

Unit Operations II Final Exam – Model Answer

Exercise 1: (Graded out of 12 points)

Given data:

Water side
 $T_{w,in} = 40 \text{ }^\circ\text{C}; \Delta T_{app} = 3 \text{ }^\circ\text{C}$
 $\dot{m}_w = 5000 \text{ kg/min}; c_{p,w} = 4.187 \text{ kJ/(kg.K)}$

Air side
 Inlet : $T_{DB,1} = 35 \text{ }^\circ\text{C}; T_{WB,1} = 28 \text{ }^\circ\text{C}$
 Outlet: $T_{DB,2} = 38 \text{ }^\circ\text{C}; \phi_2 = 60\%$

Atmospheric pressure: $P = 101.325 \text{ kPa}$

PART A — PSYCHROMETRIC CHART READING

Using the psychrometric chart:

Property	Symbol	State 1 — air inlet 35 °C DB · 28 °C WB	State 2 — air outlet 38 °C DB · 60% RH
Specific enthalpy, h	h (kJ/kg d.a.)	90	104
Humidity ratio, ω	ω (kg/kg d.a.)	0,0212	0,0255
Wet-bulb temperature	$T_{WB,2}$ (°C)	28 °C	30,7
Dew-point temperature, T_{DP}	T_{DP} (°C)	26	29

PART B — WATER-SIDE ANALYSIS

1) Cold water outlet temperature, $T_{w,out}$:

$$\Delta T_{app} = T_{w,out} - T_{WB,1} \Rightarrow T_{w,out} = \Delta T_{app} + T_{WB,1} = 3 \text{ }^\circ\text{C} + 28 \text{ }^\circ\text{C} \Rightarrow T_{w,out} = 31 \text{ }^\circ\text{C}$$

1 pts

2) Total heat rejected by water stream, Q_w :

$$Q_w = \dot{m}_w \cdot c_{p,w} \cdot (T_{w,in} - T_{w,out}) \Rightarrow Q_w = 5000 \text{ kg/min} \times 4.187 \text{ kJ/(kg.K)} \times (40 \text{ }^\circ\text{C} - 31 \text{ }^\circ\text{C})$$

$$\Rightarrow Q_w = 1.884 \times 10^5 \text{ kJ/min} = 3140 \text{ kW}$$

1 pts

PART C — ENERGY BALANCE AND AIR FLOW

1) Steady-state energy balance for the tower: For the cooling tower:

$$Q_w = \dot{m}_{da} \cdot (h_2 - h_1)$$

1 pts

2) Required dry-air flow rate, \dot{m}_{da} , through the cooling tower in: $\text{kg dry air} \cdot \text{min}^{-1}$:

$$\dot{m}_{da} = Q_w / (h_2 - h_1) \Rightarrow \dot{m}_{da} = 1.884 \times 10^5 \text{ kJ/min} / (104 \text{ }^\circ\text{C} - 90 \text{ }^\circ\text{C}) \Rightarrow \dot{m}_{da} = 1.35 \times 10^4 \text{ kg dry air} \cdot \text{min}^{-1}$$

1 pts

3) Amount of make-up water, $\dot{m}_{\text{make-up}}$, required to compensate for evaporative losses:

$$\text{Evaporated water: } \dot{m}_{\text{make-up}} = \dot{m}_{\text{evap}} = \dot{m}_{da} \cdot (\omega_2 - \omega_1) \Rightarrow \dot{m}_{\text{make-up}} = 1.35 \times 10^4 \times (0,0255 - 0,0212)$$

$$\Rightarrow \dot{m}_{\text{make-up}} = 58.05 \text{ kg} \cdot \text{min}^{-1}$$

1 pts

As a percentage of the inlet water flow:

$$\% \text{ make-up} = (58.05/5000) \times 100 \Rightarrow \% \text{ make-up} = 1.16\%$$

1 pts

PART D — DISCUSSION

1) Simultaneous heat and mass transfer processes occurring as air rises through the tower:

