

RATIONAL MECHANICS

Fundamentals of Statics and Engineering Mechanics

For Physics and Engineering Students

1-Rational Mechanics is the Branch of Science which deals with forces and their effects when the bodies are at rest or in Motion.

Statics

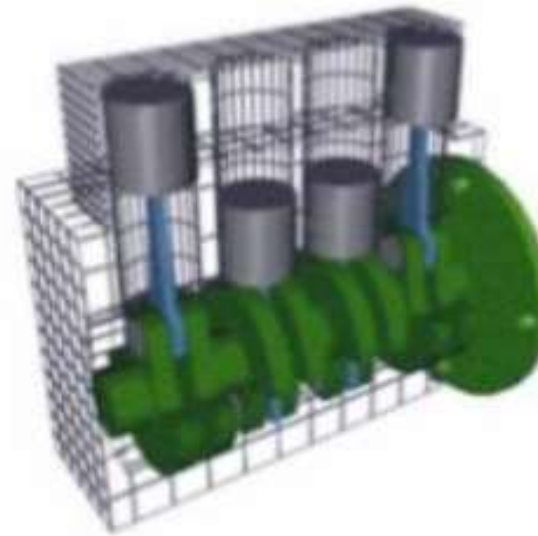
❖ Deals with forces and its effects when the body is at rest



Truss Bridge

Dynamics

❖ Deals with forces and its effects when the body is in moving condition



IC Engine

2. Fundamental Concepts and Principles

Particle: an idealized body whose dimensions are neglected.

Particle
Mass concentrated at a point

Rigid body: a body whose deformation is neglected in the analysis.

Rigid body
Distance between points
remains fixed

Force: an interaction capable of changing motion or producing deformation.

Equilibrium: the resultant force and resultant moment are both zero.

Equilibrium
No acceleration / no rotation

Newton's laws provide the physical basis for mechanics.

3. Scalars and Vectors

Scalar quantity: described by magnitude only.

Examples: mass, time, temperature, energy.

Vector quantity: has magnitude and direction.

Examples: force, velocity, acceleration, displacement.

- **Conversions**

$$1\text{ m} = 1000\text{ mm}$$

$$1\text{ mm} = 10^{-3}\text{ m}$$

$$1\text{ cm} = 10\text{ mm}$$

$$1\text{ mm} = 10^{-2}\text{ cm}$$

$$1\text{ m} = 100\text{ cm}$$

$$1\text{ cm} = 10^{-2}\text{ m}$$

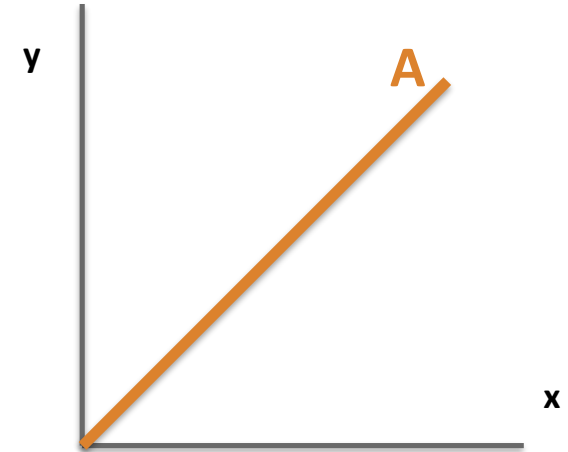
$$1\text{ MPa} = 10^6\text{ N/m}^2$$

$$1\text{ N/m}^2 = 10^{-6}\text{ N/mm}^2$$

$$1\text{ MPa} = 1\text{ N/mm}^2$$

$$1\text{ GPa} = 10^9\text{ N/m}^2$$

$$= 10^3\text{ N/mm}^2$$



$$|A| = \sqrt{A_x^2 + A_y^2}$$

4. Vector Operations

Vector addition is performed component by component.

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j}$$

Scalar multiplication changes the magnitude and may reverse direction.

$$\mathbf{A} \times \mathbf{B} = AB \sin\theta \mathbf{n}$$

Dot product produces a scalar: $\text{vec } \mathbf{A} \cdot \text{vec } \mathbf{B} = AB \cos \theta$.

Used for work and projection.

Cross product produces a vector: $\text{vec } \mathbf{A} \times \text{vec } \mathbf{B} = AB \sin \theta$.

Used for moments and rotational effects.

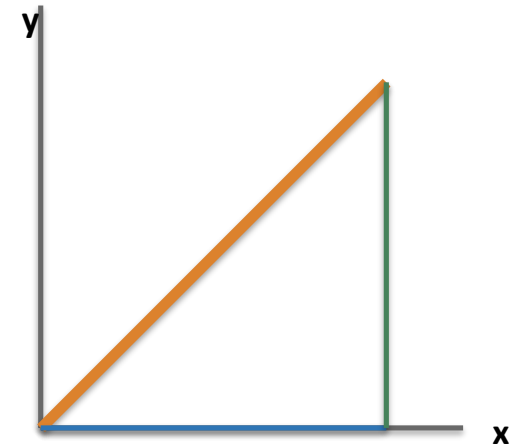
5. Coordinate Systems and Components

Cartesian coordinates are widely used in engineering mechanics.

A vector is resolved into mutually perpendicular components.

In 2D: $F_x = F \cos \theta$ and $F_y = F \sin \theta$.

In 3D, direction cosines describe the orientation of a force.



$$F_x = F \cos \theta \quad F_y = F \sin \theta$$

6. Force and Its Characteristics

A force is characterized by:

Magnitude — measured in newtons (N).

Direction — orientation of the force.

Sense — the direction along its line of action.

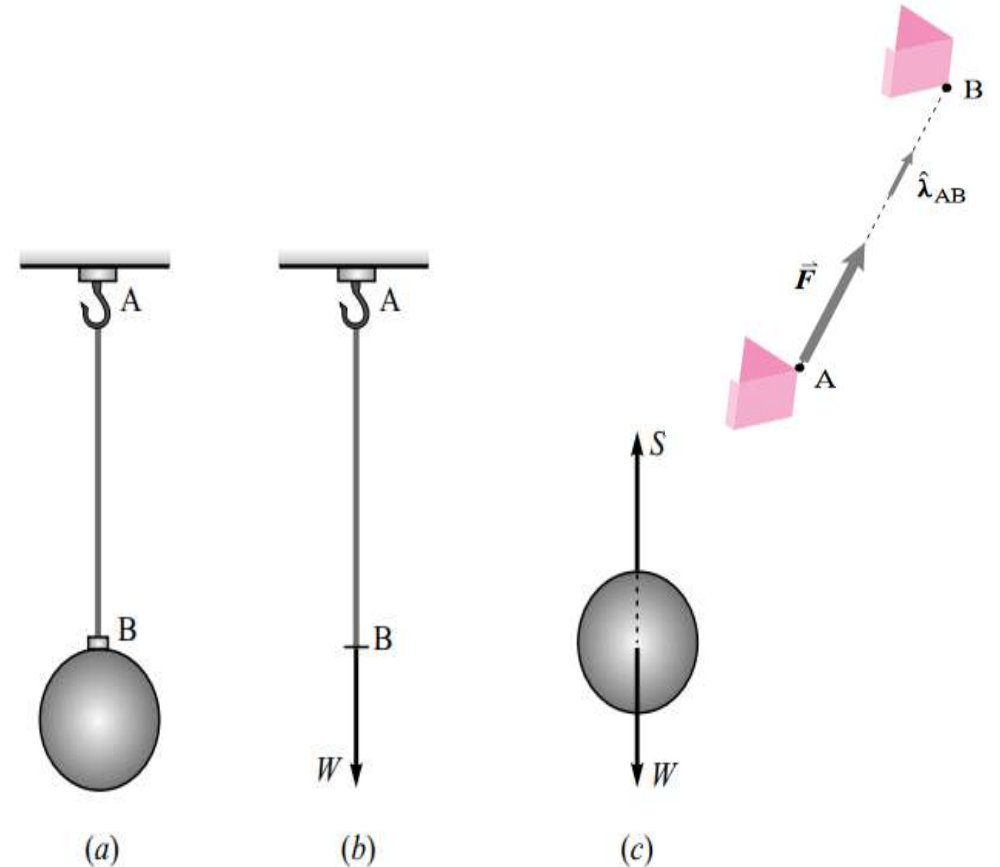
Point of application / line of action.

