# Chapitre 02

## Conductors in Electrostatic Equilibrium

**Définition**

A conductor is a material medium in which electrical charges exist that are very weakly bound to their atoms. These mobile charges are capable of moving under the action of an electric field. In the case of metals, only the electrons are mobile.

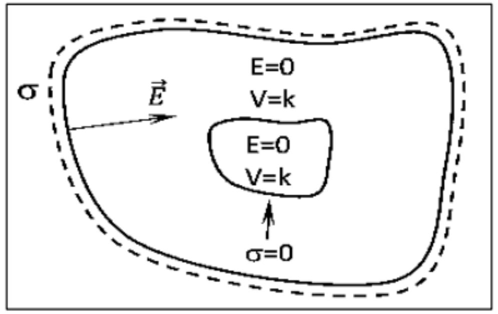
## Law of conservation of charge

Any isolated body contains a certain number of charge carriers: these are the protons bound to the nuclei of atoms and the electrons that gravitate around the nuclei. The electric charge of an isolated system can neither be created nor annihilated. In an isolated system, the electric charge is conserved:

$$\sum \boldsymbol{q} = 0$$

## Conductor in electrostatic equilibrium

A conductor is any body in which electrons can move easily (Figure I.14). - When charges can only make small movements around their equilibrium (bound) positions, the conductor is said to be in electrostatic equilibrium.



## Properties of a conductor in equilibrium

- There is no charge inside the conductor **(q=0)**; All charges are distributed on the surface of the material.

- In a conductor in equilibrium, the charges are stationary, it is not subject to any force, which means that the electrostatic field inside the balanced conductor is zero.

$$\vec{F} = q\vec{E} = \vec{0} \quad \Longrightarrow \quad \vec{E} = \vec{0}.$$

- From the relationship: $\vec{E} = -\overrightarrow{grad}V \quad \Rightarrow \quad dV = -\vec{E}.\vec{dl} = 0 \quad \Rightarrow \quad V = cte$ This means that the potential is constant at every point inside the balanced conductor and, therefore, the outer surface of the conductor is the equipotential surface.

(Equipotential surface: This is the set of points which have the same potential). $\boldsymbol{V(M) = V_0 = C^{te}}$.

- Since the potential of the outer surface of the conductor is an equipotential surface, the electrostatic field vector is perpendicular to the equilibrium surface of the conductor $\vec{E} \perp \vec{dS}$

$$\oiint \vec{E}.d\vec{S} = \frac{\sum Q_{int}}{\varepsilon_0} = \frac{\iiint \rho.dV}{\varepsilon_0} = 0 \quad \Rightarrow \quad \boxed{\rho = 0}$$

That is to say: the charge inside the conductor is zero and it is on the surface of the conductor."

"This result means that in each volume element there will be positive charges equal to the negative charges, so the charge inside the conductor is zero.

The total free charges are distributed on the surface, occupying a thickness formed by a few layers of atoms (the charge in a conductor can only be superficial, with a surface density σ). The mobile electric charges accumulate on the surface until the field they create exactly compensates for the external field applied to this surface, which leads to a state of equilibrium."

- Through the outer base of the conductor, the flux $\boldsymbol{\phi = \int \vec{E}.\vec{dS} \neq 0.}$

