

18. Moment of a Force About a Point

The moment measures the rotational effect of a force about a point.

Vector definition: $M_O = r \times F$.

The magnitude is $M_O = rF \sin\theta$.

The perpendicular distance d gives $M_O = Fd$.

Use a sign convention, such as counterclockwise positive.

$$M_O = r \times F$$

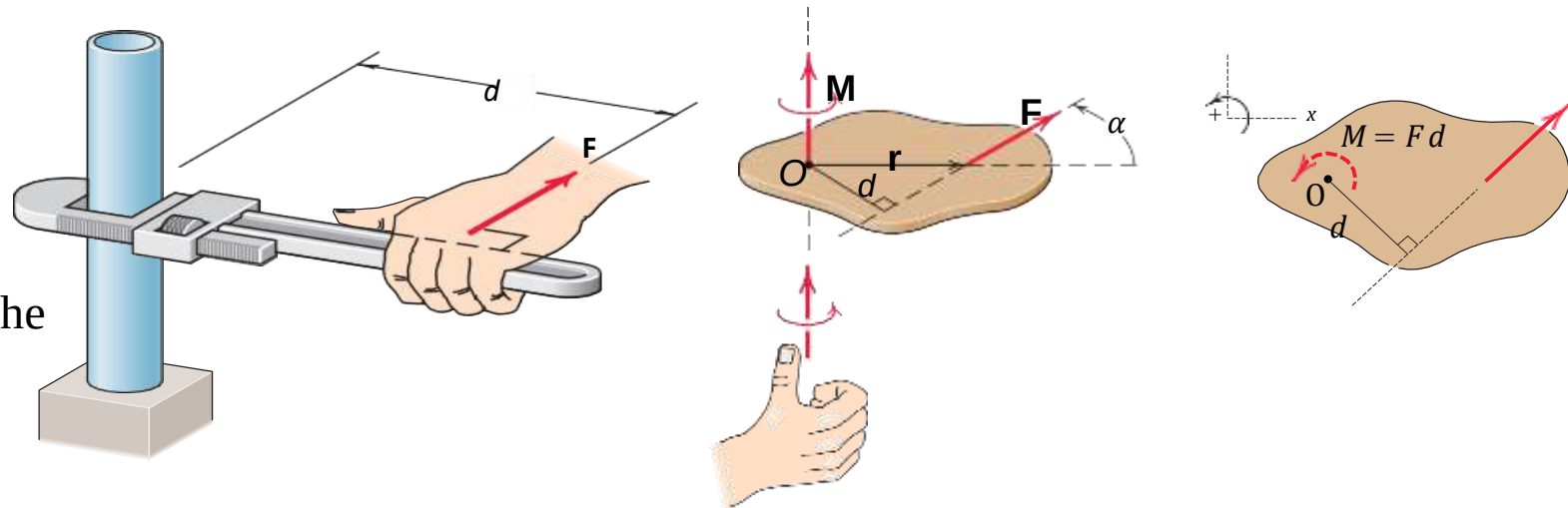


Fig.11. The moment of the force.

19. Varignon's Theorem

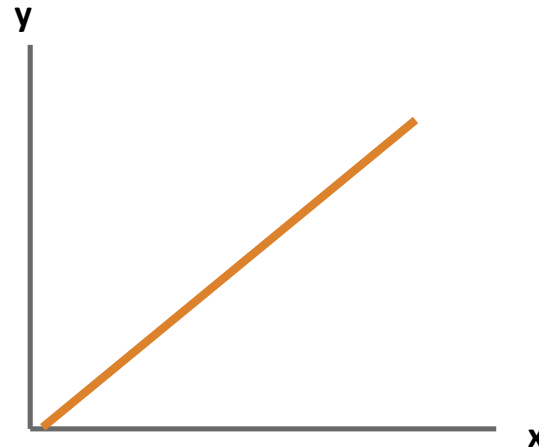
The moment of a force about a point equals the sum of the moments of its components about that point.

