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Institute of Mathematics and Computer Science

probability and statistics

3rd year computer science

Serie N 4

Note: questions marked (∗) left to the students

Exercise 1 :

Let X be a discrete random variable that takes integer values between 1 and 9 with the probabilities:

$$p_k = \mathbb{P}(X = k) = a(10 - k).$$

(1) Deduce the value of a.

(2) Calculate $E(X)$, $E(X^2)$ and deduce $Var(X)$.

Exercise 2 :

A box contains 5 tokens numbered 1, 5 tokens numbered 2, and 5 tokens numbered 3. A token is randomly drawn from the box and X is the number of the token drawn.

(1) Determine the probability distribution of X and its cumulative distribution function (C. D. F.).

(2) Calculate $E(X)$ and $Var(X)$.

(3) Calculate $\mathbb{P}(1 \leq X \leq 2)$.

Exercise 3 : (∗)

Let X be a discrete random variable and F_X its distribution function, show that.

1) F_X is increasing on $\mathbb{R}$.

2) F_X is right continuous and has a left limit at every point.

3) $\lim_{x\longrightarrow -\infty} F_X(x) = 0$.

4) $\lim_{x\longrightarrow +\infty} F_X(x) = 1$.

5) The two distribution functions $F_X = F_Y$ if and only if the random variables X and Y have the same law.

6) $\mathbb{P}(X > a) = 1 - \mathbb{P}(X \leq a) = 1 - F_X(a)$.

7) $\mathbb{P}(a < X \leq b) = F_X(b) - F_X(a)$.

Exercise 4 : (∗)

A fair die is rolled twice. Let X be a random variable defined by "the maximum number of two numbers obtained."

(1) Determine the probability distribution of X and its distribution function.

(2) Calculate $E(X)$ and deduce $Var(X)$.

(3) Calculate $\mathbb{P}(X \leq 4)$, $\mathbb{P}(X > 3)$, $\mathbb{P}(2 < X \leq 5)$.

Exercise 5 : ($*$)

Consider the following game: the player first rolls a fair die. If he gets 1, 2 or 3, he wins the equivalent in Algerian dinars (DZ) (that is, 1 DZ if he gets 1, for example). Otherwise, he loses 2 DZ.

We denote X the random variable corresponding to the player's gain (negative in case of loss).

(1) Give the law of X and its distribution function F_X.

(2) Calculate the expected value and variance of X.

Exercise 6:

Can the following expressions be considered as probability densities of random variables:

(1) $f_1(x) = \frac{1}{b-a}$ if $a \leq x \leq b$ and $f_1(x) = 0$ otherwise.

(2) $f_2(x) = \frac{|x|}{a^2}$ if $-a \leq x \leq a$ and $f_2(x) = 0$ otherwise.

Exercise 7:

Determine a, b, and c such that the following functions are probability densities.

(1) $f_1(x) = \frac{a}{x}$ if $1 \leq x \leq e$ and $f_1(x) = 0$ otherwise.

(2) $f_2(x) = \frac{1}{2}e^{-bx}$ if $x \geq 0$ and $f_2(x) = 0$ otherwise.

(3) $f_3(x) = \frac{c}{4}$ if $0 \leq x \leq c$ and $f_3(x) = 0$ otherwise.

Exercise 8:($*$)

Let X be a continuous random variable and F_X its cumulative distribution function. Show the following relationships:

(1) $\mathbb{P}(X = a) = 0$.

(2) $\mathbb{P}(X \geq a) = 1 - F_X(a)$.

(3) $\mathbb{P}(a \leq X \leq b) = \mathbb{P}(a \leq X < b) = \mathbb{P}(a < X \leq b) = \mathbb{P}(a < X < b)$.

Exercise 9 :

Let X be a continuous random variable with probability density function:

$$f(x) = \begin{cases} \alpha x(4-x), & if \ x \in [0,4] \\ 0, & if \ x \notin [0,4] \end{cases}$$

(1) Calculate the constant α.

(2) Determine the cumulative distribution function of X.

(3) Determine the probability of the events : $[1 \leq X \leq 2]$, $[X > 3]$.

(4) Calculate $E(X)$.

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