Democratic and Popular Republic of Algeria

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Analysis 2

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Exercises Serie N° 2

Note: questions marked $(*)$ ***left to the students***

Exercise 1

Using the Riemann sums calculate the following integrals:

1. $\int_0^1 x^2 \, dx.$
2. $\int_0^1 e^x \, dx.$
3. $\int_0^{\frac{\pi}{2}} \sin x \, dx \quad (*).$
4. $\int_0^{\frac{\pi}{2}} \cos x \, dx \quad (*).$

Such that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Exercise 2

Using the Riemann sum of an appropriate function, determine, in each of the following cases, the limit of the sequence $(u_n)_{n\in\mathbb{N}^*}$*:*

$$1.u_n = n\sum_{k=0}^{n-1}\frac{1}{k^2+n^2} \qquad 2.u_n = \sum_{k=0}^{n}\frac{n}{(n+k)^2} \qquad 3.u_n = \sum_{k=1}^{n}\frac{1}{\sqrt{n}\sqrt{n+k}} \quad (*)$$

$$4.u_n = \prod_{k=1}^{n}\left(1+\frac{k^2}{n^2}\right)^{\frac{1}{n}} \qquad 5.u_n = \frac{1}{n}\sum_{k=0}^{n-1}\cos\left(\frac{k\pi}{2n}\right)$$

Exercise 3

1. *Prove that the sequence* $(u_n)_{n\in\mathbb{N}}$ *whose general term is*

$$u_n = \sum_{k=1}^{n}\frac{n+k}{n^2+k^2}$$

   is a sequence of Riemann sums which converges and compute its limit.

2. *Compute the limit when* n *tends to* $+\infty$ *of* $\sum_{k=n+1}^{2n}\frac{1}{k}$.
3. *For which real number* α *is the sequence* $(v_n)_{n\in\mathbb{N}}$ *with general term*

$$v_n = \frac{1}{n^2}\sum_{k=1}^{n}k^{\alpha}\sin\left(\frac{k}{n}\right)$$

   a sequence of Riemann sums? What is its limit?

Exercise 4

Using integration by parts, calculate the following integrals:

1. $\int x^2 \ln(\frac{x-1}{x})\ dx$ 2. $\int_0^1 x \arctan x\ dx$ 3. $\int \cos x e^x\ dx$

4. $\int x^2 e^{2x}\ dx$ $(*)$ 5. $\int_0^{\frac{\pi}{2}} \cos(2x) \sin x\ dx$ $(*)$

Exercise 5

Using integration by changing the variable, calculate the following integrals:

1. $\int \frac{1}{3\ \sqrt[3]{x+1} - x + 1}\ dx$ 2. $\int_{-1}^{\frac{1}{2}} \sqrt{x^2 + 2x + 5}\ dx$ 3. $\int \frac{\sin x}{1 + \sin x}\ dx$

4. $\int \frac{x}{\sqrt{x+1}}\ dx$ 5. $\int \frac{1}{\sin x}\ dx$ 6. $\int \frac{1}{\cos x}\ dx$ $(*)$

Exercise 6

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2(x)}{\cos(x) + \sin(x)}\ dx, \text{ and } J = \int_0^{\frac{\pi}{2}} \frac{\sin^2(x)}{\cos(x) + \sin(x)}\ dx$$

1. *Without calculating I and J, show that $I = J$.*
2. *Check that* $\forall x \in \mathbb{R},\ \cos(x) + \sin(x) = \sqrt{2} \cos\left(x - \frac{\pi}{4}\right)$.
3. *Deduce that*

$$I + J = \frac{\sqrt{2}}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\cos(x)}.$$

4. *Calculate $I + J$. Deduce I and J.*

Exercise 7

Calculate the following integrals:

1. $\int_0^1 \frac{1}{(1+x^2)^2}\ dx$ 2. $\int_{-1}^1 \frac{1}{x^2 + 4x + 7}\ dx$ 3. $\int_0^1 \frac{3x+1}{(1+x)^2}\ dx$

4. $\int \frac{dx}{\sin^3 x}$ 5. $\int \frac{\sin 2x}{\sin^2 x - 5 \sin x + 6}\ dx$ 6. $\int \frac{x^3}{(x+1)(x^2+4)^2}\ dx$ $(*)$