1. Introduction:

The estimation of stresses generated within a soil mass is essential for verifying two major problems encountered in civil engineering structures: soil deformations (changes in soil volume resulting from loading) and strength (stability of foundations).

The distribution of stresses in the soil is generally caused by the following two factors:

- Stresses due to the self-weight of the soil (initial stresses).
- Stresses due to loads applied at the surface.

2. Notion of Stress:

Stress is a vector quantity defined by an origin, a direction, a sense, and a magnitude. It represents the ratio between a force **F** and a surface element **dS** on which it is applied.

It is expressed in bars or in pascals. (1 bar = 10^5 Pa)

➢ Total Stress:

Let **S** be a unit section within a soil mass. The resultant of the forces acting on this section, due to external loads and the self-weight of the soil, is the total stress **F**.

This stress can be decomposed into:

- A normal stress **σ** (perpendicular to the plane of section **S**),
- A shear stress **τ** (acting within the plane of section **S**), see Figure 1.

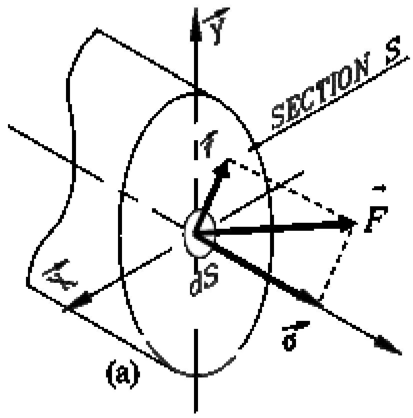


Figure 1. Components of total stress

The point of application of a force in a soil mass may correspond either to a particle or to a pore. It is obvious that a pore cannot support any load, whereas if the force is applied directly to a particle, the stresses can be very high.

Therefore, when referring to stress in a geotechnical context, it is defined as a force per unit area, where the area considered is the total area; this area includes both grain-to-grain contacts and pores (Figure 2).

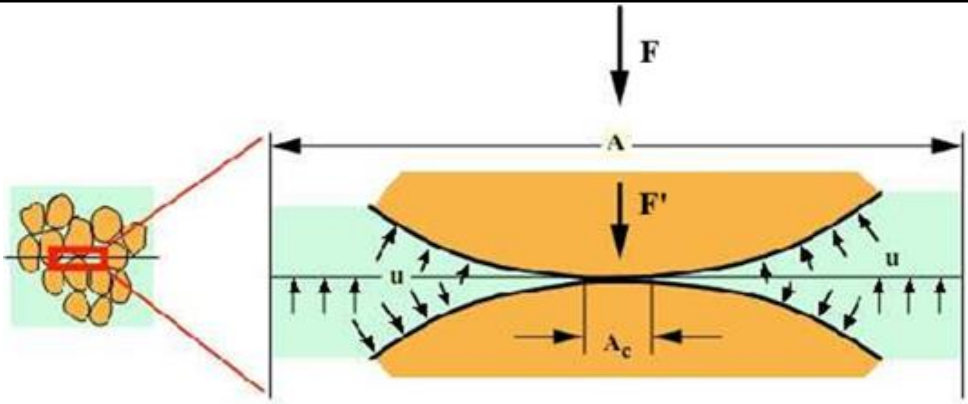


Figure 2. Forces acting at the contact points between soil particles.
after Alec Skempton, 1960

- **F** = total normal force
- **F'** = intergranular normal force
- **u** = pore water pressure
- **A** = total area
- **Ac** = contact area between grains

➢ Effective Stress:

Effective stress is the stress transmitted to the skeleton of solid grains through intergranular contact, as shown in Figure 2.

The concept of effective stress was developed by Professor Karl Terzaghi in 1923 (Terzaghi's postulate). The corresponding symbols are marked with a prime, giving the normal stress **σ'** and the shear stress **τ'**.

➢ Pore Water Pressure:

Pore water pressure is the pressure of the water occupying the voids or pores between the solid soil grains. In other words, it is the pressure existing in the pore water.

It is a hydrostatic-type pressure, meaning it acts normally to the considered section. Pore water pressure is denoted by the symbol **μ**.

