# Tutorial Worksheet No.3

## Exercise 1.

1. Find the eigenvalues and the eigenvectors of the following matrices :
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix},\ B = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix},\ \text{and } C = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$$
2. Determine the characteristic polynomial $P(\lambda)$ of $A$ and $C$. Then compute $P(A)$ and $P(C)$, and state your conclusion.

## Exercise 2.

Consider the matrix :
$$A = \begin{pmatrix} 3 & -2 & 1 \\ 0 & -3 & 3 \\ 1 & 1 & 3 \end{pmatrix}$$
1. Find the eigenvalues of $A$, and deduce $det(A)$.
2. Find the eigenvalues of $A^{-1}$ and $A^5$.

## Exercise 3.

A) Consider two ordered bases of the vector space $\mathbb{R}^3$

$$\mathcal{B} = \{(1,0,0),(0,1,0),(0,0,1)\}$$
$$\mathcal{B}' = \{(1,-2,0),(1,8,5),(1,8,-2)\}$$
1. Determine the matrix $\mathcal{S}$ that describes the change of basis $\mathcal{B}' \to \mathcal{B}$.
2. Compare the matrix $\mathcal{S}$ with the eigenvectors matrix of
$$C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 8 \\ 2 & 1 & 4 \end{pmatrix}$$

B) Consider the following linear system written relative to the basis $\mathcal{B}$
$$C \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$
1. Rewrite this system in terms of the basis $\mathcal{B}'$.
2. Solve this system.

## Exercise 4.

1. Are the following matrices diagonalizable ?
$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 13 \\ 2 & 1 & 3 \end{pmatrix}, \; B = \begin{pmatrix} 5 & 0 & 3 \\ 0 & 6 & 0 \\ 0 & 3 & 5 \end{pmatrix}$$
2. Calculate $A^{100}$ and $det(A^{10})$.

## Exercise 5.

Consider the system of linear differential equations :

$$\begin{cases} \dfrac{dx}{dt} = -x + 2y \\ \dfrac{dy}{dt} = x - 2y \end{cases}$$

1. Write this system on the form $\dfrac{d}{dt}u = Au$, where $u = \begin{pmatrix} x \\ y \end{pmatrix}$ and $A$ is a $2 \times 2$ matrix.
2. Find the eigenvalues and the eigenvectors of $A$
3. Solve this system subject to the initial condition $u(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

## Exercise 6 (Home Work).

Consider the matrix :

$$A = \begin{pmatrix} 5 & 0 & 3 \\ 0 & 6 & 2 \\ 0 & 3 & 5 \end{pmatrix}$$

1. Find the eigenvlaues and eigenvectors of $A$.
2. Is $A$ diagnalizable. If yes, calculate $A^{120}$.
3. Find the eigenvlaues and eigenvectors of $A - 5I$.