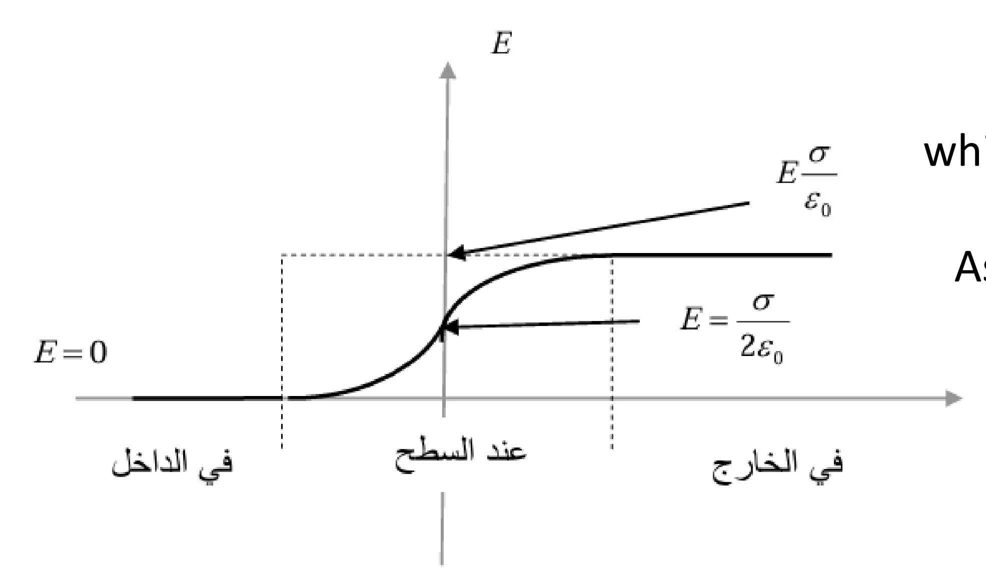
# The field in the immediate vicinity of a conductor in equilibrium: Coulomb theory

The field near a conductor is given from Gauss's theorem: $\boldsymbol{\phi = E.dS = \frac{\sigma dS}{\varepsilon_0}} \rightarrow \boldsymbol{E = \frac{\sigma}{\varepsilon_0}}$

with: σ is the surface density of the electric charge, $\boldsymbol{\varepsilon 0}$ is the permittivity.
- On the surface of a conductor, electrostatic pressure (repulsive forces) is applied between the external charges.

$$\boldsymbol{p = \sigma E_m = \frac{\sigma^2}{2\varepsilon_0}} \;;\; \boldsymbol{sachant\ que}: \boldsymbol{E_m = \frac{\sigma}{2\varepsilon_0}}$$

with: $\mathbf{E_m}$ is the average electric field.




فعند عبور السطح يتغير الحقل وفقا للشكل التالي:

The value of the field at a point near the surface is $\boldsymbol{E_{ext} = \frac{\sigma}{\varepsilon_0}}$ while inside it is zero. $\boldsymbol{E_{int}} = 0$

As for on the surface, the field takes an average value $\boxed{E_{moy} = \frac{\sigma}{2\varepsilon_0}}$

**Two cases can arise depending on the electrical nature of the body:**

Case of a neutral conductor A neutral conductor is a conductor with zero charge, it is characterized by:

- A volume density of charges $\boldsymbol{\rho_{int}} = \mathbf{0}$.
- A surface density of charges $\boldsymbol{\sigma} = \mathbf{0}$
- $\overrightarrow{E_{int}} = \vec{0} \implies V_{int} = C^{te} = V_0,$ which indicates that the surface of this conductor is equipotential.

## The electric field outside a neutral conductor

We will apply Gauss's theorem to calculate the electric field outside a neutral conductor. The Gaussian surface is a cylinder (see Figure I.15):

$$\oiint \overrightarrow{\boldsymbol{E_{ext}}}.\overrightarrow{\boldsymbol{dS}} = \frac{\sum \boldsymbol{Q_{int}}}{\boldsymbol{\varepsilon_0}}$$

There are no charges inside the cylinder, which implies that $\sum \boldsymbol{Q_{int}} = \mathbf{0} \Rightarrow \overrightarrow{\boldsymbol{E_{ext}}} = \vec{\mathbf{0}}.$