External forces may be concentrated or distributed.

Representation of force

Graphically a force may be represented by the segment of a straight line. In mechanics as in everyday experiences, forces are usually produced by the action of one body on another.

Since forces are vector quantities, they will be represented by bold-face letters such as **S** and **W**.



A ball of weight W hanging by a string.

External and Internal Effects

We can separate the action of a force on a body into two effects, **external** and **internal**. When external forces are applied to an object from outside, internal forces are induced in the object to counteract the externally applied forces and maintain equilibrium (Figure 11.(a)). Think of the forces you apply to stretch a rubber band. Your externally applied forces result in internal forces being set up in the rubber band. So when you let go of the band these internal forces pull the band back to its original unstretched state. Internal forces are produced in reaction to the application of external forces and so are often called *the reactive forces or reactions*.

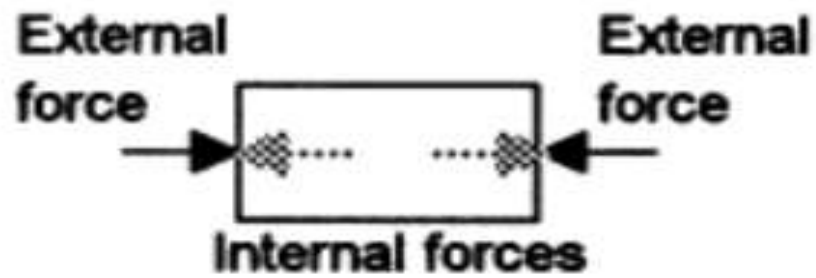
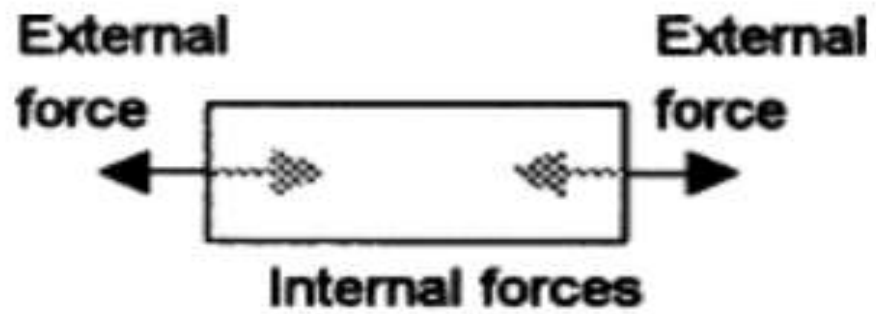
Division in the system

-Internal force:

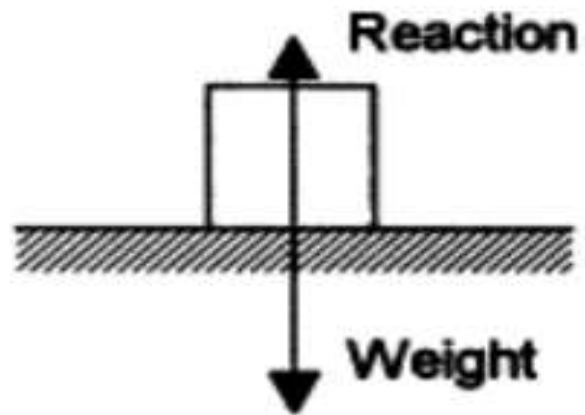
obtained by notionally cutting the body (Figure 1.(c)). This force acts between the parts of a body or system (normal force N , shear force Q and bending moment M).

-External force:

acts on a body from the outside (e.g. weight, wind pressure, snow load, adhesive force and support force).

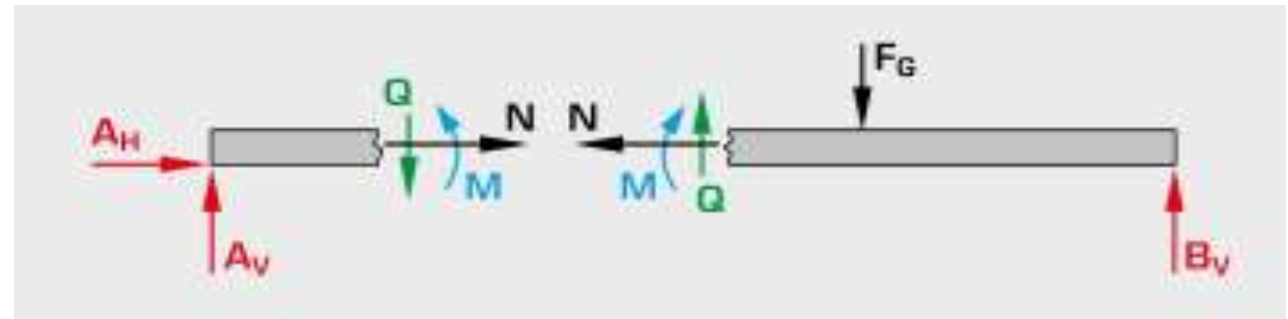


(a)



(b)

Fig.1



(c)

Types of forces acting on a body

Forces on a system can be applied either by direct physical contact or by remote action. According to Newton's third law, force applied is always accompanied by reactive force. Both applied force and reactive force may be either **concentrated** or **distributed**. We can be the following three types of forces acting on a body in equilibrium.

1. **Applied forces** There are the forces of push or pull applied externally to the body. Each force has got a point of contact with the body.
2. **Self –Weight** The weight $W (= mg)$ of the body acts vertically downward.
3. **Reactions** These are the forces developed by other bodies in contact with the body under consideration. If the body under equilibrium presses (called *action* force) against another body, it is also pressed (called *reaction* force) by the other body. As per Newton's third law, the reaction is equal and opposite to the action. The reactions adjust

themselves to keep the body in equilibrium. It is of utmost importance to correctly identify the action-reaction pair of forces. The reaction forces are supplied by ground, walls, rollers, knife-edges, cables pins, or other means. A roller or knife-edges support gives a reaction perpendicular to the member. However, a pin connection can give the reaction at any angle.

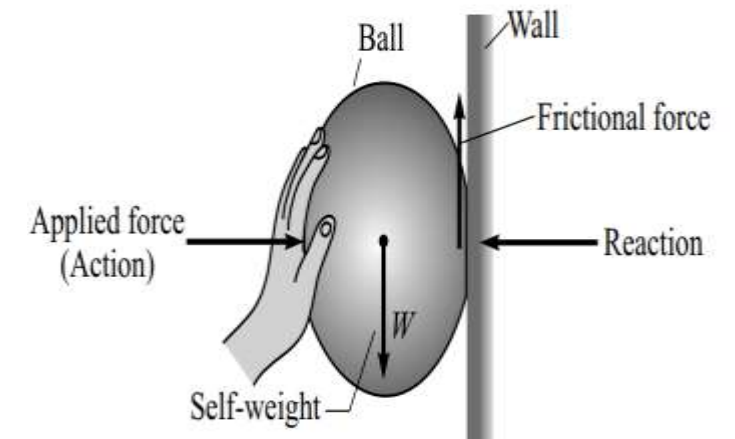


Fig.2. Types of forces acting on a body.