On the psychrometric chart, the air process follows: **State 1** → **State 2**

Observed changes:

- **T_{DB} increases:** $35\text{ }^{\circ}\text{C} \rightarrow 38\text{ }^{\circ}\text{C} \Rightarrow$ Heating (Water transfers heat to air, increasing air temperature)
- **ω increases:** $0.0212 \rightarrow 0.0255 \Rightarrow$ Humidification (Part of the water evaporates into the air)

Thus, simultaneous:

- **Cooling of water;**
- **Heating and humidification of air.**

1 pts

2) Effect of increasing inlet RH to 80%:

Since the entering air is already more humid ($RH_{in} = 80\%$), it has **less capacity to absorb additional water vapor**. This reduces evaporation, which lowers cooling efficiency.

- **Approach temperature increases:**

0.5 pts

→ less evaporation means weaker cooling, so outlet water temperature, $T_{w,out}$, becomes higher.

- **Make-up water requirement decreases:**

0.5 pts

→ less water evaporates, so less replacement water is needed.

- **Required air mass flow rate increases:**

0.5 pts

→ each kg of humid air removes less heat, so more air is required to reject the same heat load.

Exercise 2: (Graded out of 08 points)

1) Saturation temperatures: from steam tables:

- Live steam, at: **198.5 kPa** $\Rightarrow T_s = 120\text{ }^{\circ}\text{C}$
- First effect boiling temperature, at: **90 kPa** $\Rightarrow T_1 = 96\text{ }^{\circ}\text{C}$
- First effect boiling temperature, at: **17.9 kPa** $\Rightarrow T_1 = 58\text{ }^{\circ}\text{C}$

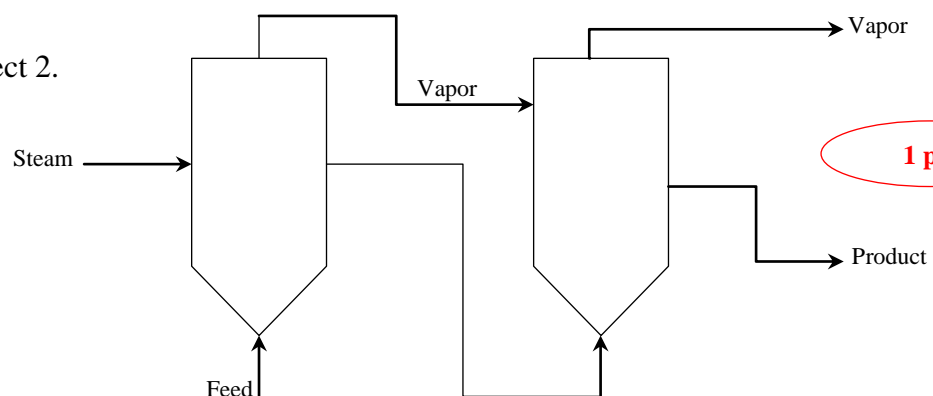
0.5 pts $\times 3$

2) Schematic forward-feed arrangement:

Forward-feed arrangement: **Feed** → **Effect 1** → **Effect 2** → **Product**

Steam: **Steam** → **Effect 1**

Vapor from effect 1 heats effect 2.



1 pts

3) Total solids flow rate in the system, \dot{m}_s :

$$\dot{m}_s = x_F \cdot F \Rightarrow \dot{m}_s = 0.11 \times 5.5 \Rightarrow \dot{m}_s = 0.605 \text{ kg/s}$$

0.5 pts

4) Final mass flow rate of concentrated product, P: $P = F - V_1 - V_2$

- **Determine vapor produced in effect 1, V_1 (already question 7) :**

$$\text{Feed sensible heating: } Q_{sens} = F \cdot c_{p,f}(T_1 - T_f) \Rightarrow Q_{sens} = 5.5 \times 3.9 \times (96 - 58) \Rightarrow Q_{sens} = 815 \text{ kW}$$

$$\text{Latent heat available: } Q_{evap,1} = 4800 - 815 \Rightarrow Q_{evap,1} = 3985 \text{ kW}$$

$$\Rightarrow V_1 = Q_{evap,1} / \lambda_1 \Rightarrow V_1 = 3985 \text{ kW} / 2270 \text{ kJ/kg} \Rightarrow V_1 = 1.76 \text{ kg/s}$$

0.5 pts

Latent heat at 96°C: $\lambda_1 = 2270 \text{ kJ/kg}$, from tables.

