Tutorial Worksheet No.1

Exercise 1.

Consider the matrices :

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 2 & 1 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 1 & -2 & 2 \end{pmatrix},\ \text{and } C = \begin{pmatrix} 1 & 6 & 5 \\ 4 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

1. Calculate $E_{ij} = iB_{ij} + jC_{ij}$,$BA$, A^T, $D_{ij} = B_{ji} + C_{ij}$
2. Check the following :
   (a) $(B + C)^2 = B^2 + C^2 + 2BC$
   (b) $(2B)A = B(2A)$
   (c) $(BA)^T = A^T B^T$
   (d) $tr(C + B) = tr(C) + tr(B)$

Exercise 2.

Consider a point with coordinates $(1, 1, 1)$ in fixed coordinate system.

– Determine the coordinates of a point in the spacecraft system performes a pitch roll and yaw in sequence through the angles $\alpha = 30,\ \beta = 45,\ \gamma = 60$

Exercise 3.

Consider the matrices :

$$A = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 4 & 7 \\ 2 & 1 & 2 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 1 & 2 \\ 3 & 4 & 7 \end{pmatrix},\ \text{and } C = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 2 & 2 \\ 4 & 3 & 7 \end{pmatrix}$$

1. Calculate $det(A)$, $det(5A)$,$det(B)$, $det(C)$, $det(A + B),$ and $det(AB)$. What is your conclusion ?
2. The matrix M is just A where R_2 in A is replaced by R_2+3R_3 in M, then calculate $det(M)$.

Exercise 4.

Consider the matrices :

$$A = \begin{pmatrix} e & e+1 & e+2 \\ f & f+1 & f+2 \\ g & g+1 & g+2 \end{pmatrix},\ B = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a+c & b+c & a+b \end{pmatrix}$$

- Without expanding evaluate $det(A)$ and $det(B)$.

Exercise 5.

Consider the matrix :

$$A = \begin{pmatrix} 6 & 2 & 5 \\ 10 & 4 & 7 \\ 2 & 3 & 2 \end{pmatrix}$$

1. Show that A is nosingular (invertible) and deduce $rank(A)$.
2. Calculate the determinant of A using the row reduction method.
3. Calculate $adj(A)$ and deduce that $A\,adj(A) = \begin{pmatrix} det(A) & 0 & 0 \\ 0 & det(A) & 0 \\ 0 & 0 & det(A) \end{pmatrix}$.
4. Determine A^{-1}.

Exercise 6.

Let T be a map defined by :

$$T : R^3 \longrightarrow R^3$$
$$(x, y, z) \longrightarrow (3x + y + z, 2x + 2y, 3z + 4y)$$

1. Determine the matrix representation of T and the rank of this matrix.
2. Is T bijective ?
3. Determine T^{-1} and its matrix representation .

Exercise 7.

Consider the matrix :

$$A = \begin{pmatrix} 6 & 3 & 8 \\ 3 & \frac{3}{2} & 4 \\ 7 & 5 & 1 \end{pmatrix}$$

- Determine the rank of A.
- Is A invertible ?

Exercise 8 (Homework).

Consider the matrix

$$B = \begin{pmatrix} (1+k) & 2 & 2k \\ 0 & k & 4 \\ 2 & 6 & 3 \end{pmatrix}$$

- For what value(s) of k is B singular ?
- Calculate A^{-1} for $k = 2$.

Exercise 9

Consider two ordered bases of the vector space $\mathbb{R}^3$

$$\mathcal{B} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$
$$\mathcal{B}' = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$$

1. Determine the matrix $\mathcal{S}$ that describe the change of basis $\mathcal{B}' \to \mathcal{B}$.
2. Write the vector $v = (1, 2, 3)$ in terms of the basis $\mathcal{B}'$.