Force Classification

Forces are classified as either *contact* or *body* forces. A contact force is produced by direct physical contact; an example is the force exerted on a body by a supporting surface.

On the other hand, a body force is generated by virtue of the position of a body within a force field such as a gravitational, electric, or magnetic field. An example of a body force is your weight. Forces may be further classified as either ***concentrated or distributed***

Concentrated force/point load

when the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an *area*, as in the case of mechanical contact, over a *volume* when a body force such as weight is acting, or over a *line*, as in the case of the weight of a suspended cable.

Distributed force

Every contact force is actually applied over a finite area and is therefore really a distributed force (a).

Uniformly distributed force. This load is a system of forces having continuous and uniform distribution along a straight line (b).

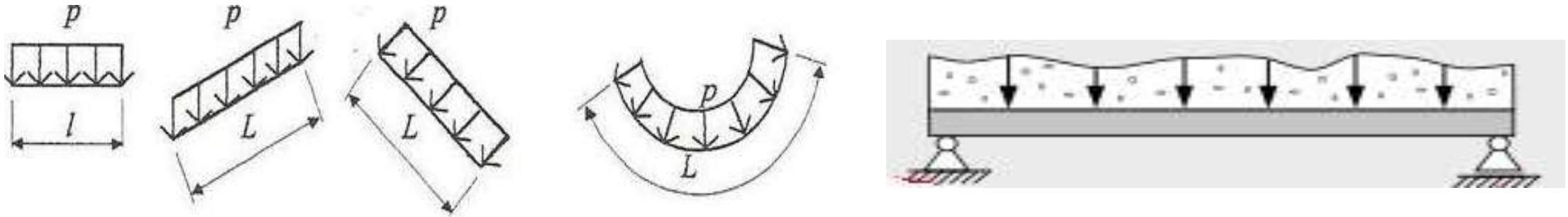
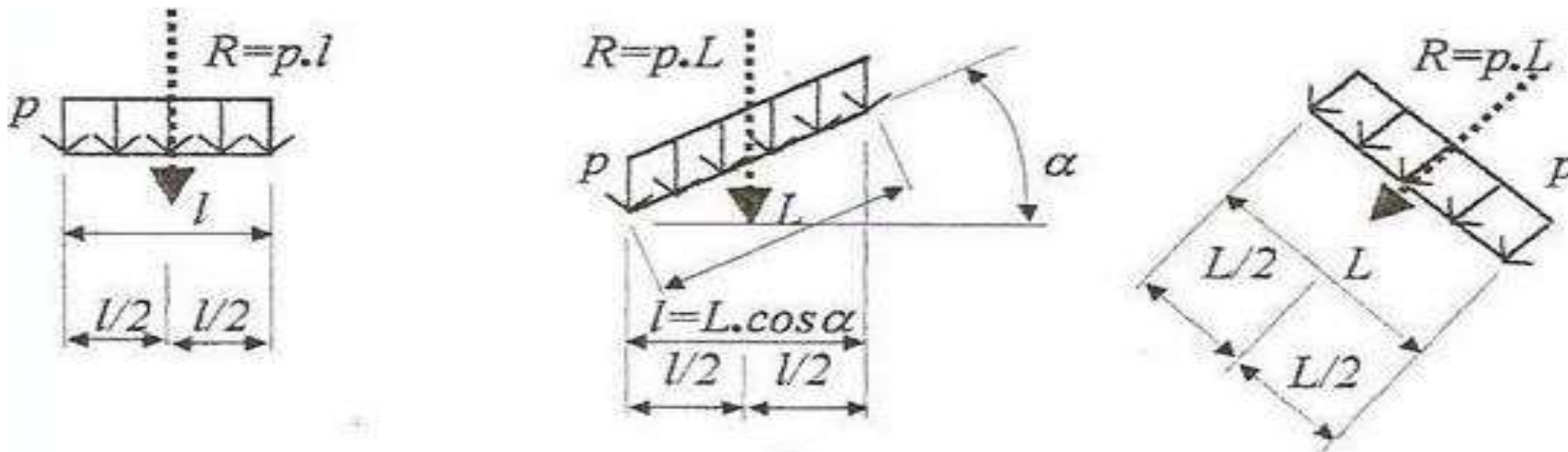


Fig.3. The uniformly distributed force .

Being a system of forces, the uniformly distributed force can be replaced with a resultant force having as magnitude the product between the intensity of it and the length of distribution: $R = p \cdot l$ or $R = p \cdot L$

The direction and the sense of the resultant is the same of the components of the system and the position is in the middle of the distribution length.



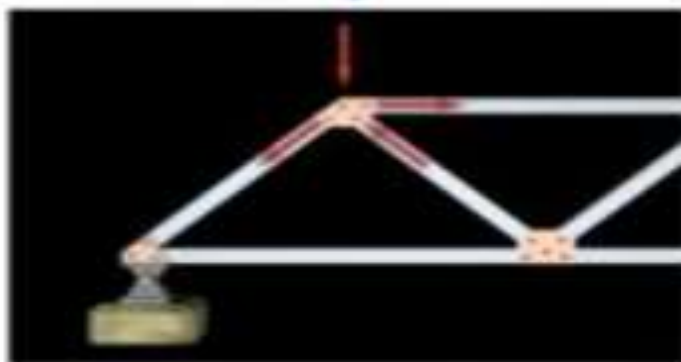
7. System of Forces

Coplanar forces: *The forces, whose lines of action lie on the **same plane**, are known as coplanar forces.*

- **Collinear forces:** *The forces, whose lines of action lie on **the same line**, are known as collinear forces.*
- **Concurrent forces:** The forces, **which meet at one point**, are known as concurrent forces. The concurrent forces may or may not be collinear.
- **Coplanar concurrent forces:** The forces, which meet at one point and their line of action also lay on the same plane, are known as coplanar concurrent forces.
- **Coplanar non-concurrent forces:** The forces, which do not meet at one point, but their lines of action lie on the same, are known as coplanar non-concurrent forces.
- **Non-Coplanar concurrent forces:** The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.
- **Non-Coplanar non-concurrent forces:** *The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar nonconcurrent force.*

Types of forces

Concurrent coplanar forces



Collinear forces

Non Concurrent coplanar
(Parallel)



Concurrent non-coplanar



8. Characteristics of the Forces Systems

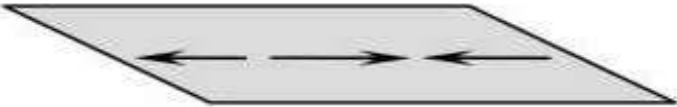
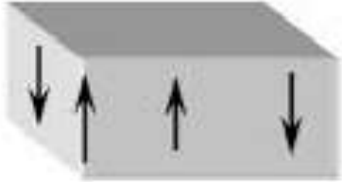
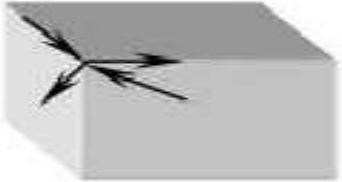
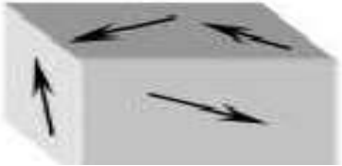
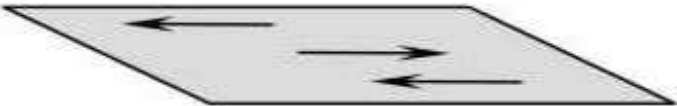
DESCRIPTION	COPLANAR FORCES	FORCES IN SPACES
i) Collinear Forces		No to possible have   
ii) Parallel Forces		
ii) Concurrent Forces		
iv) Non-Concurrent Forces and non-Parallel Forces		

Fig.5 Graphical representation of various system of forces.

