0 r}{\lambda} = 0$$

- 1) Find the solution $T(r)$ to this equation:
- 2) Write the appropriate boundary conditions,
- 3) Deduce the temperature $T(r)$,
- 4) Calculate the maximum temperature T_m ,
- 5) Write the temperature $T(r)$ in a dimensionless form.

Exercise No. 18

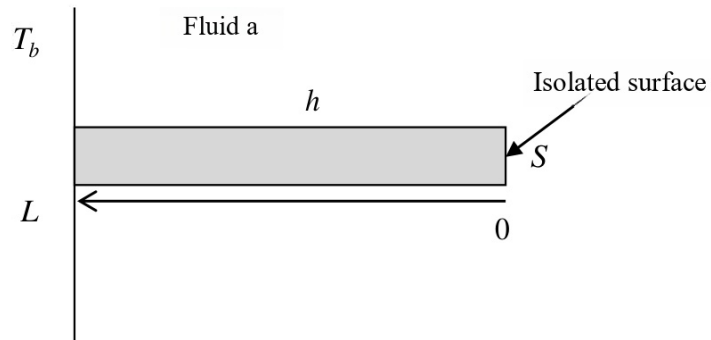
A cylindrical bar of length L and cross-section S is connected to two plane walls at respective temperatures T_1 and T_2 . It loses heat by convection to the surrounding air at temperature T_∞ .

Write the differential equation for the temperature distribution $T(x)$ and determine its solution.



Exercise No. 19

Let the following winglet



- 1) Write the heat equation for this fin,
- 2) Write the boundary conditions,
- 3) Find the dimensionless temperature $\frac{T(x) - T_\infty}{T_b - T_\infty}$,
- 4) Calculate the amount of heat dissipated by the fin Q_a .

Final Exam

Course Questions

- a) State the phenomenological laws of heat transfer.
- b) Write the special forms of the heat equation.

Exercise No. 1

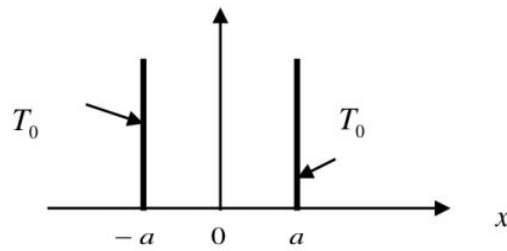
A cylinder with inner radius R_1 and outer radius R_2 is made of a material with thermal conductivity $\lambda = \alpha + \beta T$.

- a) What are the units of the constants α and β ,
- b) If the temperatures of the two walls are T_1 and T_2 , calculate the quantity of heat exchanged Q as a function of the thermal conductivity λ_m evaluated at the mean temperature of the cylinder material.

Exercise No. 2

A plane wall of thickness $2a$ and thermal conductivity λ , whose two sides are maintained at a constant temperature T_0 , is the site of internal heat generation $Q_s = Q_0 \cos(\alpha x)$ where α is a positive constant.

- What are the units of Q_0 and β ,
- Write the Poisson equation in this case,
- Determine the temperature $T(x)$,
- Calculate the heat flux passing through the right wall.



Replacement Exam

Course Questions

- What is the definition of the thermal efficiency of a fin,
- Write the different forms of the boundary conditions for the heat equation.

Exercise No. 1

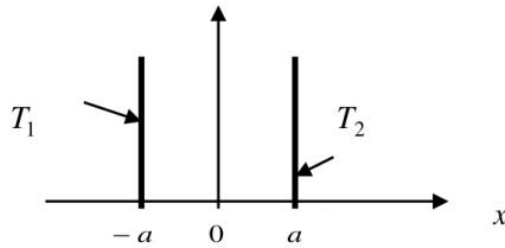
A cylinder with inner radius R_1 and outer radius R_2 is composed of a material with thermal conductivity $\lambda = \alpha + \beta T + \gamma T^2$.

- What are the units of the constants α , β , and γ ,
- If the temperatures of the two walls are T_1 and T_2 , calculate the exchanged heat flux q .

Exercise No. 2

A plane wall of thickness $2a$ and thermal conductivity λ , whose two sides are maintained at constant temperatures T_1 and T_2 , is subject to internal heat generation $q_s = q_0 \cos(\alpha x)$ where α is a positive constant.

- What are the units of q_0 and α ,
- Write the Poisson equation in this case,
- Determine the temperature $T(x)$,
- Calculate the heat flux passing through the right wall.