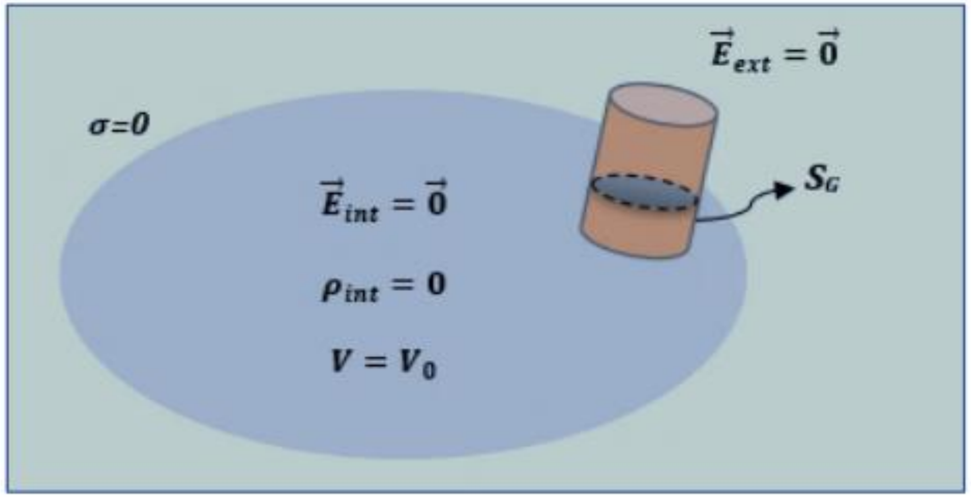



Figure I.15. A neutral conductor in electrostatic equilibrium

## Case of a charged (isolated) conductor

An isolated conductor is an unconnected conductor; it is not connected to any other body and its charge is constant. It is characterized by:

- $\overrightarrow{\boldsymbol{E}_{int}} = \vec{\mathbf{0}}$; The charge can only be distributed over the surface.
- $\boldsymbol{V}_{int} = \boldsymbol{c}^{te} = \boldsymbol{V}_0$ ; The surface of this conductor is equipotential.

## The electric field outside an insulated conductor:

We will apply Gauss's theorem to calculate the electric field outside an isolated conductor with surface charge density σ. We choose a cylinder as the Gaussian surface (see figure I.16):

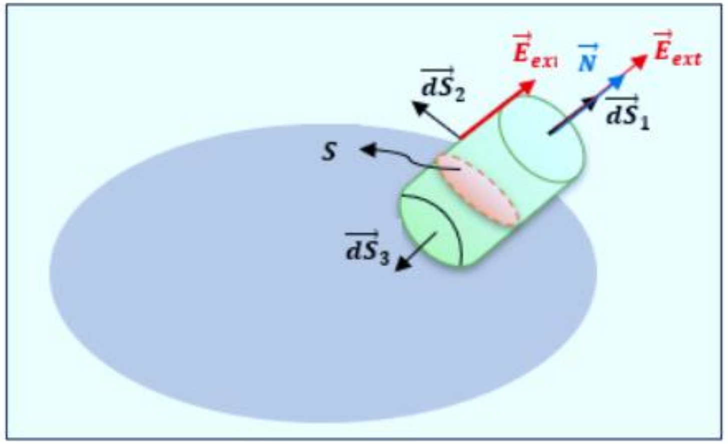
$$\oiint \overrightarrow{\boldsymbol{E}_{ext}}.\overrightarrow{\boldsymbol{dS}} = \frac{\sum \boldsymbol{Q}_{int}}{\boldsymbol{\varepsilon}_0}$$

$$\phi_T = \phi_1 + \phi_2 + \phi_3$$

$$\phi_1 = E_{ext}.S_1 \quad (\vec{E}_{ext} \perp \overrightarrow{dS_2}) \Rightarrow \phi_2 = 0$$

$$\vec{E}_{int} = \vec{0} \Rightarrow \phi_3 = 0$$

$$\phi_T = \phi_1 = E_{ext}.S_1$$




So : $$E_{ext}.S_1 = \frac{\iint \sigma.dS}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}S$$

Figure I.16 A charged conductor in electrostatic equilibrium.

