**University Center Abdelhafid Boussouf-Mila-**
Institute of Science and Technology
1st year ST+ENG

Algebra 2
2025/2026

Exam duration: 1 h 15 m

# Assessment Exam (T02)

**Exercise 1 : (5.5pts)**

Consider the following matrices :

$$\boldsymbol{A} = \begin{pmatrix} -2 & 1 & 1 \\ -3 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

1. Show that for any sequare matrix $\boldsymbol{Q}$, $\det(\boldsymbol{Q}^{-1}) = \frac{1}{\det(\boldsymbol{Q})}$
2. By using row reduction method compute $det(\boldsymbol{A})$.
3. Is the matrix $\boldsymbol{B}$ diagonal or triangular?
4. Calculate : $\det(\boldsymbol{A}^{-1})$, $\det(\boldsymbol{B})$, $\boldsymbol{AB}$, $\boldsymbol{A}^2$ and $\boldsymbol{B}^2 - k\boldsymbol{I}$.
5. Determine $\alpha$ and $\beta$ such that $\boldsymbol{B}^2 = \alpha\boldsymbol{B} + \beta\boldsymbol{I}$ .

**Exercise 2 : (5 pts)**

Let $T$ be a linear transformation defined by :

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$T(x, \quad y, \quad z) = (x - 2y, \quad -x + ay + z, \quad 2x - y + az), \quad a \in \mathbb{R}$$

1. Determine $\boldsymbol{A}$ the matrix representation of $\boldsymbol{T}$
2. Show that : $\boldsymbol{T}$ is not bijective $\Longrightarrow (a = 3) \vee (a = -1)$
3. Determine $\boldsymbol{A}^{-1}$ for $a = 2$, and deduce $\boldsymbol{T}^{-1}$
4. Determine $\delta$ such that $\boldsymbol{A}\begin{pmatrix} \delta \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$