Make-up Exam

Course Questions

- Cite the three physical situations encountered during the study of a uniform-section airfoil.
- Give the definition of the thermal efficiency η_a of a fin.

Exercise No. 1

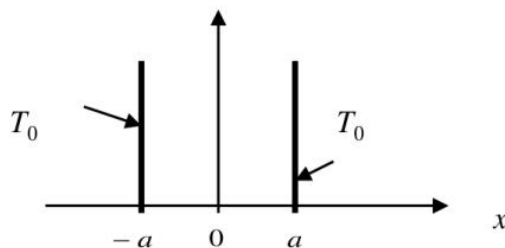
A sphere with inner radius R_1 and outer radius R_2 is made of a material with thermal conductivity $\lambda = \alpha + \beta T + \gamma T^2$.

- What are the units of the constants α , β , and γ ,
- If the temperatures of the two walls are T_1 and T_2 , calculate the amount of heat exchanged Q .

Exercise No. 2

A plane wall of thickness $2a$ and thermal conductivity λ , whose two sides are maintained at the constant temperature T_0 , is subject to an internal heat generation $q_s = q_0 \sin(\alpha x)$ where α is a positive constant.

- What are the units of q_0 and α ,
- Write the Poisson equation in this case,
- Find the integration constants C_1 and C_2
- Deduce the temperature $T(x)$.



Final Exam

Course Questions

- Write Fourier's law and the Fourier equation.
- What is the role of a stir bar.

- c) Provide the definition and formula for the thermal efficiency η_a of a fin.

Exercise No. 1

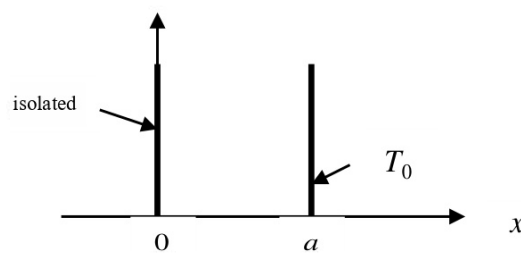
A cylinder with inner radius R_1 , outer radius R_2 , and length L is made of a material with thermal conductivity $\lambda(T) = \alpha + \beta T + \gamma T^2$.

- c) What are the units of the constants α , β , and γ ,
d) If the temperatures of the two walls are T_1 and T_2 , calculate the quantity of heat exchanged Q .

Exercise No. 2

A planar wall of thickness a and thermal conductivity λ , with one side maintained at a constant temperature T_0 and the other insulated, is subject to an internal heat generation rate $q_s = q_0 \cos(\alpha x + \beta)$, where α and β are positive constants.

- a) What are the units of q_0 , α and β ,
b) Write the Poisson equation,
c) Write the boundary conditions,
d) Find the integration constants C_1 and C_2 ,
e) Deduce the temperature $T(x)$ and
f) Calculate the heat flux q removed



Make-up Exam

Course Questions

- a) Write Fourier's law and the Fourier equation.
b) Provide the definition and formula for the thermal efficiency η_a of a fin.

Exercise No. 1

A hollow sphere with inner radius R_1 and outer radius R_2 is made of a material with thermal conductivity $\lambda(T) = \lambda_0 e^{\beta T}$.

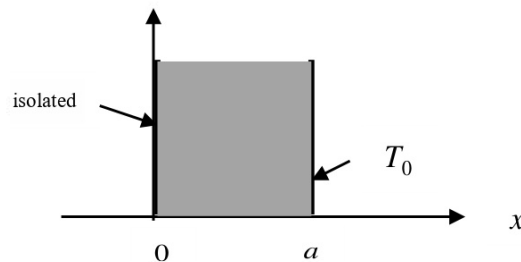
- a) What are the units of the constants λ_0 and β ,

- b) If the temperatures of the two walls are T_1 and T_2 , calculate the amount of heat Q passing through the wall of the sphere and
- c) The quantities of heat Q_1 and Q_2 passing through the inner surface and the outer surface of the sphere.

Exercise No. 2

A plane wall of thickness a and thermal conductivity λ , with one side maintained at a constant temperature T_0 and the other insulated, is subject to internal heat generation $q_s = q_0 \sin(\alpha x + \beta)$, where α and β are positive constants.

- What are the units of q_0 , α , and β ,
- Write the Poisson equation,
- Write the boundary conditions,
- Find the integration constants C_1 and C_2 ,
- Deduce the temperature $T(x)$ and
- Calculate the heat flux q removed.