The total flux through the three surfaces shown in Figure I.16 is:

With : $\phi_T = \phi_1 + \phi_2 + \phi_3$ $\quad \phi_1 = E_{ext}.S_1$ $\quad \vec{E}_{ext} \perp \overrightarrow{dS_2} \Longrightarrow \phi_2 = 0$ $\quad \vec{E}_{int} = \vec{0} \Longrightarrow \phi_3 = 0$

So : $\phi_T = \phi_1 = E_{ext}.S_1$ $\quad E_{ext}.S_1 = \dfrac{\iint \sigma.dS}{\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}S$

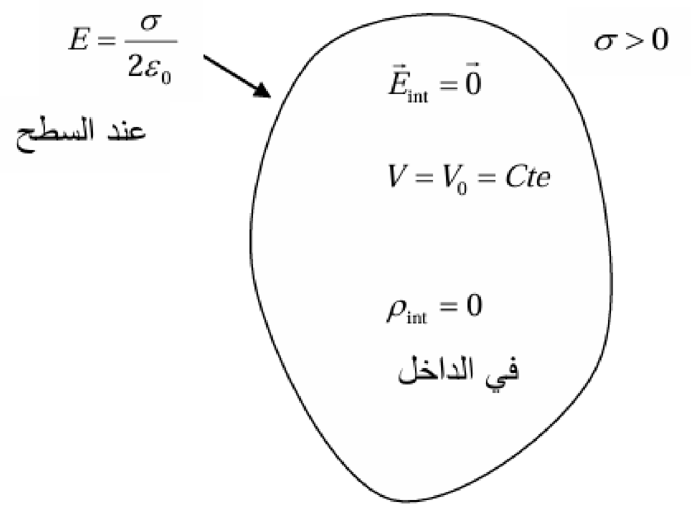
The internal charges on the Gaussian surface are found on the surface S (indicated in pink in Figure I.16) where S = S1, which gives: $E_{ext} = \dfrac{\sigma}{\varepsilon_0}$

Let $\vec{N}$ be a unit vector normal to the surface S1 of this conductor, then $\boxed{\vec{E}_{ext} = \dfrac{\sigma}{\varepsilon_0}\vec{N}}$

- If $\sigma > 0$ , the electric field is deflected outwards

- If $\sigma < 0$ , the electric field is deflected inwards

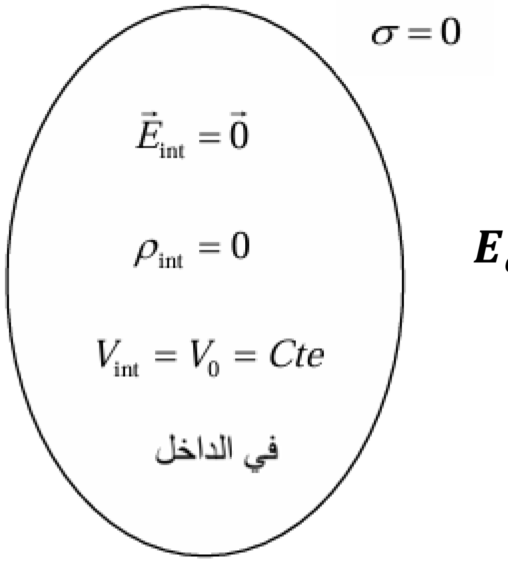
خلاصة : يبن الشكل خصائص الناقل المتزن ***An isolated conductor***




$$\boldsymbol{E_{ext}} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\varepsilon_0}}$$

في الخارج

أما بالنسبة االناقل المعتدل فإن عند السطح $\sigma = 0$ و عليه فالحقل على السطح و خارج الناقل معدوم ***A neutral conductor***

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$\boldsymbol{E_{ext}}=0$

## Coulomb's theorem

**The electric field outside a charged conductor, in electrostatic equilibrium, is:**

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{N}$$

**The lines of this field are perpendicular to the surface of the conductor.**

## Problem:

A **spherical conductor** C, with a **total charge** Q and **radius**, is in **electrostatic equilibrium**.