9.The resultant of system of forces

The resultant of addition of two forces depends not only on their magnitudes, but also on their directions.

The resultant action of a group or system of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduce the system to its simplest form to describe its action. The resultant of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Resultant of Coplanar concurrent forces

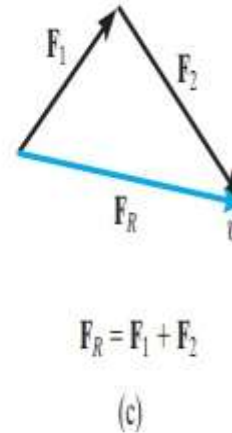
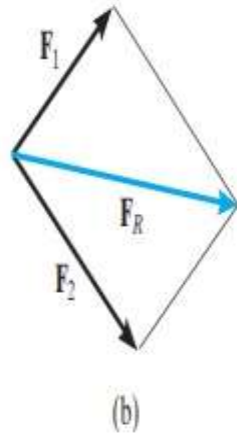
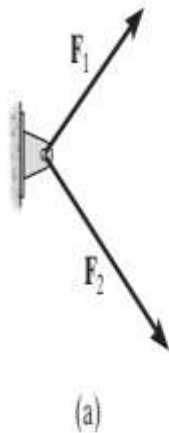
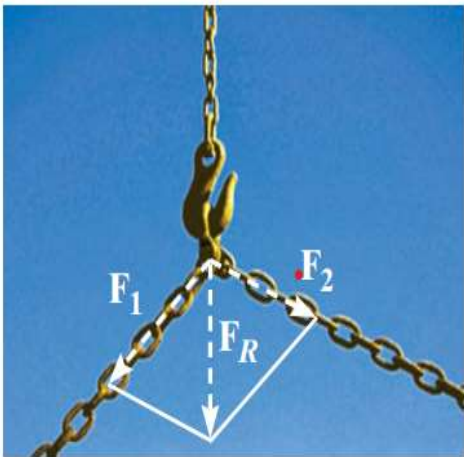
Various methods are employed to determine the resultant of concurrent forces in a plane. They are described below :

- a) Graphical methods: Parallelogram law, triangle law and polygon law;
- b) Trigonometric methods: Cosine law and sine law;
- c) Analytical : Vector approach.

10.Resultant force & Parallelogram law of forces

Parallelogram law

The two component forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the pin in Fig.6a can be added together to form the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$, as shown in Fig.6.b. From this construction, or using the triangle rule, Fig. c, we can apply **the law of cosines** or **the law of sines** to the triangle in order to obtain the magnitude of the resultant force and its direction.



$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$
$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

Where, R = Resultant force
 θ = angle between P and Q
 α = angle between P and R

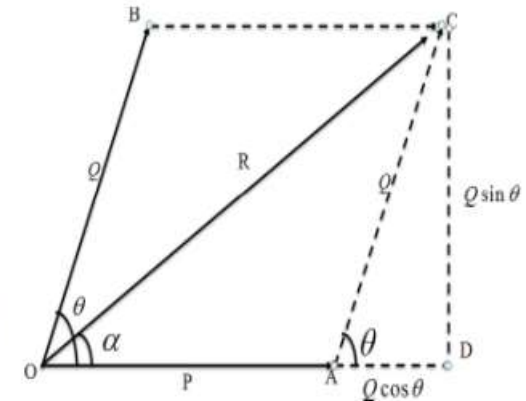
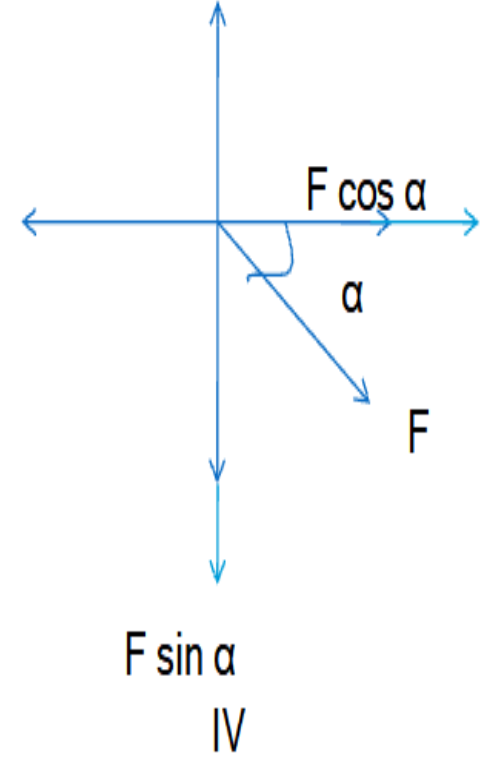
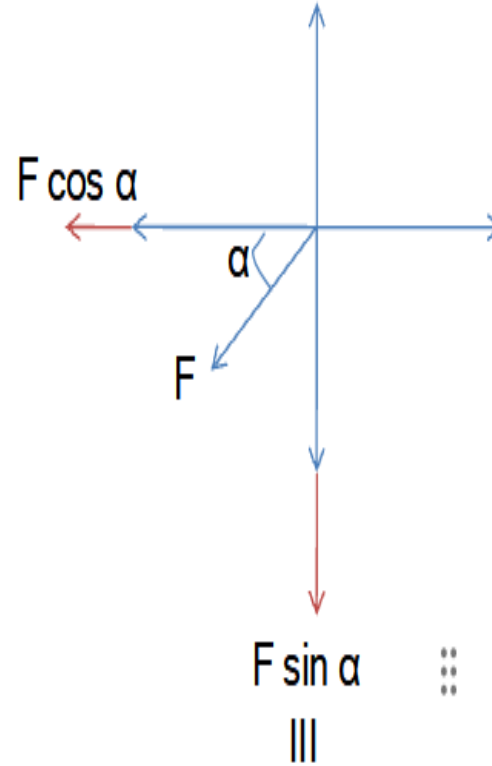
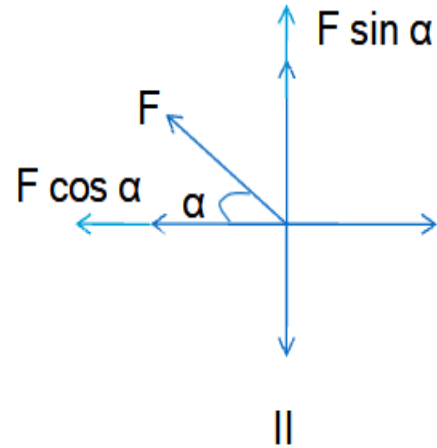
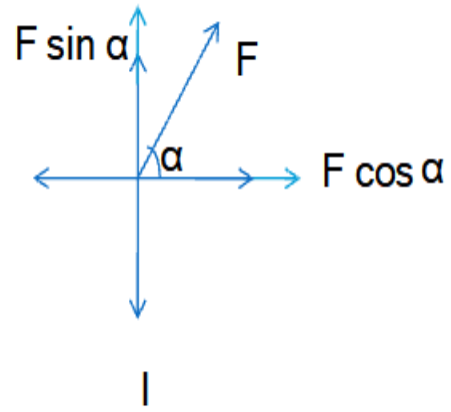
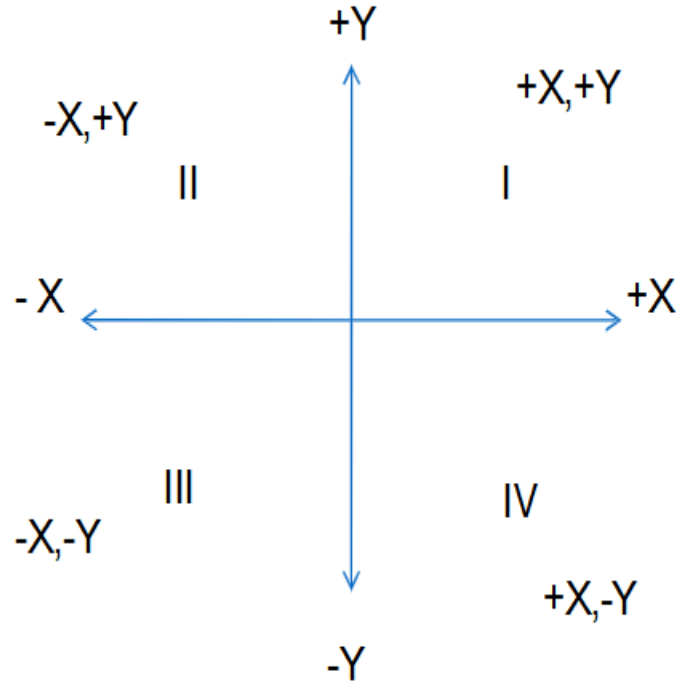


