

Subject : *Algebra 4*
Instructor : *Y. Halim*

TUTORIAL EXAM NO. 1

Exercise 1 : 4 Pts

Let ψ_1, ψ_2, ψ_3 be linear forms on $\mathbb{R}_2[X]$ defined by

$$\psi_1(P) = P(1), \quad \psi_2(P) = P'(0), \quad \psi_3(P) = \int_{-1}^0 P(t) dt.$$

1. Show that the family $\{\psi_1, \psi_2, \psi_3\}$ is a basis of the dual of $\mathbb{R}_2[X]$.
2. Determine its antidual basis.

Exercise 2 : 2 Pts

Let f be a nonzero linear form on a vector space E of dimension n .
Prove that

$$\dim(\ker f) = n - 1.$$

Good luck