3. Stress Tensor (Mohr's Circle):

The stress at any point **M** can be vectorially decomposed into a component perpendicular to the surface (SS'), as shown in Figure 3, denoted by **σ** and called the normal stress, and a component parallel to the surface (SS'), denoted by **τ** and called the shear (tangential) stress.

This representation is commonly analyzed using the concept of Mohr Circle.

$$\vec{F} = \vec{\sigma} + \vec{\tau}$$

If we change the orientation of the plane (SS') passing through point **M**, we obtain a new stress vector $\mathbf{F_1}$ at point **M**, and therefore new values of the stress components, as shown in Figure 3.

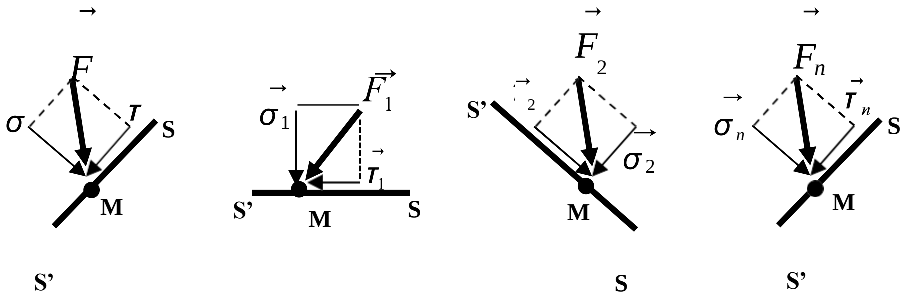


Figure 3. Stress state when a plane SS' rotates around a point M

The Mohr representation is a graphical method used to represent all stress vectors acting at a point **M** for different planes (facets) within a coordinate system (**σ, τ**), where **σ** represents the normal stress component and **τ** represents the shear (tangential) stress component of each stress state.

In a graph where **σ** is plotted on the horizontal axis (abscissa) and **τ** on the vertical axis (ordinate), each stress state can be represented by a point N_i (Figure 4).

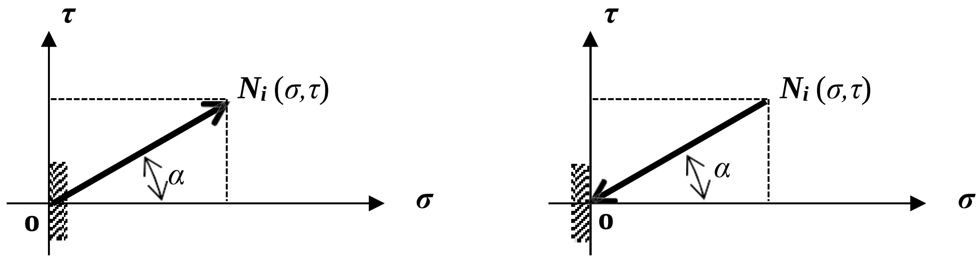


Figure 4. Mohr Representation

Mohr showed that this domain is bounded by three tangent circles centered on the **σ-axis** (Figure 2.5a). These three circles are defined by the three principal stresses σ_1, σ_2, **and** σ_3, represented by the points N_1, N_2, **and** N_3.

The largest of these three circles is called the **Mohr circle**. It is important to note that it does not depend on the intermediate principal stress σ_2, but only on the minor and major principal stresses σ_3 **and** σ_1.

It can also be characterized by the abscissa σ_m of its center and its radius τ_m (Figure 5b).