Fig.4 The parallelogram law must be used to determine the resultant of the two forces acting on the hook.

11.Resolution of forces



12.Two-Dimensional Force Systems

The Components of a Force

Sometimes it is necessary to resolve a force into **two components** in order to study its pulling or pushing effect in two specific directions. For example, in Fig.6.a, $\mathbf{F}$ is to be resolved into two components along the two members, defined by the u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first.

The force components $\mathbf{F}_u$ and $\mathbf{F}_v$ are then established by simply joining the tail of $\mathbf{F}$ to the intersection points on the u and v axes, Fig.6.b. This parallelogram can then be reduced to a triangle, which represents the triangle rule, Fig.6.c. From this, the law of sines can then be applied to determine the unknown magnitudes of the components.

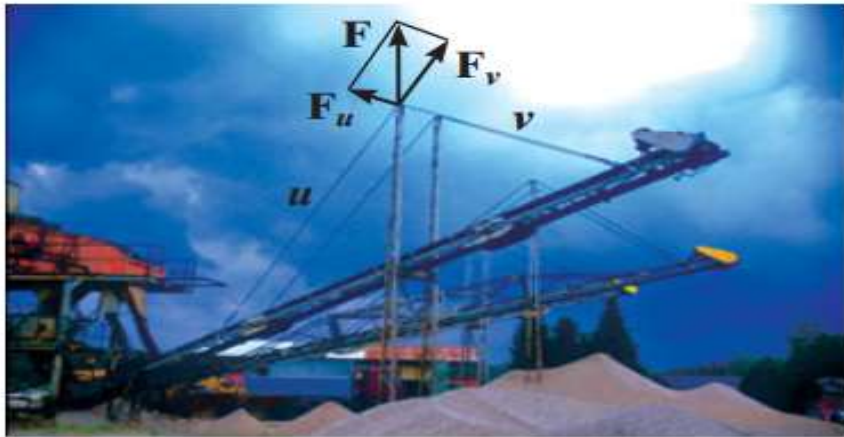
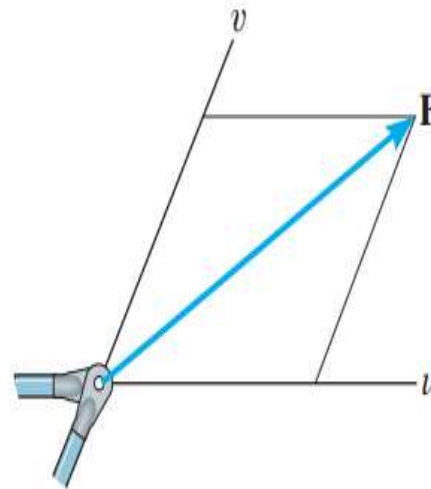
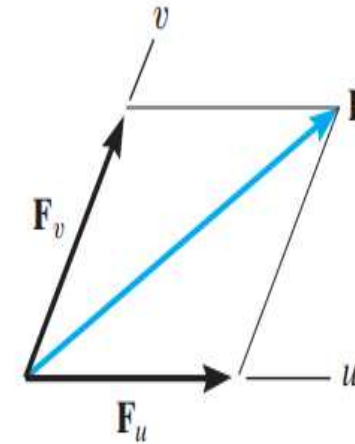


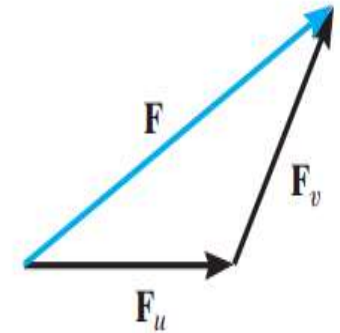
Fig.6 Using the parallelogram law force $\mathbf{F}$ caused by the vertical member can be resolved into components acting along the suspension cables a and b .



(a)



(b)



(c)

13. Resultant of a System of Forces

The resultant force is the vector sum of all applied forces.

In Cartesian form:

Equilibrium requires the resultant force to be zero.

Graphically, forces can be added head-to-tail.

$$\mathbf{R} = \Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j}$$

$$R_x = \Sigma F_x, \quad R_y = \Sigma F_y$$

14.Rectangular Components

Analytical method

Decomposition of two forces

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector $\mathbf{F}$ of Fig.7. may be written as: $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of $\mathbf{F}$ in the x and y directions.

$\mathbf{F}_x = F_x \mathbf{i}$ and $\mathbf{F}_y = F_y \mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

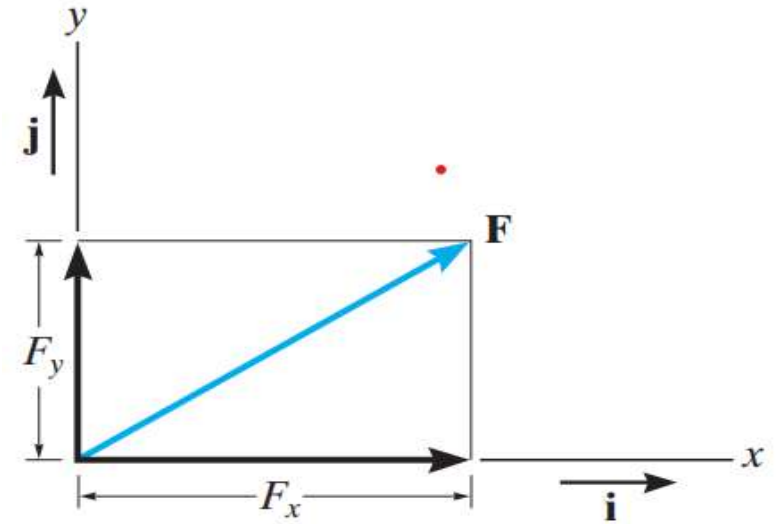
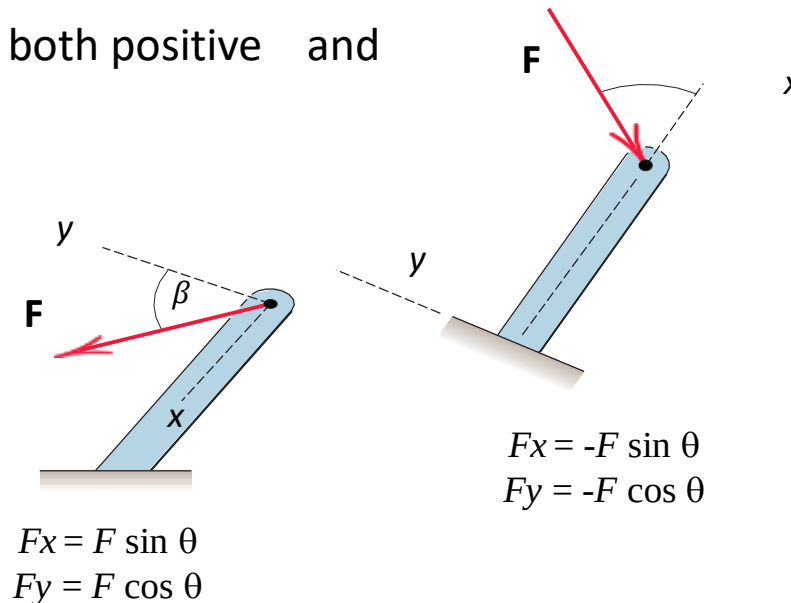


Fig.7.