- **Determine vapor produced in effect 2, V_2 (already question 10) :**

Latent heat at 58°C: $\lambda_2 = 2350 \text{ kJ/kg}$, from tables.

$$V_2 = Q_2 / \lambda_2 \Rightarrow V_2 = 3800 / 2350 \Rightarrow V_2 = 1.62 \text{ kg/s}$$

0.5 pts

- **Final product flow rate, P: $P = F - V_1 - V_2 \Rightarrow P = 5.5 - 1.76 - 1.62 \Rightarrow P = 2.12 \text{ kg/s}$**

0.5 pts

5) Heat duty in the first effect:

$$Q_1 = U_1 A_1 \Delta T_1 \Rightarrow Q_1 = 2000 \times 100 \times (120 - 96) \Rightarrow Q_1 = 4800 \text{ kW}$$

0.5 pts

$$\text{Where: } \Delta T_1 = T_s - T_1 \Rightarrow \Delta T_1 = 120 - 96 \Rightarrow \Delta T_1 = 24 \text{ }^\circ\text{C}$$

6) Amount of steam required, \dot{m}_{steam} :

Latent heat of steam at 198.5 kPa: $\lambda_s = 2200 \text{ kJ/kg}$

$$\dot{m}_{\text{steam}} = Q_1 / \lambda_s \Rightarrow \dot{m}_{\text{steam}} = 4800 \text{ kW} / 2200 \text{ kJ/kg} \Rightarrow \dot{m}_{\text{steam}} = 2.18 \text{ kg/s}$$

0.5 pts

8) Solids fraction of the liquid leaving the first effect:

$$\text{Liquid leaving effect 1: } L_1 = F - V_1 \Rightarrow L_1 = 5.5 - 1.76 \Rightarrow L_1 = 3.74 \text{ kg/s}$$

$$\text{Solids fraction: } x_1 = \dot{m}_s / L_1 \Rightarrow x_1 = 0.605 \text{ kg/s} / 3.74 \text{ kg/s} \Rightarrow x_1 = 0.162 = 16.2\%$$

0.5 pts

9) Heat duty in the second effect:

$$Q_2 = U_2 A_2 \Delta T_2 \Rightarrow Q_2 = 1000 \times 100 \times (96 - 58) \Rightarrow Q_2 = 3800 \text{ kW}$$

0.5 pts

$$\text{Where: } \Delta T_2 = T_1 - T_2 \Rightarrow \Delta T_2 = 96 - 58 \Rightarrow \Delta T_2 = 38 \text{ }^\circ\text{C}$$

11) Final mass flow rate of product:

$$P = L_1 - V_2 \Rightarrow P = 3.74 \text{ kg/s} - 1.62 \text{ kg/s} \Rightarrow P = 2.12 \text{ kg/s}$$

0.5 pts

12) Final solids fraction:

$$x_p = \dot{m}_s / P \Rightarrow x_p = 0.605 \text{ kg/s} / 2.12 \text{ kg/s} \Rightarrow x_p = 0.285 = 28.5\%$$

0.5 pts

$$\text{13) Steam economy: } S_{\text{Economy}} = (V_1 + V_2) / \dot{m}_{\text{steam}} \Rightarrow S_{\text{Economy}} = (1.76 + 1.62) / 2.18 \Rightarrow S_{\text{Economy}} = 1.54$$

0.5 pts