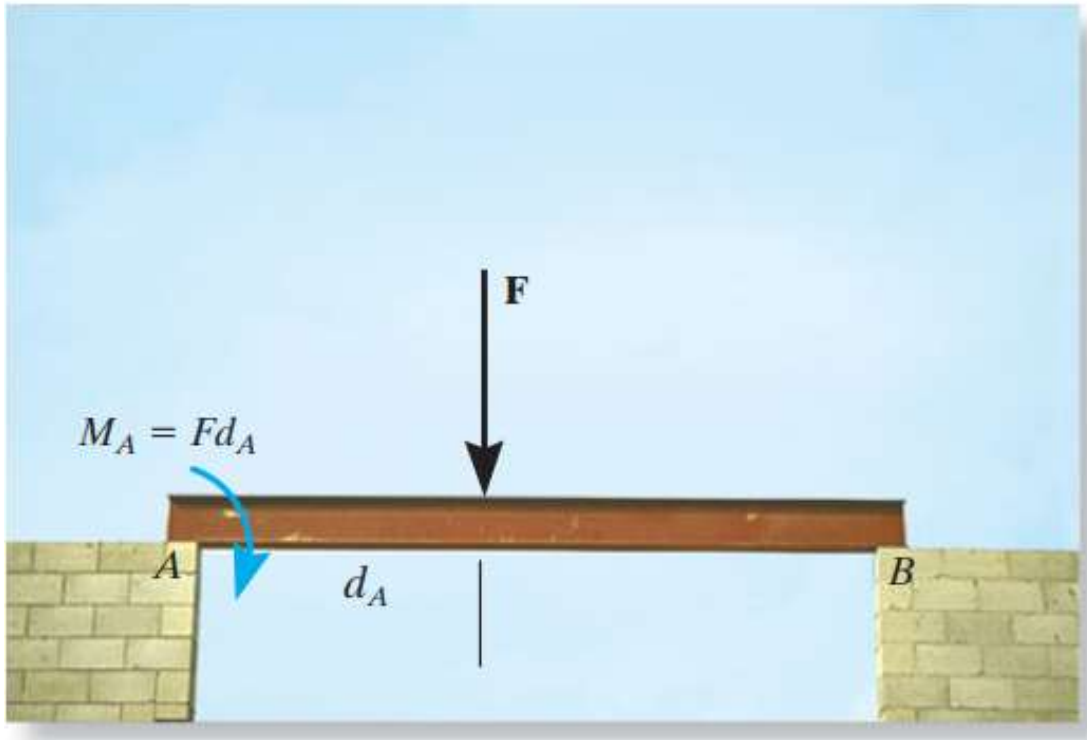
This theorem is especially useful when a force is resolved into x and y components.

For a 2D system:

$$M_O(F) = M_O(F_x) + M_O(F_y)$$

$$M_O = xF_y - yF_x$$





As illustrated by the example problems, the moment of a force does not always cause a rotation. For example, the force $\mathbf{F}$ tends to rotate the beam clockwise about its support at A with a moment. The actual rotation would occur if the support at B were removed.



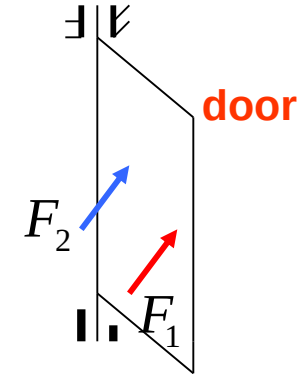
The ability to remove the nail will require the moment of about point O to be larger than the moment of the force $\mathbf{F}_N$ about O that is needed to pull the nail out.

20.Moment of Force

(1) By basic definition

- Forces distributed over a rigid body may cause rotation.

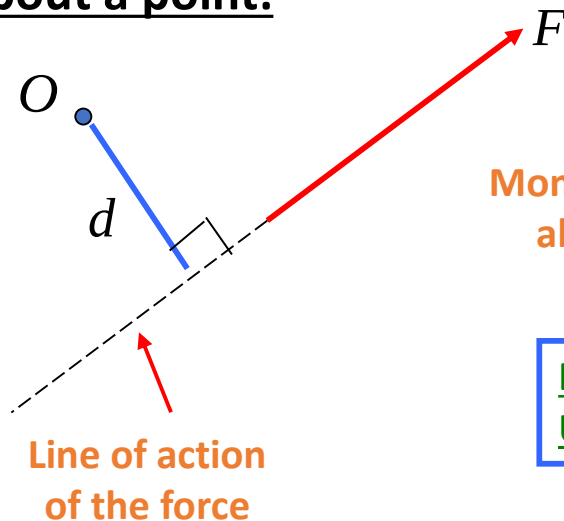
Effects of a force on rotation depend not only on magnitude and direction of the force but also on where it applies.



Concept of
moment

← To open the door where is better to apply the force?

Definition of Moment of a Force about a point:



Magnitude of
the force

Distance of the force to its
line of action

$$M_F^{(O)} = \pm Fd$$

Moment of a force
about point O

Sign convention:

“+” --- counterclockwise

“-” --- clockwise

Dimension: $[M] = [F][L]$

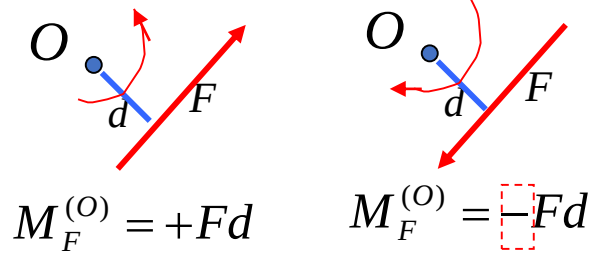
Unit: N-m; lb-ft

General Comments:

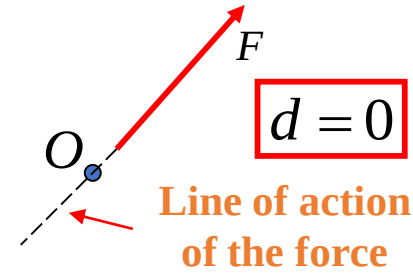
Sign convention:

"+"---- counterclockwise

"-" --- clockwise



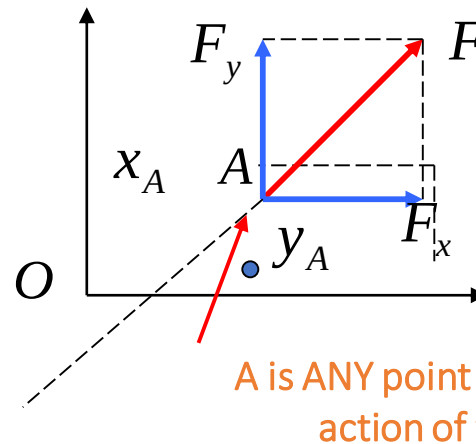
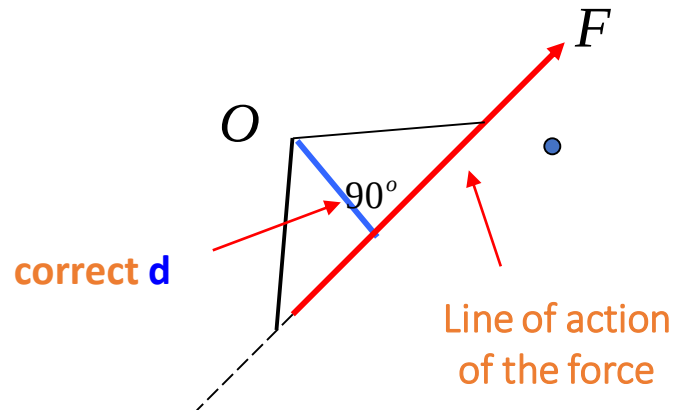
Moment of a force about a point is zero if the point is on the line of action of the force.