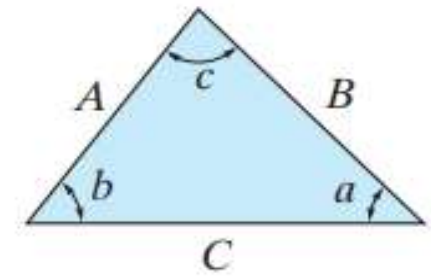
For the force vector of Fig.7 the x and y scalar components are both positive and are related to the magnitude and direction of $\mathbf{F}$ by

$$\begin{aligned} F_x &= F \cos \theta & F &= \sqrt{F_x^2 + F_y^2} \\ F_y &= F \sin \theta & \theta &= \tan^{-1} \frac{F_y}{F_x} \end{aligned}$$



Trigonometry.

- Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.
- From this triangle, the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig.7.



Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Fig.7.

The polygon Law

To add more than two vectors, the parallelogram can be used by adding two vectors at a time and then continuing the process. Fig.5 shows the process of adding four P, Q, R, R_f.

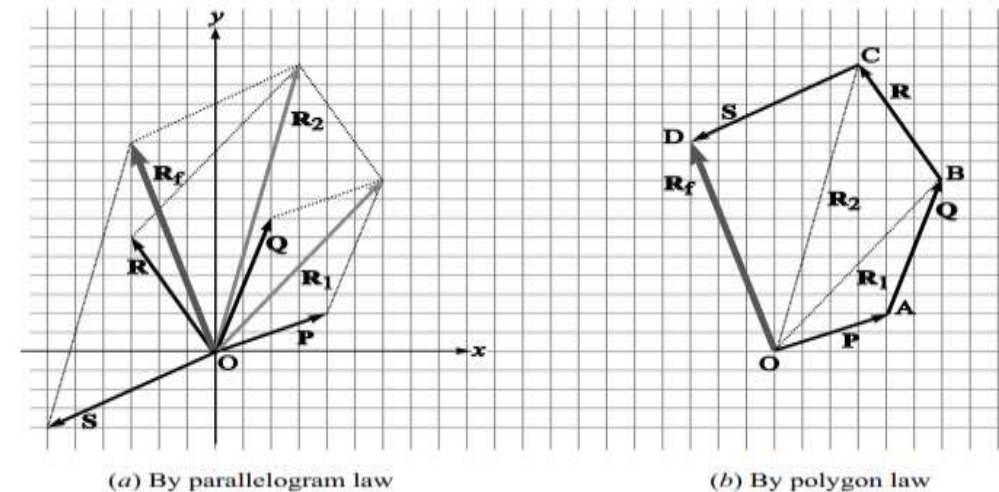
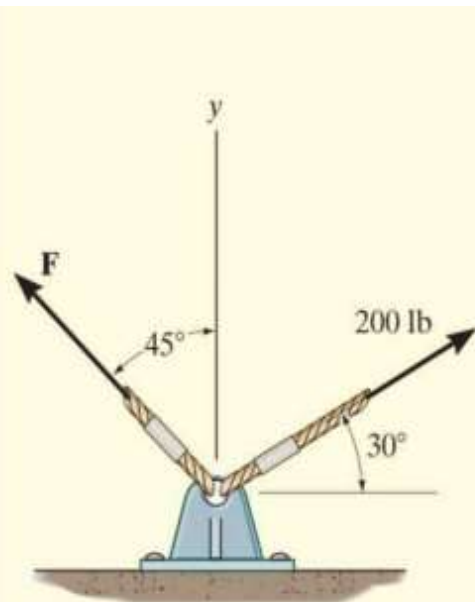


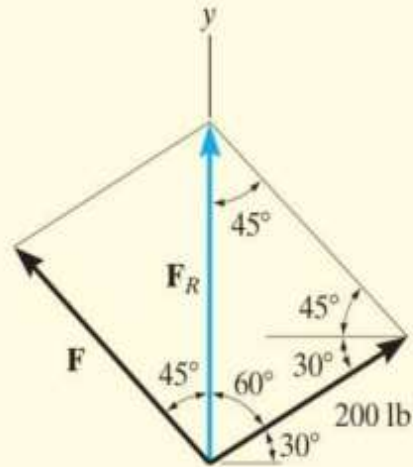
Fig.8. Addition of more than two vectors.

Example 2

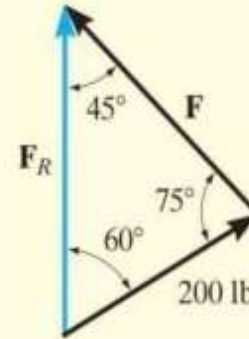
Determine the magnitude of the component force F in Figure.9 and the magnitude of the resultant force F_R if F_R is directed along the positive y axis.



(a)



(b)



(c)

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb}$$

15.Coplanar Force Resultants.

- We can use either of the two methods just described to determine the resultant of several *coplanar forces*.

$$\mathbf{F}_1 = F_{1x} \mathbf{i} + F_{1y} \mathbf{j}$$

$$\mathbf{F}_2 = -F_{2x} \mathbf{i} + F_{2y} \mathbf{j}$$

$$\mathbf{F}_3 = F_{3x} \mathbf{i} - F_{3y} \mathbf{j}$$

- The vector resultant is therefore

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= F_{1x} \mathbf{i} + F_{1y} \mathbf{j} - F_{2x} \mathbf{i} + F_{2y} \mathbf{j} + F_{3x} \mathbf{i} - F_{3y} \mathbf{j}$$