Final Exam

Exercise No. 1

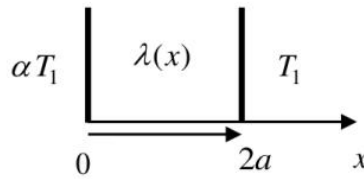
Show that the wall friction coefficient given by the relation

$$Cf_P = \frac{2\nu}{U_\infty^2} \left(\frac{\partial V_x(x,y)}{\partial y} \right)_{y=0} \text{ is dimensionless.}$$

Exercise No. 2

A plane wall is composed of a material with thermal conductivity $\lambda(x) = \lambda_0 e^{-\frac{x}{2a}}$, where λ_0 is a positive constant. If the two faces of the wall are at temperatures αT_p and T_p , calculate:

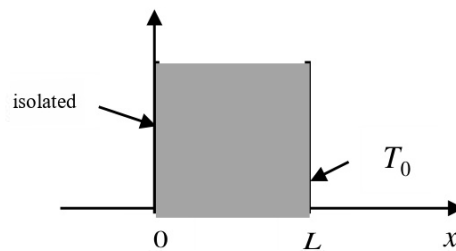
- The exchanged heat flux q ,
- The temperature $T(x)$.
- The temperature at the center of the wall T_m .



Exercise No. 3

A plane wall of thickness L and thermal conductivity λ , with one side maintained at a constant temperature T_0 and the other insulated, is subject to internal heat generation $q_s = Q_0 \sin(\alpha x + \beta)$, where α and β are positive constants.

- What are the units of Q_0 , α , and β ,
- Write the Poisson equation,
- Write the appropriate boundary conditions,
- Determine the integration constants C_1 and C_2 ,
- Deduce the temperature $T(x)$ and
- the heat flux q dissipated by the wall.



Replacement Exam

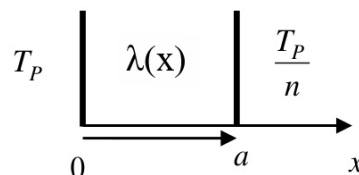
Exercise No. 1

Find the units of the shear stress $\tau_P = \frac{1}{2} \frac{\mu U^2 C_{fp}}{\nu}$ in the SI system.

Exercise No. 2

A plane wall is made of a material with thermal conductivity $\lambda(x) = \lambda_0 e^{\frac{x}{a}}$, where λ_0 is a positive constant. If the two faces of the wall are at temperatures $\frac{T_P}{n}$ and T_P , where n is a positive integer different from zero, calculate:

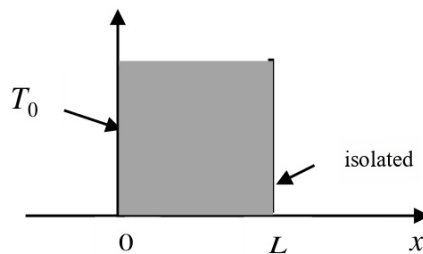
- The quantity of heat exchanged Q ,
- The temperature $T(x)$.



Exercise No. 3

A plane wall of thickness L and thermal conductivity λ , with one side maintained at a constant temperature T_0 and the other insulated, is subject to internal heat generation $Q_s = Q_0 \sin(\alpha x + \beta)$, where α and β are positive constants.

- What are the units of Q_0 , α and β ,
- Write the Poisson equation,
- Write the appropriate boundary conditions,
- Determine the integration constants C_1 and C_2 ,
- Deduce the temperature $T(x)$ and
- the heat flux q dissipated by the wall.



Make-up Exam

Course Questions

Let the system of equations

$$\begin{aligned} A_1 x + B_1 y &= C_1 \\ A_2 x + B_2 y &= C_2 \end{aligned}$$

using Cramer's method, calculate the unknowns x and y .

Exercise No. 1

A cylinder of length H , inner radius R_1 and outer radius R_2 is made of a material with thermal conductivity $\lambda = a + bT + cT^2$.

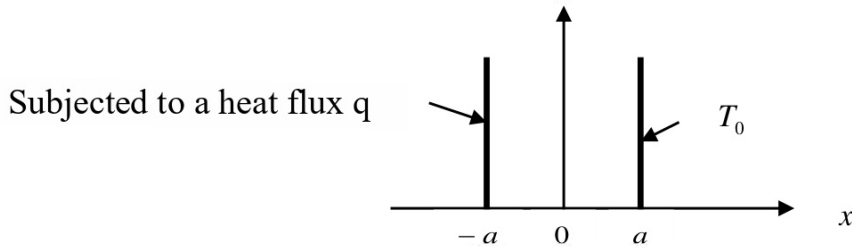
- What are the units of the constants a , b and c ,
- If the temperatures of the two walls of the cylinder are T_1 and T_2 , calculate the quantity of heat exchanged Q .