1. Express the **surface charge density** σ on the conductor.
2. What can be concluded?

### 1. Expression of the surface charge density σ

By definition, the surface charge density is: $\boldsymbol{\sigma} = \frac{\boldsymbol{Q}}{\boldsymbol{S}}$

where S is the **surface area** of the sphere, given by: $\boldsymbol{S} = \boldsymbol{4\pi R^2}$

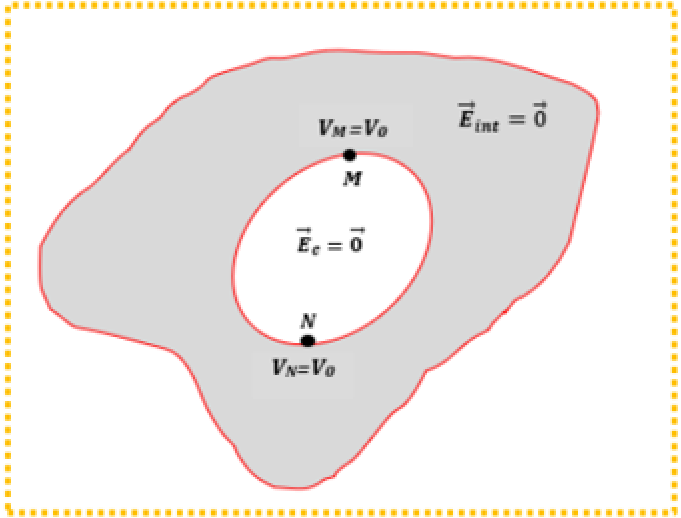
Thus: $\boldsymbol{\sigma} = \frac{\boldsymbol{Q}}{\boldsymbol{4\pi R^2}}$

Moreover, the electric field just outside the surface of the sphere is given by: $\vec{E}_{\text{ext}} = \frac{\sigma}{\varepsilon_0}\vec{N} = \frac{Q}{4\pi\varepsilon_0 R^2}\vec{N}$

where $\vec{\boldsymbol{N}}$ is the unit vector normal to the surface (pointing outward), and $\boldsymbol{\varepsilon_0}$ is the vacuum permittivity.

## Conductor with a cavity

Let C be a conductor in electrostatic equilibrium that contains a cavity. Whether the body carries charge or not, both its outer and inner surfaces are equipotential **(i.e.\ V=constant=$V_0$)**.




Conductor with a cavity

Let M and N be two points on the **inner surface** of the conductor, and let E⃗c\vec{E}_cEc represent the **electric field inside the cavity** (see Figure I.17). We have:

$$V_M = V_N$$

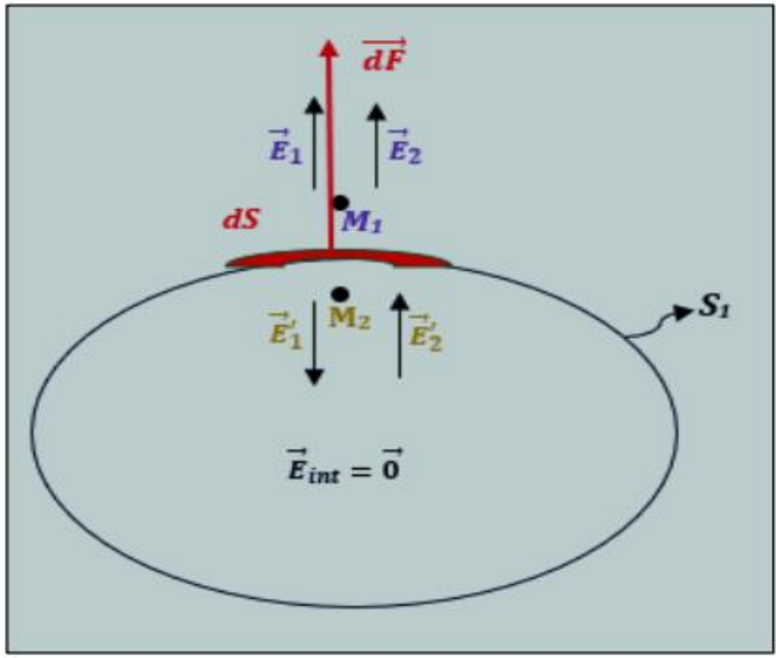
$$V_M - V_N = -\int_{\widehat{MN}} \vec{E_c} \cdot \vec{dl} = 0$$

$$\vec{E_c} = \vec{0}$$

**Whatever the external conditions around the hollow conductor, the electric field is zero inside the cavity and within the bulk of the conductor.**