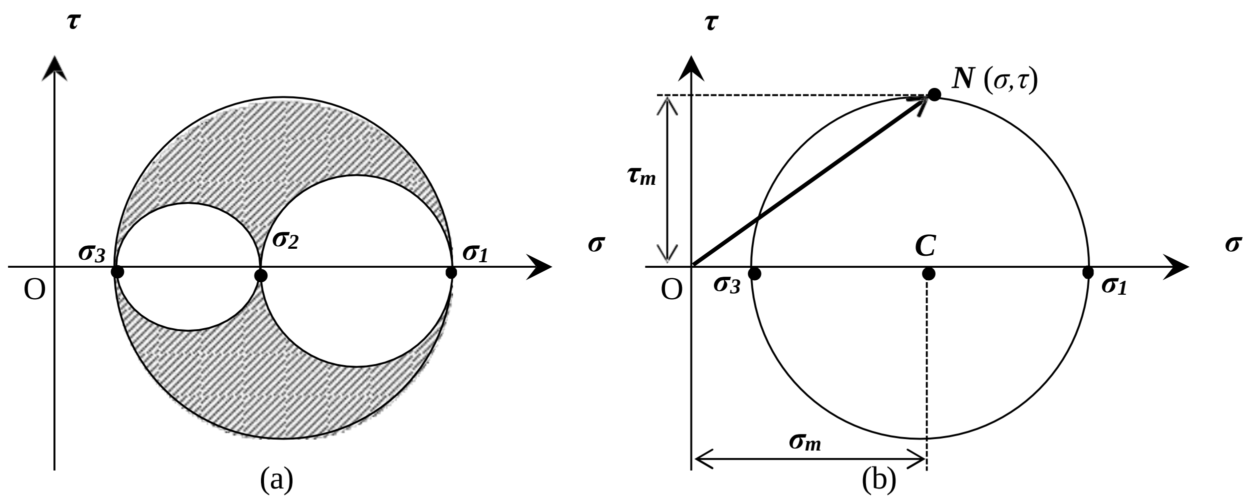


Figure 5. Characteristics of Mohr's Circle

From the graphical representation of Mohr, we define the following:

- **Mean stress:** $(p = (\sigma 1 + \sigma 2 + \sigma 3) / 3)$ This mean stress is also called the **octahedral normal stress** (σ_{oct}).
- **Deviatoric stress (stress deviator)**: $q = \sigma 1 - \sigma 3$

  This stress deviator corresponds to the **diameter of Mohr's circle**

- **Lambe's parameters** :

  $S = \sigma m = (\sigma 1 + \sigma 3)/2$ (It is the position of the **center of Mohr's circle**)

  $t = \tau m = (\sigma 1 - \sigma 3)/2$ (It is the **radius of Mohr's circle**)

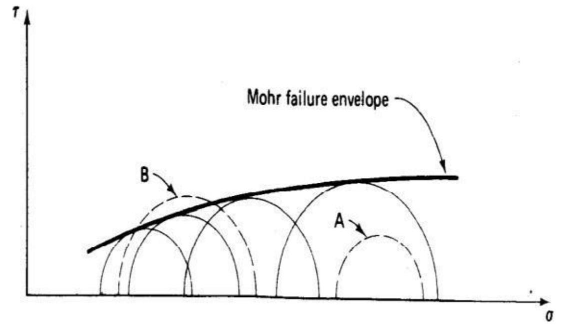
It should be noted that **Mohr's circle is completely defined by these two parameters**

• **Note:** In effective stress analyses, the quantities ($\sigma 1$, $\sigma 3$, p, q, τm, σm) are replaced by ($\sigma' 1$, $\sigma' 3$, p', q', $\tau' m$, $\sigma' m$).

• In the (σ,τ) plane, the limit of the elastic domain can be represented by a curve called the **"intrinsic curve"** or the **Mohr failure envelope**. It is the envelope of Mohr's circles corresponding to failure.

• **Note:**
– Any Mohr's circle located **below the failure envelope (circle A)** represents **stable conditions (before failure)**;
– **Failure occurs only when the circle is tangent to the envelope**;
– There cannot be any circle **above the envelope (circle B)**.

Enveloppe de rupture de Mohr



• **Figure 6. Intrinsic curve (Mohr failure envelope)**

1. Calculation of stresses within a soil mass

1.1 Stresses due to the self-weight of soil

Soil is a porous medium composed of a solid skeleton whose voids are filled with air and/or water. In its natural state, it is generally at rest and in equilibrium under the effect of gravity.