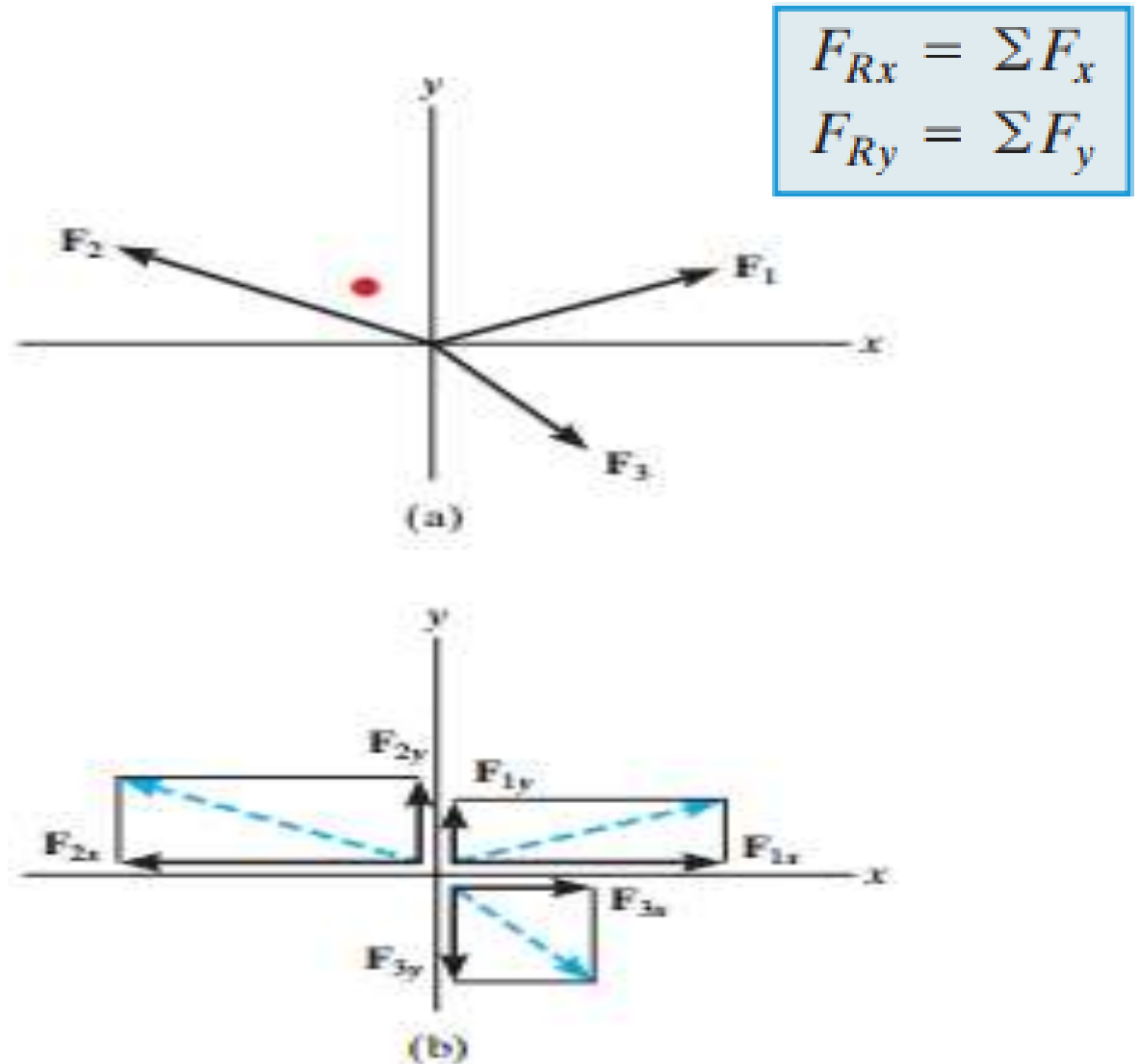
$$= (F_{1x} - F_{2x} + F_{3x}) \mathbf{i} + (F_{1y} + F_{2y} - F_{3y}) \mathbf{j}$$

$$= (F_{Rx}) \mathbf{i} + (F_{Ry}) \mathbf{j}$$

- If *scalar notation* is used, then we have

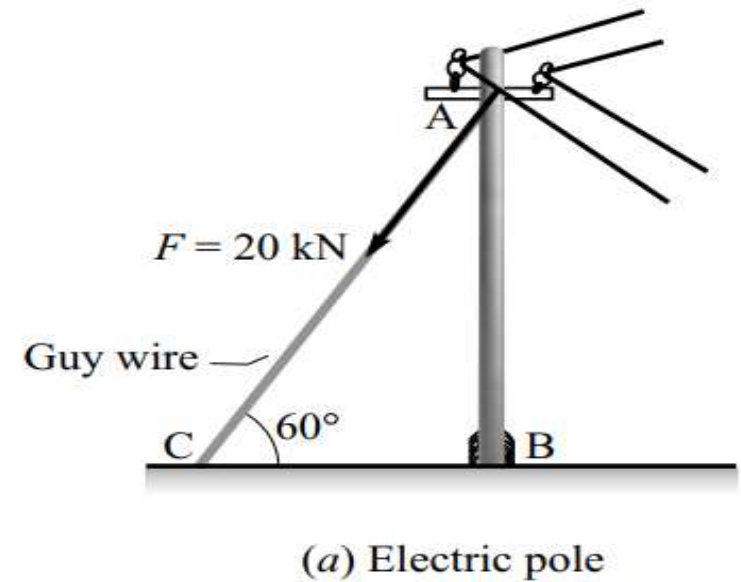
$$F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$

$$F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$



Example.3:

To maintain an electric pole AB upright, a guy wire AC is tied to it, as shown in figure 9. The guy wire makes an angle of 60° to the horizontal and exerts a force of 20 kN on the pole. Find the horizontal and vertical components of this force.

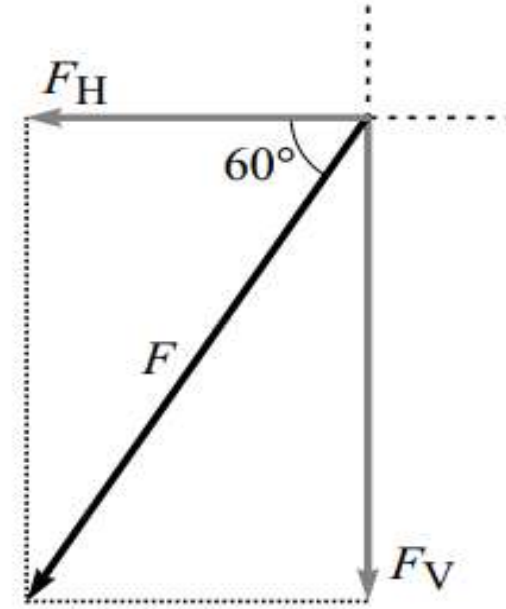


Solution: The resolution of the force F exerted by the guy wire is shown in

figure.(b).The two components are given as:

$$F_H = F \cos 60^\circ = 20 \cos 60^\circ = 10 \text{ kN.}$$

$$F_V = F \sin 60^\circ = 20 \sin 60^\circ = 17.32 \text{ kN.}$$



(b) Resolution of force F

16. Resultant of a System of Forces

- The resultant force is the vector sum of all applied forces.

In Cartesian form:

- Equilibrium requires the resultant force to be zero.
- Graphically, forces can be added head-to-tail.

$$\mathbf{R} = \Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k}$$

$$R_x = \Sigma F_x, \quad R_y = \Sigma F_y, \quad R_z = \Sigma F_z$$

17.Three-Dimensional Force Systems

Concurrent force in space

Rectangular Components

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force $\mathbf{F}$ acting at point $\mathbf{O}$ in Fig.7 has the rectangular components F_x , F_y , F_z , where

$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

$$\mathbf{F} = F(\cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k})$$

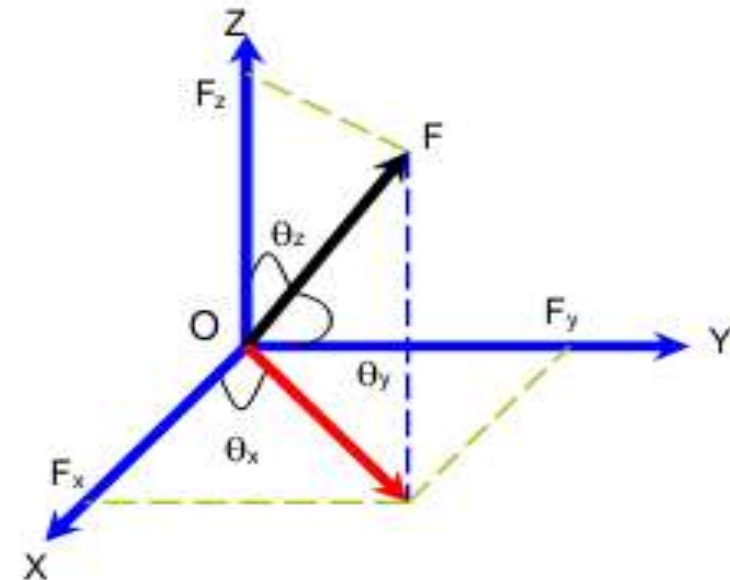
•Cartesian Vectors

The unit vector $\mathbf{u}$ has a length of one, no units, and it points in the direction of the vector $\mathbf{F}$.

