Serie 4

Exercise 1:

Determine the nature (convergent or divergent) of the following series:

$$\sum_{n=1}^{\infty} \frac{1}{n^2}\left(\frac{3}{4}\right)^n \quad , \quad \sum_{n=1}^{\infty} \frac{n}{2^n} \quad , \quad \sum_{n=1}^{\infty} \frac{1}{n} x^n \quad , \quad \sum_{n=1}^{\infty} \frac{2n}{(2n)!}$$

Exercise 2:

- Calculate the trigonometric Fourier series of the 2π periodic function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \pi - |\, x \,|$ on $]-\pi, \pi]$. Does the series converge to f?
- Calculate the Fourier series, in trigonometric form, of the 2π periodic function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ on $[0, 2\pi[$. Does the series converge to f?

Exercise 3:

Compute the Fourier transforms of the following functions:

$$f_1(t) = \begin{cases} 1\,, & if \ \ |t| < 1 \\ \frac{1}{2}\,, & if \ |t| = 1 \\ 0\,, & if \ \ |t| > 1 \end{cases} \quad , \quad f_2(t) = \begin{cases} -1\,, & if \ -1 < t < 0 \\ 1\,, & if \ \ 0 < t < 1 \\ 0\,, & if \ \ |t| > 1 \end{cases}$$

Exercise 4:

Compute the Laplace transform of the following functions.

1- $f(t) = e^{-at}$ for $a > 0$
2- $f(t) = t^n$ $with\; n \in \mathbb{N}$
3- $f(t) = \cos(\omega_0 t)$