Tutorial Worksheet No.2

Exercise 1.

Which of the following systems is consistent or inconsistent

$$S_1 = \begin{cases} -w_1 + 2w_2 + w_3 = 2 \\ w_1 - 2w_3 = 1 \end{cases} \quad S_2 = \begin{cases} -r_1 + 2r_2 + r_3 = 2 \\ r_1 - 2r_2 - r_3 = 1 \end{cases} \quad S_3 = \begin{cases} x_1 + 3x_2 - 2x_3 = 2 \\ \frac{1}{2}x_1 + 4x_2 + 8x_3 = 3 \\ 2x_1 - 2x_2 - x_3 = 1 \end{cases}$$

$$S_4 = \begin{cases} 3x + 2y + z = 2 \\ -x - 3y + 2z = 1 \\ 2x - y + 3z = 3 \end{cases}$$

Exercise 2.

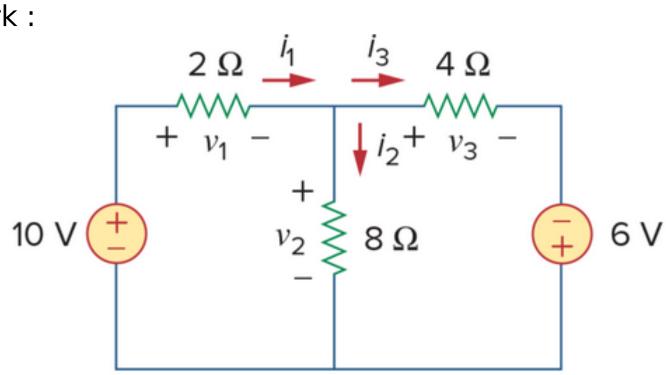
Consider the linear system :

$$\begin{aligned} bx_1 + 3(b-1)x_2 - 3x_3 &= 1 \\ x_1 + 2x_2 - 2bx_3 &= 5 \\ 3x_1 - 2x_2 - x_3 &= 2 \end{aligned}$$

1. For what value of b does the system have a unique solution ?
2. Using Cramer's Rule, solve the system for $b = 1$.

Exercise 3.

Consider the following electrical network :



- By using Kirchhoff's point and loop rules determine $i_1,\ i_2$ and i_3

Exercise 4.

Let T be a linear transformation defined by

$$\begin{aligned} T:\ & \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \\ & (x,y,z) \longrightarrow (\kappa(x+y),\ (\kappa-1)(y+z),\ 2z+3x) \end{aligned}$$

1. Determine the matrix representation A of the linear transformation T.
2. Determine κ such that T is bijective and determine T^{-1}.
3. Using three different methods, solve the linear system $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ for $\kappa = 2$.

Exercise 5. (Home work)

Let A and B be (3×3)matrices given by :

$$A = \begin{pmatrix} 2 & 9 & 1 \\ 6 & 1 & 6 \\ 0 & 2 & 1 \end{pmatrix} \quad , \quad B = \begin{pmatrix} -2 & 4 & -1 \\ 1 & 5 & 2 \\ 7 & 2 & 1 \end{pmatrix}$$

Let $X = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ be a three dimensional vector.

1. Which of the following systems is consistent :
$A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = X$, $B\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = X,\ A^T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2X,\ AB\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = X$
2. Do the following two systems $AB\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = X,\ (AB)^T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = X$ have the same solution ?