We assume that soil behaves as a **continuous medium** subjected to its self-weight (the soil weight increases with depth)

- The **total vertical stress** (σ_v), for a soil unit weight (γ) (kN/m³) at a depth (z) (m), is given by:

$$\sigma_v = \frac{F}{S} = \frac{\gamma.V}{S} = \frac{\gamma.S.z}{S}$$

Therefore $\sigma_v = \gamma .z$

It increases with soil depth

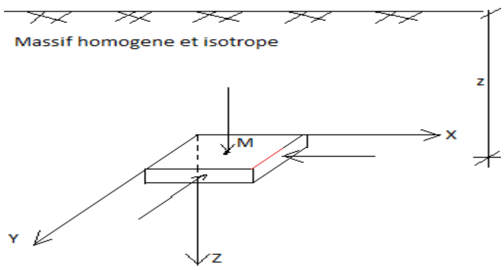
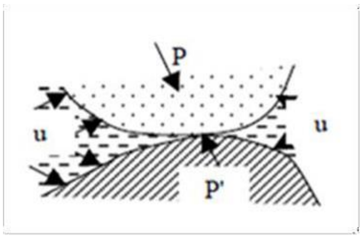


Figure 7. Stress in soil due to its self-weight

• **Effective stress:**
The different phases that make up a soil, whether saturated or not, are not governed by the same laws. The study of the gaseous or liquid phases falls within the field of fluid mechanics or hydraulics.

For the study of the strength and deformation of the solid phase, **Terzaghi (1923)** introduced the concept of effective pressure, which is the pressure actually applied to the solid skeleton; it is the intergranular pressure. Hence, **Terzaghi's postulate**, or the **principle of effective stresses**, which states that the mechanical behavior of soil depends only on effective stresses, and this is very important in geotechnical engineering.



P = σ: Total pressure
P′ = σ′: Effective pressure
U: Pore water pressure

Figure 8. Stresses in the soil

Terzaghi's Law or Terzaghi's Postulate

$$\sigma = \sigma' + u \; ; \qquad \tau = \tau'$$

σ is the total normal stress, and τ is the shear stress.
σ′ is the effective normal stress, and τ′ is the effective shear stress.
u is the pore water pressure.

σ′ cannot be measured directly; it can only be calculated as:
σ′ = σ − u

In an unsaturated soil, the liquid phase is neglected and capillary stresses are zero everywhere:
U = 0 and **σ = σ′, τ = τ′.**

Distribution of stresses in soil
For the calculation of stresses in soil masses due to their self-weight, a distinction is made between dry or moist layers and saturated layers.
We assume a saturated soil. The groundwater level coincides with the ground surface.

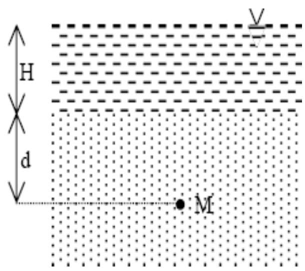


Figure 9. Saturated soil mass

σ = γsat · h ; γsat is the saturated unit weight
u = γw · h ; **γw** is the unit weight of water
σ′ = σ − u = (γsat − γw)·h = γ′ · h

where **γ′** is the submerged (buoyant) unit weight

For a dry soil, the effective stress is equal to the total stress.
$\sigma' = \sigma = \gamma d\ h$; **γd** is the dry unit weight

For a moist soil: **$\sigma = \gamma h\ h$; γh** is the moist unit weight

For stratified (layered) soils or multiple layers:
$\sigma = \Sigma\ \gamma i\ hi$;
$U = \gamma w\ \Sigma\ hi$ et $\sigma' = \sigma - u$

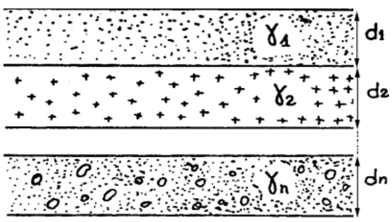


Figure.10. sol lité

2. Example of stress calculation:

2.1. Stress state of soil under its self-weight:
Consider a semi-infinite soil mass with a horizontal surface and not subjected to any external force. Let **σv** be **the vertical stress** on a horizontal plane at an arbitrary point **M** within the medium, and **σh** the **horizontal stress** on a vertical face of this plane (Figure 11).