d must be the perpendicular distance to the line of action of the force

(2) By Cartesian components

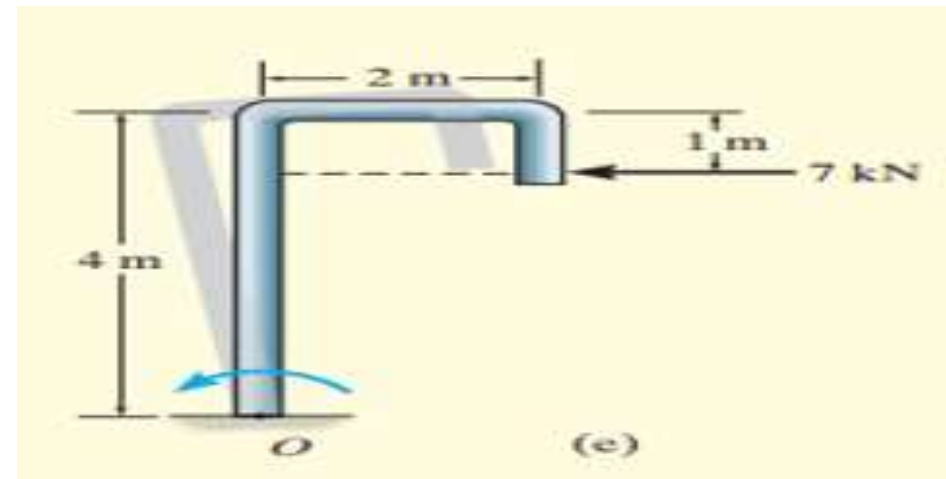
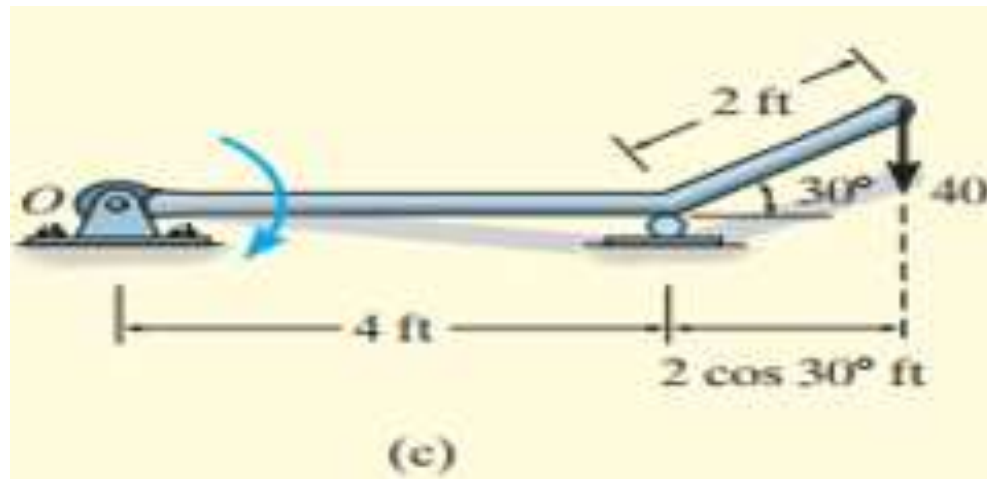
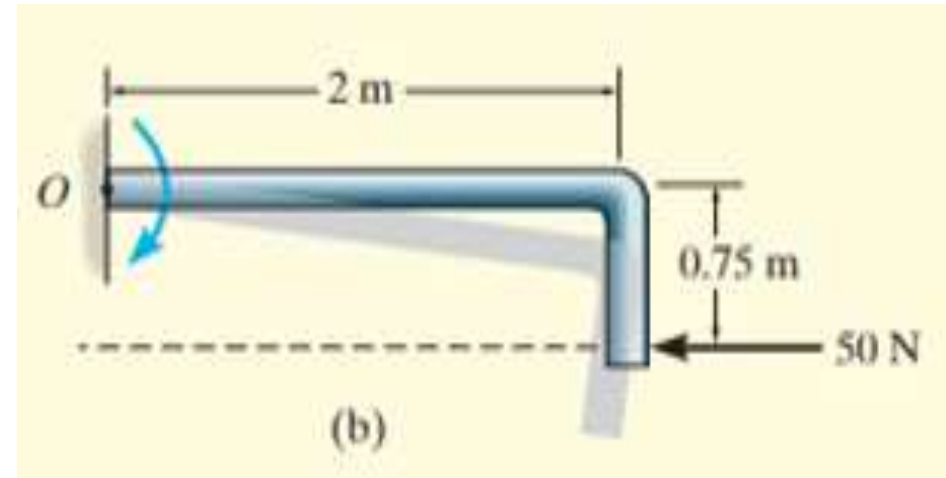
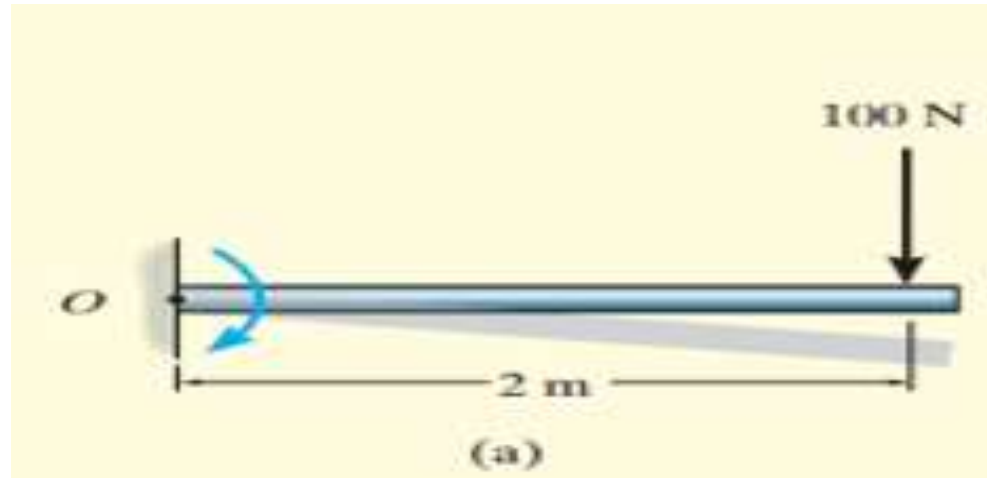
This method is generally easier than finding the same moment using $\mathbf{M}_O = \mathbf{F} \mathbf{d}$.



$$M_F^{(O)} = F_y x_A - F_x y_A$$

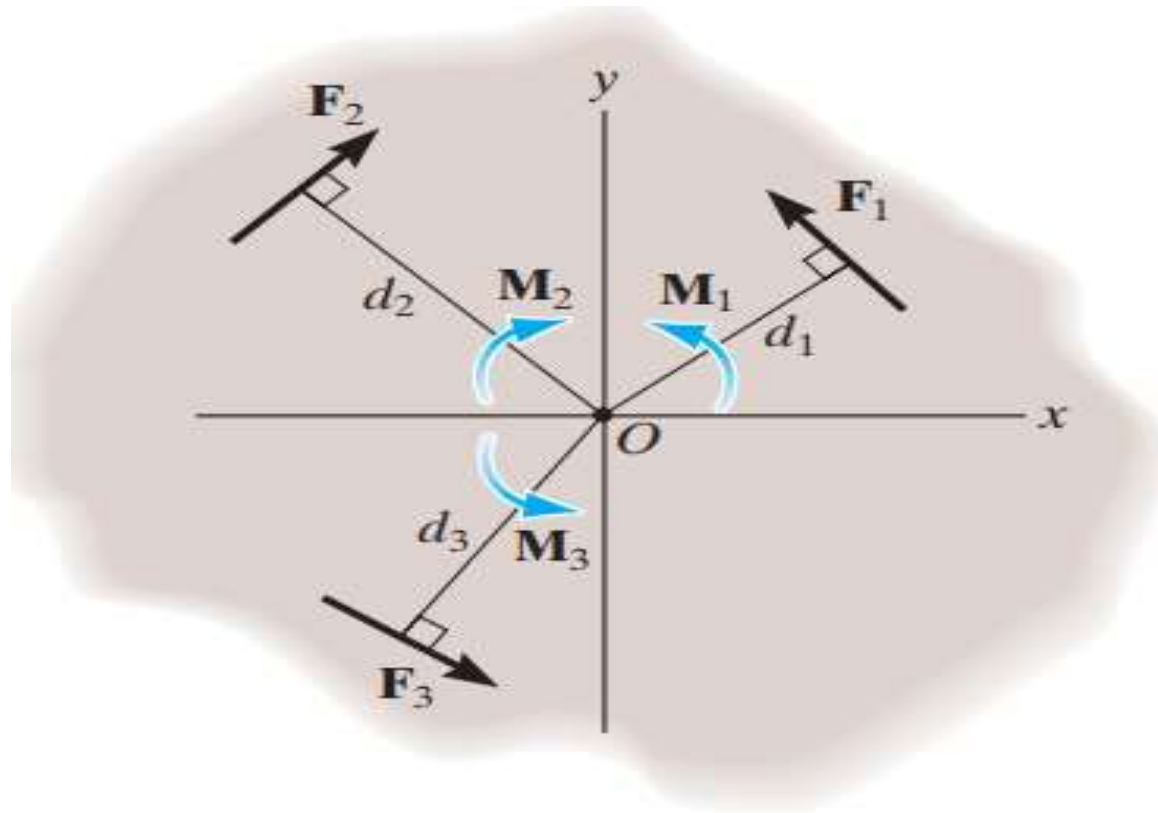
Equivalence?