A force can be resolved into its Cartesian components along the x , y , z axes so that

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$\|\vec{F}\| = F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



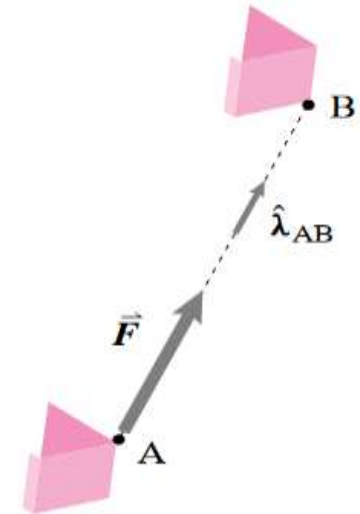
- To find the resultant of concurrent force system, express each force as a Cartesian vector and add the i, j, k components of all the forces in the system.
- If the line of action of a force passes through points A and B , then the force acts in the same direction as the position vector $\vec{r}_{AB}$. Which is defined by the unit vector $\hat{\lambda}_{AB}$. The force can then be expressed as a Cartesian.
- An easy way to find a unit vector in the direction of a vector $\vec{A}$ is to divide $\vec{A}$ by its magnitude. Thus is a unit vector in the direction $\vec{A}$

$$\hat{\lambda}_A \equiv \frac{\vec{A}}{|\vec{A}|}$$

A common situation is to know that a force $\vec{F}$ is a yet unknown scalar F multiplied by a unit vector pointing between known points A and B . (fig.10).

We can then write as

$$\vec{F} = F\hat{\lambda}_{AB} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = F \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = F \left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}} \right)$$

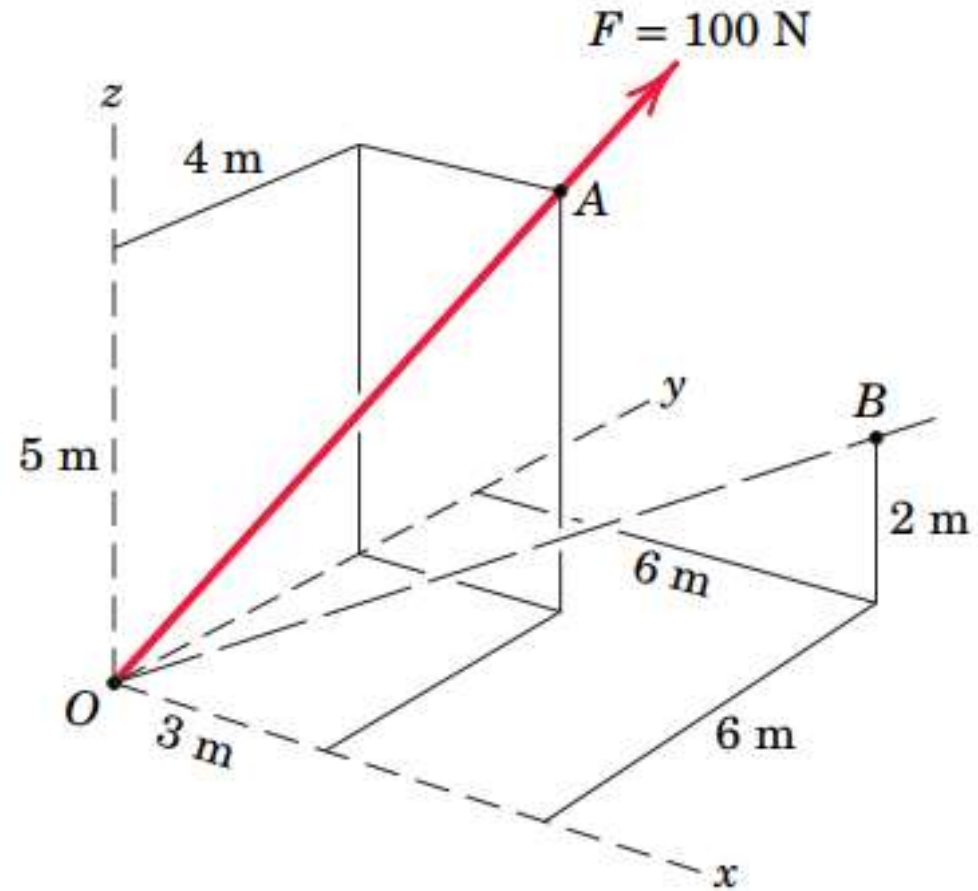


Example 3:

(a) Determine the x , y and z scalar components of the projection F_{xy} of $\mathbf{F}$ on the x - y plane.

(b) Projection F_{xy} of $\mathbf{F}$ on the x - y plane.

(c) Projection F_{OB} of $\mathbf{F}$ along the line OB .



Solution Part (a). We begin by writing the force vector $\mathbf{F}$ as its magnitude F times a unit vector $\mathbf{n}_{OA}$.

$$\begin{aligned}\mathbf{F} &= F\mathbf{n}_{OA} = F \frac{\vec{OA}}{OA} = 100 \left[\frac{3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}}{\sqrt{3^2 + 4^2 + 5^2}} \right] \\ &= 100[0.424\mathbf{i} + 0.566\mathbf{j} + 0.707\mathbf{k}] \\ &= 42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k} \text{ N}\end{aligned}$$

The desired scalar components are thus

$$F_x = 42.4 \text{ N} \quad F_y = 56.6 \text{ N} \quad F_z = 70.7 \text{ N} \quad \textcircled{2} \quad \text{Ans.}$$

Part (b). The cosine of the angle θ_{xy} between $\mathbf{F}$ and the x - y plane is

$$\cos \theta_{xy} = \frac{\sqrt{3^2 + 4^2}}{\sqrt{3^2 + 4^2 + 5^2}} = 0.707$$

so that $F_{xy} = F \cos \theta_{xy} = 100(0.707) = 70.7 \text{ N}$ Ans.

Part (c). The unit vector $\mathbf{n}_{OB}$ along OB is

$$\mathbf{n}_{OB} = \frac{\vec{OB}}{OB} = \frac{6\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}}{\sqrt{6^2 + 6^2 + 2^2}} = 0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}$$

The scalar projection of $\mathbf{F}$ on OB is

$$\begin{aligned}F_{OB} &= \mathbf{F} \cdot \mathbf{n}_{OB} = (42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k}) \cdot (0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}) \quad \textcircled{2} \\ &= (42.4)(0.688) + (56.6)(0.688) + (70.7)(0.229) \\ &= 84.4 \text{ N}\end{aligned} \quad \text{Ans.}$$

If we wish to express the projection as a vector, we write

$$\begin{aligned}\mathbf{F}_{OB} &= \mathbf{F} \cdot \mathbf{n}_{OB} \mathbf{n}_{OB} \\ &= 84.4(0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}) \\ &= 58.1\mathbf{i} + 58.1\mathbf{j} + 19.35\mathbf{k} \text{ N}\end{aligned}$$