Soil surface

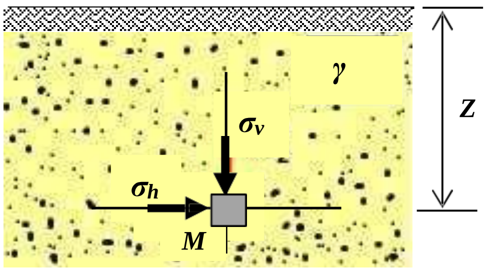


Figure 11. Principal stresses in a soil with a horizontal surface

• **Calculation of vertical stress σv and effective stress σ′v:**

- The vertical component of the stress due to the self-weight of the soil is given by:
  $\sigma v = \gamma \cdot z$

where **z** denotes the depth at which the vertical stress is calculated, and **γ** is the apparent unit weight of the soil in question.

- For a multilayer soil, the vertical stress is calculated as follows:

For a multilayer soil, the vertical stress is calculated as follows:

$$\sigma v = \Sigma \gamma i \cdot hi$$

where **hi** and **γi** are respectively the thickness and the apparent unit weight of layer **i.**

- In the case of a groundwater table located at depth **H** with (**H < Z**), as illustrated in Figure 2.9, the vertical stress is calculated as follows:

$$\sigma v = \gamma h \cdot H + \gamma sat\ (Z - H)$$

$$\sigma' v = \gamma h \cdot H + \gamma' (Z - H)$$

Soil surface

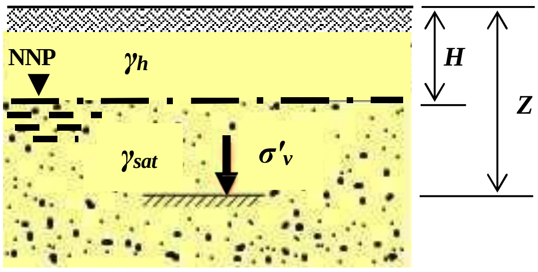


Figure 12. Stress below a groundwater table

1.2. Flooded soil with a horizontal surface:

We consider a horizontal plane located at a depth **Z** in the soil, with the groundwater free surface located at a height **hw** above the ground surface, as shown in Figure 13. **Costet and Sanglerat (1981)** and **Mc Carthy (2007)** show that, from Figure 13, we have:

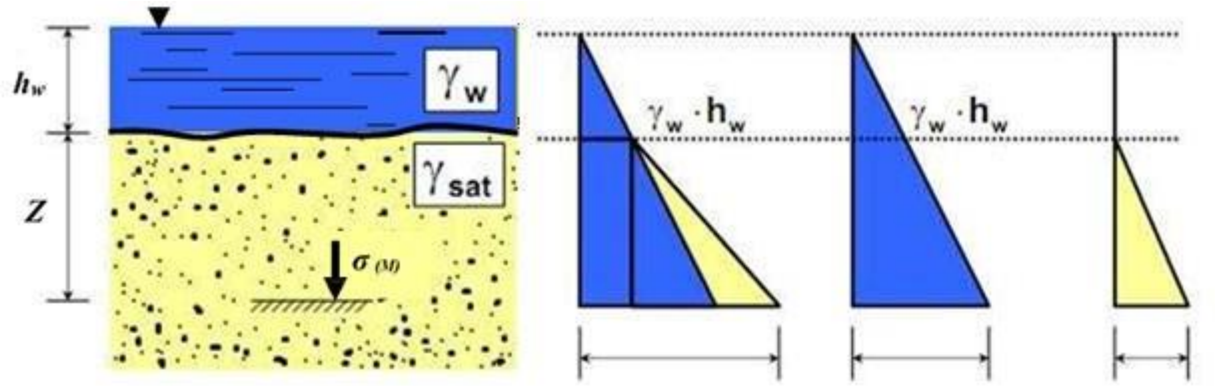
$$\sigma(M) = \gamma w \cdot hw + \gamma sat \cdot Z$$

where **γsat** is the saturated unit weight of the soil. The pore water pressure is:

$$\mu(M) = \gamma w\ (hw + Z)$$

and therefore, the effective stress is:

$$\sigma'(M) = \sigma(M) - \mu(M) = (\gamma sat - \gamma w)\ Z = \gamma' \cdot Z$$



$\gamma_w \, . \, h_w + \gamma_{sat} \, . \, Z$ $\gamma_w(h_w + Z)$ $\gamma' \, . \, Z$

Figure 13. Flooded soil with a horizontal surface