Example.1. Determine the moment of the force about point O for each case illustrated in Figure.



Resultant Moment. For two-dimensional problems, where all the forces lie within the x - y plane. Using this sign convention, the resultant moment in Fig.12 is therefore

$$\zeta + (M_R)_O = \Sigma Fd; \quad (M_R)_O = F_1d_1 - F_2d_2 + F_3d_3$$



(3) By the cross-product definition

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. The moment of $\mathbf{F}$ about point $\mathbf{O}$ of Figure. may be represented by the cross-product expression

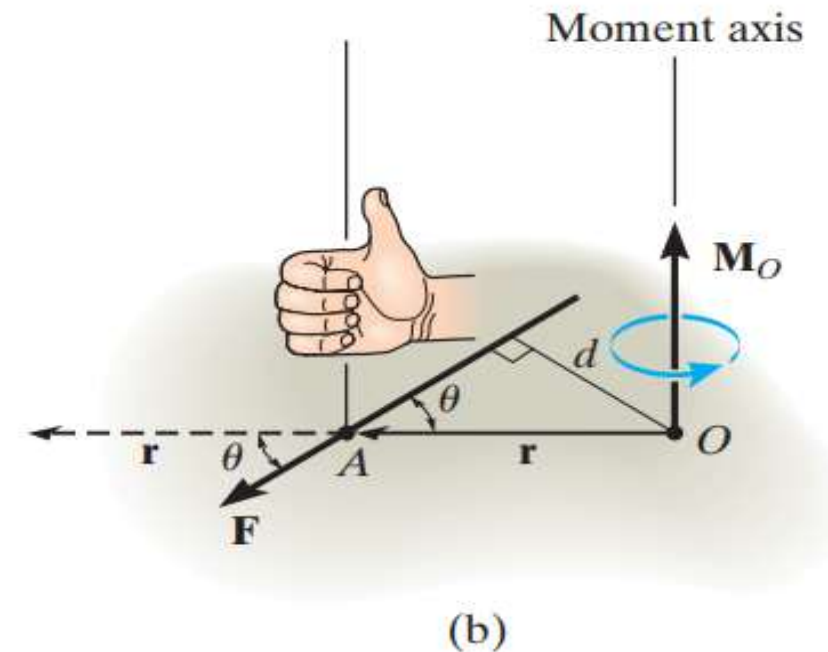
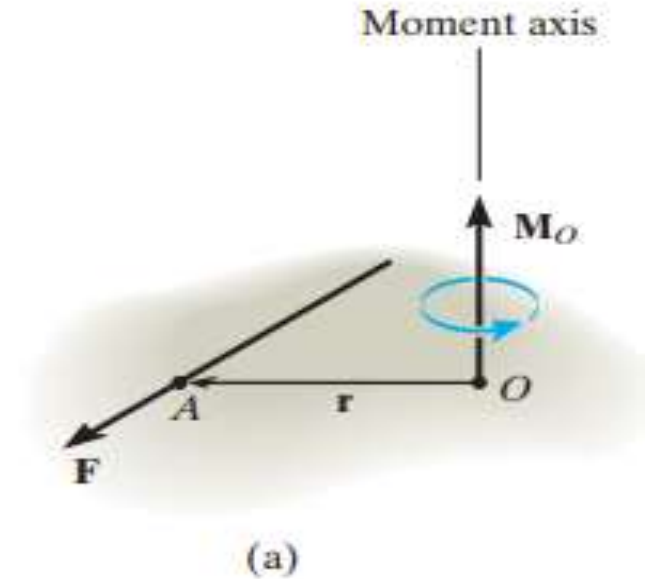
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Magnitude. where r is a position vector which runs from the moment reference point $\mathbf{O}$ to any point on the line of action of $\mathbf{F}$. The magnitude of this expression is given by defined from.