Exercise No. 2

A plane wall of thickness $2a$ and constant thermal conductivity λ , whose two sides are maintained at a constant temperature T_0 , is the site of internal heat generation $q_s = q_0 \sin(\alpha x)$ where α is a positive constant.

- What are the units of q_0 and α ,
- Write the Poisson equation,
- Find the integration constants C_1 and C_2 ,

d) Deduce the temperature $T(x)$.



Final Exam

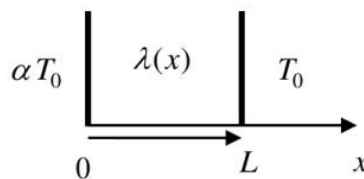
Exercise No. 1

Show that the wall friction coefficient given by the relation $\frac{C_{fP} U_{\infty}^2}{2\nu} = \left(\frac{\partial V_x(x, y)}{\partial y} \right)_{y=0}$ is dimensionless.

Exercise No. 2

A plane wall is made of a material with thermal conductivity $\lambda(x) = \lambda_m e^{-\frac{x}{L}}$, where λ_m is a positive constant. If the two faces of the wall are at temperatures αT_0 and T_0 , calculate:

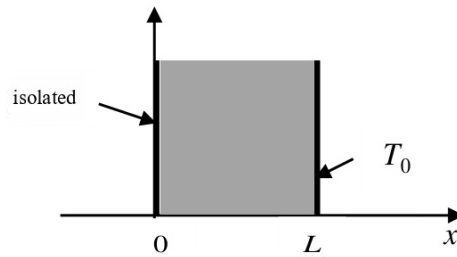
- 1) The exchanged heat flux q ,
- 2) The temperature $T(x)$.
- 3) The temperature T_r at $x = \frac{L}{4}$.



Exercise No. 3

A plane wall of thickness L and thermal conductivity λ , one side of which is maintained at a constant temperature T_0 and the other is insulated, is subject to internal heat generation $q_s = q_0 \cos\left(\frac{x}{m}\right)$ where m is a positive constant.

- 1) What are the units of q_0 and m ,
- 2) Write the Poisson equation,
- 3) Write the appropriate boundary conditions,
- 4) Determine the integration constants C_1 and C_2 ,
- 5) Deduce the temperature $T(x)$ and
- 6) Calculate the heat flux q dissipated by the wall.



Makeup Exam

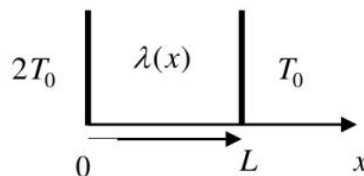
Exercise No. 1

Show that the wall friction coefficient given by the relation $\frac{\rho C_f U_\infty^2}{2\mu} = \left(\frac{\partial V_x(x,y)}{\partial y} \right)_{y=0}$ is dimensionless.

Exercise No. 2

A plane wall is composed of a material with thermal conductivity $\lambda(x) = \lambda_m e^{-\frac{x}{L}}$, where λ_m is a positive constant. If the two faces of the wall are at temperatures $2T_0$ and T_0 , calculate:

- 1) The exchanged heat flux q ,
- 2) The temperature $T(x)$.
- 3) The temperature T_r at $x = \frac{L}{4}$.

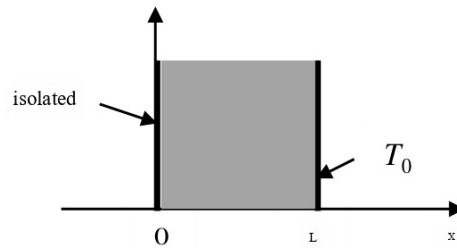


Exercise No. 3

A planar wall of thickness L and thermal conductivity λ , one side of which is maintained at a constant temperature T_0 and the other is insulated, is subject to internal heat generation

$$q_s = q_0 \cos\left(\frac{x}{m}\right) \text{ où } m \text{ est une constante positive.}$$

- 1) Write the Poisson equation,
- 2) Write the appropriate boundary conditions,
- 3) Determine the integration constants C_1 and C_2 ,
- 4) Deduce the temperature $T(x)$,
- 5) What is the temperature T_i on the insulated side and
- 6) What is the amount of heat Q dissipated by the wall?