# Pression électrostatique

Let $M_1$ and $M_2$ be two points infinitely close to the surface of a conductor carrying a surface charge density $\sigma > 0$ (see Figure I.18), where:




**$M_1$** is located outside the conductor.
**$M_2$** is located inside the conductor.
**dS** represents an infinitesimal surface element located between the two points **$M_1$** and **$M_2$**.
**$S_1$** represents the rest of the conductor's surface.
$\overrightarrow{\boldsymbol{E_1}}$ is the field created at **$M_1$** by the charges located on **dS**.
$\overrightarrow{\boldsymbol{E_2}}$ is the field created at **$M_1$** by the other charges located on **$S_1$**

The total field created at point **$M_1$** is: $\vec{E}_{ext} = \overrightarrow{E_1} + \overrightarrow{E_2}$

Let:
$\overrightarrow{\boldsymbol{E'_1}}$ : be the field created at **$M_2$** by the charges located on **dS**.
$\overrightarrow{\boldsymbol{E'_2}}$ : be the field created at **$M_2$** by the other charges located on **$S_1$.**
The total field created at point **$M_2$** is:
$\overrightarrow{E_{int}} = \overrightarrow{\boldsymbol{E'_1}} + \overrightarrow{\boldsymbol{E'_2}}$ .

$$\overrightarrow{\boldsymbol{E'_1}} = -\overrightarrow{\boldsymbol{E'_2}}$$

The points **$M_1$** and **$M_2$** are infinitely close, we have:

$$\begin{cases} \overrightarrow{E_2} = \overrightarrow{\boldsymbol{E'_2}} \\ \overrightarrow{E_1} = -\overrightarrow{\boldsymbol{E'_1}} \Longrightarrow \overrightarrow{E_1} = \overrightarrow{E_2} \\ \overrightarrow{\boldsymbol{E'_1}} = -\overrightarrow{\boldsymbol{E'_2}} \end{cases}$$

**It follows that the contribution of the entire set of charges on the conductor is equal to that of the charge located in the immediate vicinity.**

The total field created at point **$M_1$** (outside) is: $\overrightarrow{E_{ext}} = 2\overrightarrow{E_2} \implies \overrightarrow{E_2} = \frac{\overrightarrow{E_{ext}}}{2}$

According to Coulomb's law: $\overrightarrow{E_{ext}} = \frac{\sigma}{\varepsilon_0}\vec{N}$ which gives: $\boldsymbol{\overrightarrow{E_2} = \frac{\sigma}{2\,\varepsilon_0}\vec{N}}$

Let **dq** be the charge of the surface element **dS**, the force $\boldsymbol{\overrightarrow{dF}}$ exerted by the rest of the charges on the conductor on **dq** is:

$$\overrightarrow{dF} = dq.\boldsymbol{\overrightarrow{E_2}} = \boldsymbol{\sigma ds.\frac{\sigma}{2\,\varepsilon_0}\vec{N}}$$

$$\overrightarrow{dF} = \frac{\sigma^2 ds}{2\varepsilon_0}\vec{N}$$

**The force $\overrightarrow{dF}$ is normal to the surface of the conductor and always directed outward, regardless of the sign of the charge.**

The electrostatic pressure exerted on the surface of a charged conductor is defined by: $\boldsymbol{P = \frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0}}$

- ***Electrostatic pressure*** is the pressure experienced by the surface of a charged electrical conductor. It acts perpendicular to the surface of the conductor, from the inside to the outside. It thus tends to pull off the charges that are held on the conductor.