$$M_O = F (r \sin\theta) = F.d$$

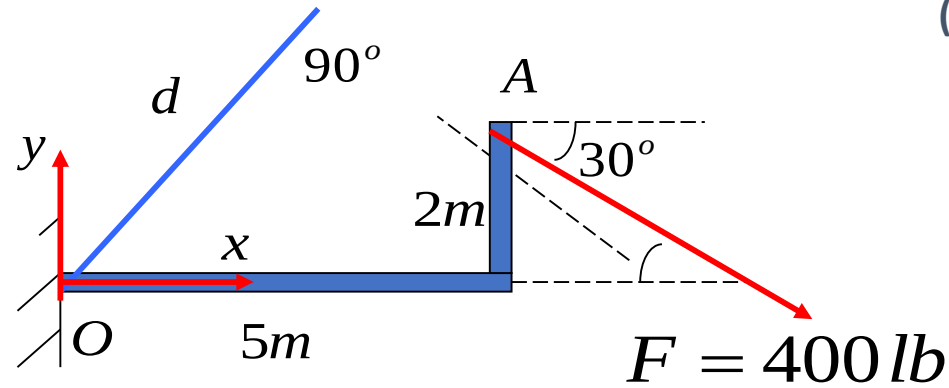
Direction.

The direction and sense are determined by the right-hand rule as it applies to the cross product.



The moment axis.

Example.2. Find the moment of the force shown about point O.



(1) By basic definition

$$d = (5 + 2 / \tan 30^\circ) \sin 30^\circ$$

$$= 4.23 \text{ (in m)}$$

$$\Rightarrow M_F^{(O)} = -Fd$$

Clockwise

$$= -(400)(4.23) = -1.69 \text{ (in kN-m)}$$

(2) By Cartesian components

$$\vec{F} = 400(\cos 30^\circ \vec{i} - \sin 30^\circ \vec{j})$$

$$\vec{r}_A = 5\vec{i} + 2\vec{j}$$

$$M_F^{(O)} = F_y x_A - F_x y_A$$

$$= (-400 \sin 30^\circ)(5) - (400 \cos 30^\circ)(2)$$

$$= -(1000 + 400\sqrt{3})$$

$$= -1690 \text{ (in N-m)}$$

(3) By the cross-product definition

$$\vec{F} = 400(\cos 30^\circ \vec{i} - \sin 30^\circ \vec{j}) \quad \vec{r}_A = 5\vec{i} + 2\vec{j}$$

$$\vec{M}_F^{(O)} = \vec{r}_A \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 400 \cos 30^\circ & -400 \sin 30^\circ & 0 \\ 5 & 2 & 0 \end{vmatrix}$$

$$= [(-400 \sin 30^\circ)(5) - (400 \cos 30^\circ)(2)]\vec{k}$$

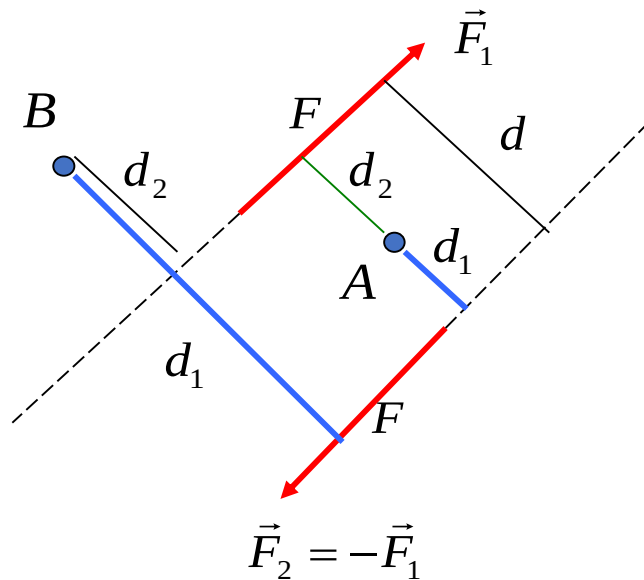
$$= -1690 \vec{k} \text{ (in N-m)}$$

21. Moment of a Couple

The moment produced by two equal, opposite, and noncollinear forces is called a couple. Couples have certain unique properties and have important applications in mechanics.

