4.4 Exercises

Exercise 4.1

Solve the following system using four different methods:

- by substitution,
- by the Gaussian elimination method (pivot method),
- by inverting the coefficient matrix,
- using Cramer's rule.

$$\begin{cases} 2x + y = 1, \\ 3x + 7y = -2. \end{cases}$$

Exercise 4.2

Depending on the values of a, choose the method you consider the most efficient to solve the following systems:

1. $$\begin{cases} ax + y = 2, \\ (a^2 + 1)x + 2ay = 1. \end{cases}$$

2. $$\begin{cases} (a + 1)x + (a - 1)y = 1, \\ (a - 1)x + (a + 1)y = 1. \end{cases}$$

Exercise 4.3

Study the existence of solutions for the following system:

$$\begin{cases} ax + by + z = 1, \\ x + aby + z = b, \\ x + by + az = 1. \end{cases}$$

Exercise 4.4

Using the Gaussian elimination method (pivot method), solve the following systems:

1.
$$S(1) = \begin{cases} x + y + z = 3, \\ 2x - y + z = 1, \\ 3x + y + 2z = 4. \end{cases}$$

2.
$$S(2) = \begin{cases} x + y + z = 6, \\ 2x - y + z = 2, \\ 3x + y + 2z = 12. \end{cases}$$

3.
$$S(3) = \begin{cases} x + y + z + w = 2, \\ 2x + 3y + z + w = 5, \\ 3x + 5y + 2z + w = 8, \\ 4x + 7y + 3z + 2w = 10. \end{cases}$$

Exercise 4.5

Consider the system:

$$\begin{cases} x + y - z = 1, \\ 2x + 3y + \lambda z = 3, \\ x + \lambda y + 3z = -3. \end{cases}$$

Determine the values of $\lambda \in \mathbb{R}$ such that the system has:

1. No solution;
2. A unique solution;
3. Infinitely many solutions.

Exercise 4.6

Using the Gaussian elimination method (pivot method), solve the following systems:

1.
$$S(1) = \begin{cases} x + y + z = 3, \\ 2x - y + z = 1, \\ 3x + y + 2z = 4. \end{cases}$$

2.
$$S(2) = \begin{cases} x + y + z = 6, \\ 2x - y + z = 2, \\ 3x + y + 2z = 12. \end{cases}$$

3.
$$S(3) = \begin{cases} x + y + z + w = 2, \\ 2x + 3y + z + w = 5, \\ 3x + 5y + 2z + w = 8, \\ 4x + 7y + 3z + 2w = 10. \end{cases}$$

Exercise 4.7

Consider the system:
$$\begin{cases} x + y - z = 1, \\ 2x + 3y + \lambda z = 3, \\ x + \lambda y + 3z = -3. \end{cases}$$

Determine the values of $\lambda \in \mathbb{R}$ such that the system has:

1. No solution;
2. A unique solution;
3. Infinitely many solutions.

Exercise 4.8

Let $m \in \mathbb{R}$. Consider the system $S(m)$:

$$S(m) = \begin{cases} (m-1)x + y - z = m, \\ 2x + my + z = 3, \\ mx + (1-m)y + mz = m^2. \end{cases}$$

1. Solve the system in the case where it is a Cramer system.

2. Solve the system in the case where it is not a Cramer system.