Final Exam

Course Questions

- 1) Show that the skin friction coefficient $C_{fP} = \frac{\tau_p}{\frac{1}{2}\rho U_\infty^2}$ is dimensionless.
- 2) If a fluid of density ρ has a velocity V with Cartesian components (V_x, V_y, V_z) , write its continuity equation in Cartesian coordinates and in steady regime.
- 3) In the equation of motion below, provide the name of each term

$$\underbrace{\frac{\partial V_i}{\partial t}}_1 + \underbrace{V_j \frac{\partial V_i}{\partial x_j}}_2 = - \underbrace{\frac{\partial U}{\partial x_i}}_3 - \underbrace{\frac{1}{\rho} \frac{\partial P}{\partial x_i}}_4 + \underbrace{\nu \Delta V_i}_5$$

Exercise No. 1

A sphere with inner radius R_1 and outer radius R_2 is made of a material with thermal conductivity $\lambda(T) = a + bT + cT^2$.

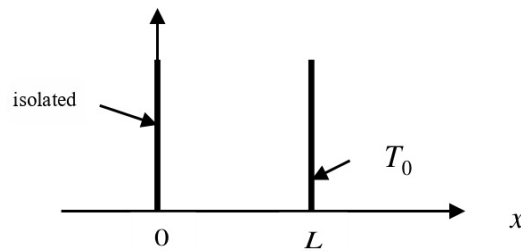
- 1) What are the units of the constants a , b , and c ,
- 2) if the temperatures of the two walls are T_1 and T_2 , calculate the quantity of heat exchanged Q .

Exercise No. 2

A planar wall of thickness L and thermal conductivity λ , one side of which is maintained at a constant temperature T_0 and the other is insulated, is the site of internal heat generation

$q_s = q_0 e^{\alpha x}$ where q_0 and α are positive constants.

- 1) what are the units of q_0 and α ,
- 2) write the Poisson equation,
- 4) find the integration constants C_1 and C_2 ,
- 5) deduce the temperature $T(x)$ and
- 6) calculate the heat flux q removed



Make-up Exam

Course Questions

- Write Fourier's law and the Fourier equation.
- Provide the definition and formula for the thermal efficiency η_a of a fin.

Exercise No. 1

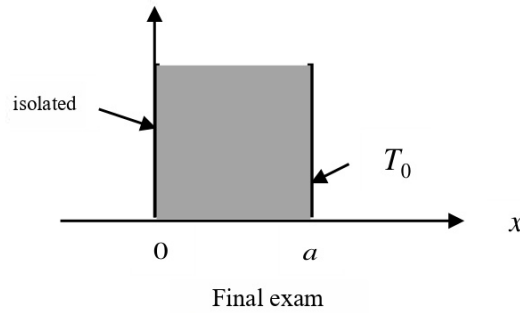
A hollow sphere with inner radius R_1 and outer radius R_2 is made of a material with thermal conductivity $\lambda(T) = \lambda_0 e^{\beta T}$.

- What are the units of the constants λ_0 and β ,
- If the temperatures of the two walls are T_1 and T_2 , calculate the quantity of heat Q passing through the spherical wall, and
- The quantities of heat Q_1 and Q_2 passing through the inner surface and the outer surface of the sphere.

Exercise No. 2

A plane wall of thickness a and thermal conductivity λ , with one side maintained at a constant temperature T_0 and the other insulated, is subject to internal heat generation $q_s = q_0 \sin(\alpha x + \beta)$, where α and β are positive constants.

- What are the units of q_0 , α , and β ,
- Write the Poisson equation,
- Write the boundary conditions,
- Find the integration constants C_1 and C_2 ,
- Deduce the temperature $T(x)$ and
- Calculate the heat flux q removed.



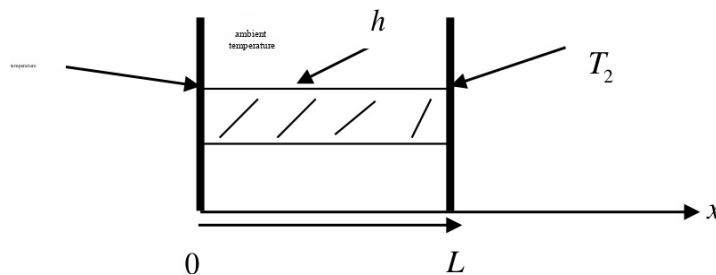
Course Questions

- a) Show that the skin friction coefficient C_{fP} given by the relation $\frac{\rho C_{fP} U_\infty^2}{2\mu} = \left(\frac{\partial V_x(x,y)}{\partial y} \right)_{y=0}$ is dimensionless.
- b) If a fluid with volumetric mass density ρ has a velocity $\vec{V}$ with components (V_x, V_y, V_z) , write its continuity equation in Cartesian coordinates.
- c) Provide the formulas for the Prandtl number Pr , the Reynolds number Re_L , and the Nusselt number Nu_L .