Couple ----- a pair of forces with the same magnitude and opposite direction

Moments of a Couple:



$$M = \pm Fd$$

Distance between lines of action of the couple of force

Moment of the couple

Sign convention

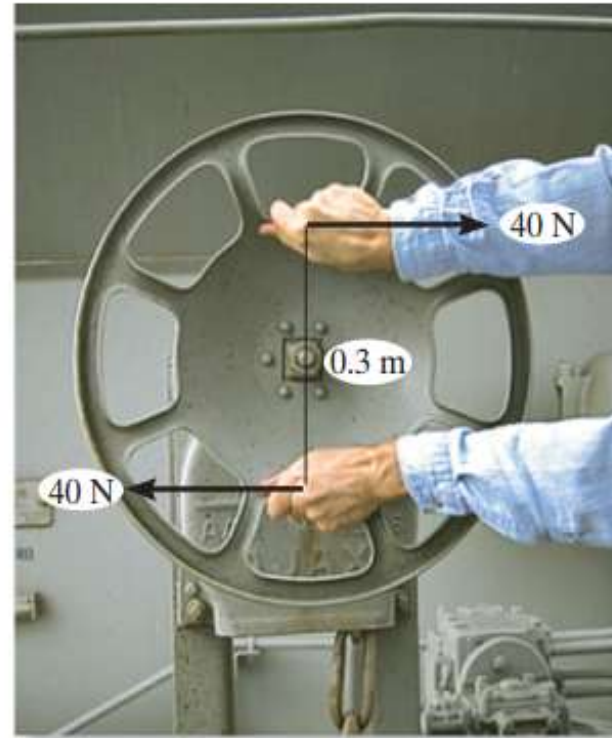
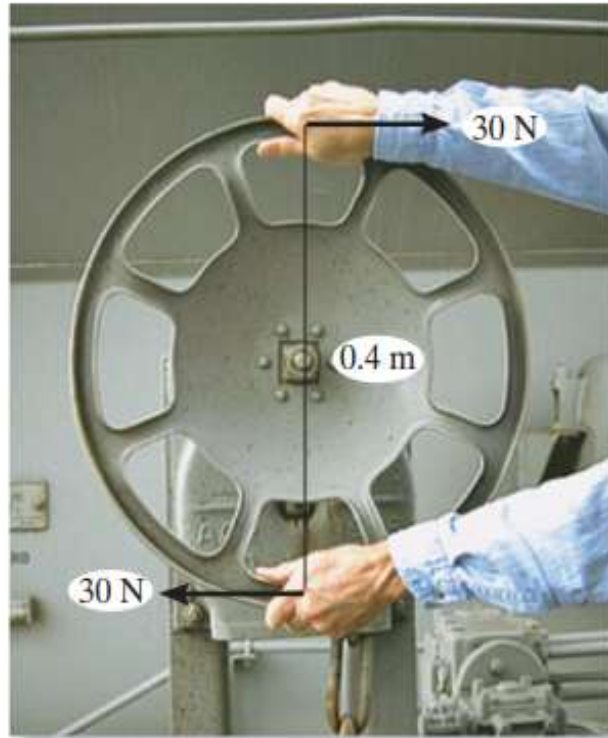
Magnitude of force

$$M^{(A)} = -Fd_1 - Fd_2 = -F(d_1 + d_2) = -Fd$$

$$M^{(B)} = -Fd_1 + Fd_2 = -F(d_1 - d_2) = -Fd$$

Moment of a couple about ANY point is the same!!

Equivalent Couples. If two couples produce a moment with the *same magnitude and direction*, then these two couples are *equivalent*. For example, the two couples shown in Fig. are *equivalent* because each couple moment has a magnitude of $M = 30 \text{ N}(0.4 \text{ m}) = 40 \text{ N}(0.3 \text{ m}) = 12 \text{ N.m}$.



Example.3.

Replace the two couples acting on the pipe column in Fig. *a* by a resultant couple moment.

Solution

The couple moment developed by the forces at *A* and *B*, can easily be determined from **a scalar formulation**

$$M_1 = F \cdot d = 150 \text{ N} (0.4 \text{ m}) = 60 \text{ N} \cdot \text{m}$$

By the right-hand rule, acts in the direction $+\mathbf{i}$, Fig. *b*. Hence, $\mathbf{M}_1 = 60\mathbf{i} \text{ N} \cdot \text{m}$

Vector analysis will be used to determine $\mathbf{M}_2$, caused by forces at *C* and *D*. If moments are computed about point *D*,