Exercise No. 1

A cylindrical rod of length L and cross-sectional area S is connected to two plane walls at respective temperatures T_1 and T_2 . This rod loses heat by convection to the ambient air at temperature T_∞ .

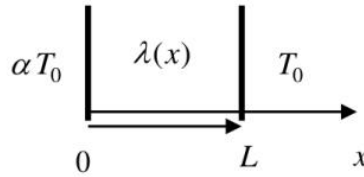
- a) Write the differential equation for the temperature distribution $T(x)$,
- b) Write the appropriate boundary conditions and
- c) Find the temperature distribution $T(x)$.



Exercise No. 2

A plane wall is composed of a material with thermal conductivity $\lambda(x) = \lambda_m e^{-\frac{x}{L}}$, where λ_m is a positive constant. If the two faces of the wall are at temperatures αT_0 and T_0 , calculate:

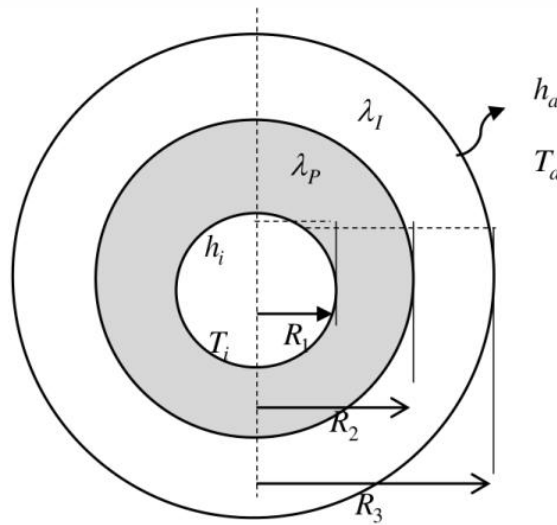
- a) The exchanged heat flux q ,
- b) The temperature $\frac{T(x)}{T_0}$.
- c) The temperature T_r at $x = \frac{L}{4}$.
- d) What is the value of α such that T_r becomes equal to T_0 ?



Make-up Exam

Exercise No. 1

A hot fluid resides within a sphere of inner radius R_1 and outer radius R_2 composed of a material with constant thermal conductivity λ_p . This sphere is covered with a thermal insulator of radius R_3 and constant thermal conductivity λ_I . Let the fluid temperature within the sphere be T_i and the average convective heat transfer coefficient between the hot fluid and the inner wall of the sphere be h_i . The insulated sphere is placed in ambient air at temperature T_a , where the average convective heat transfer coefficient between this ambient air and the insulated sphere is h_a .



- Assuming that $T_i > T_a$, list the different modes of heat transfer involved.
- In the context of heat diffusion within the sphere, provide the expressions for the different thermal resistances X_1 , X_2 , X_3 and X_4 as a function of the radii, the thermal conductivities, and the average convective exchange coefficients.
- Calculate the equivalent thermal resistance X_{th} .
- Deduce the amount of heat $\dot{Q}$ lost by the fluid to the ambient air.
- Find the critical radius of the insulator R_{3c} that minimizes the thermal resistance X_{th} .

Exercise No. 2

Given that the heat equation for a plane wall, a cylinder, and a sphere in the presence of heat generation is written as

$$\frac{d}{dx} \left(\lambda \frac{dT(x)}{dx} \right) + Q_s = 0$$
$$\frac{1}{r} \frac{d}{dr} \left(r \lambda \frac{dT(r)}{dr} \right) + Q_s = 0$$
$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \lambda \frac{dT(r)}{dr} \right) + Q_s = 0$$

where x and r are the abscissa and radius, respectively. Q_s and λ are the internal heat source and the thermal conductivity of the material (we assume they are not constant).

- By introducing a general variable ξ , write these three equations in a single form so as to recover the three previous equations,
- Find the general solution of the obtained equation,
- Deduce the solution for constant λ .

Exercise No. 3

Find the temperature distribution $T(x)$ in a plane wall subject to heat generation of the form $q_s = q_0(1 - a(T(x) - T_s))$, if both faces are at temperature T_s and separated by a distance H .