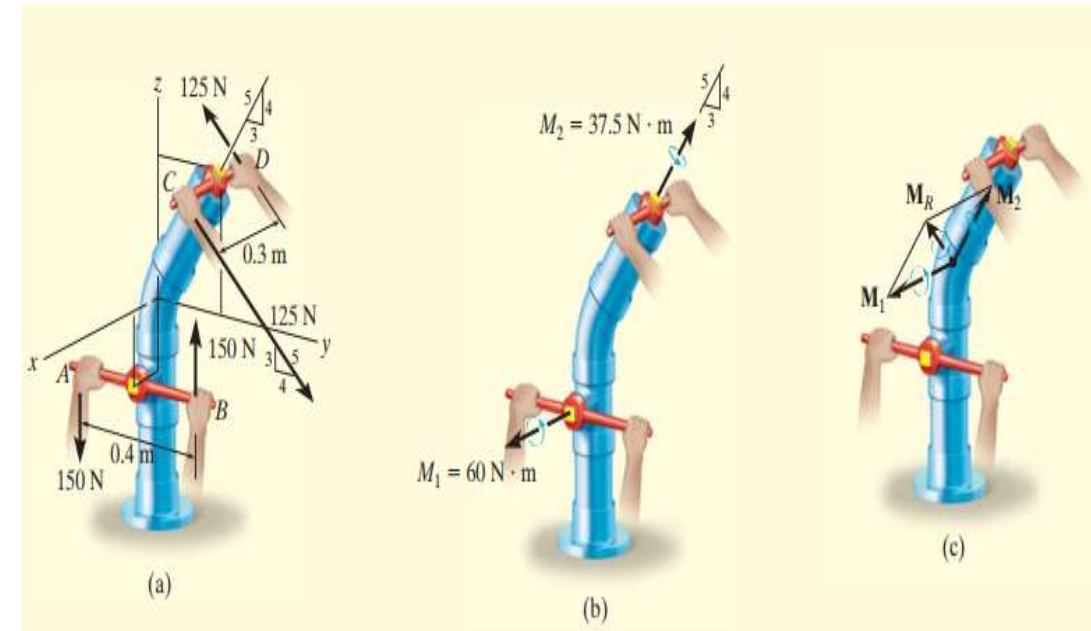
Fig.*b*,

$$\begin{aligned} \mathbf{M}_2 &= \mathbf{r}_{DC} \times \mathbf{F}_C \text{ then } \mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C = 0.3\mathbf{i} \times [125 (4/5) \mathbf{j} - 125 (3/5) \mathbf{k}] \\ &= 0.3\mathbf{i} \times [100\mathbf{j} - 75\mathbf{k}] = 30 (\mathbf{i} \times \mathbf{j}) - 22.5 (\mathbf{i} \times \mathbf{k}) \\ &= \{-22.5 \mathbf{j} + 30 \mathbf{k}\} \text{ N} \cdot \text{m} \end{aligned}$$

Since $\mathbf{M}_1$ and $\mathbf{M}_2$ are free vectors, they may be moved to some arbitrary point and added vectorially,

Fig. *c*. The resultant couple moment becomes

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = 60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k} \text{ N} \cdot \text{m}$$



22. Moments in Three Dimensions

Cartesian Vector Formulation. If we establish x, y, z coordinate axes, then the position vector $\mathbf{r}$ and force $\mathbf{F}$ can be expressed as Cartesian vectors, Fig.7.a. we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where r_x, r_y, r_z represent the x, y, z components of the position vector drawn from point O to *any point* on the line of action of the force F_x, F_y, F_z represent the x, y, z components of the force vector. If the determinant is expanded, we have

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} - (r_x F_z - r_z F_x)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

The scalar magnitudes of the moments of these forces about the positive x -, y -, and z -axes through O can be obtained from the moment arm rule, and are

$$M_x = r_y F_z - r_z F_y \quad M_y = r_z F_x - r_x F_z \quad M_z = r_x F_y - r_y F_x$$

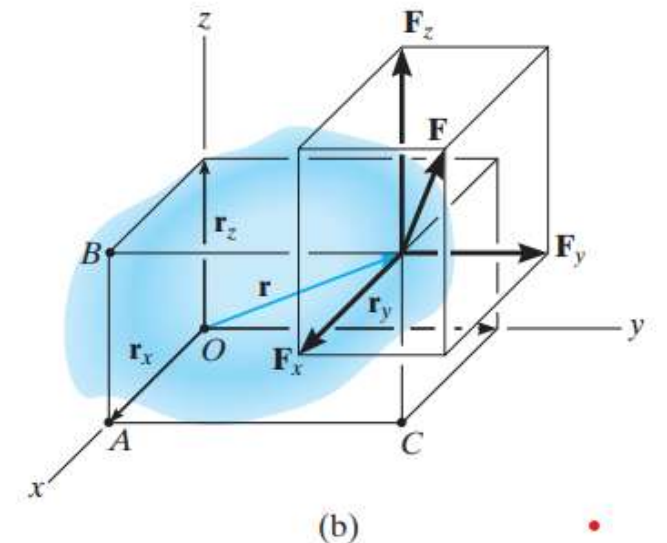
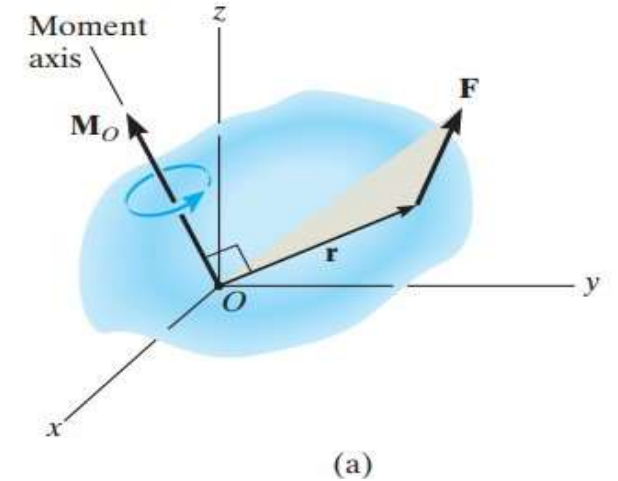


Fig.7. The moment in three dimensions.