Lecture 02

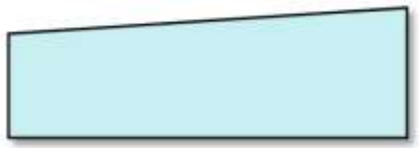
Tension and Compression

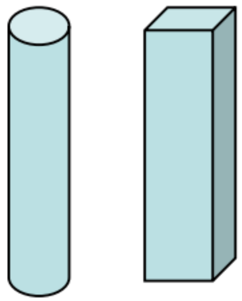
- ✓ Normal stress and strain of a prismatic bar
- ✓ Elasticity and plasticity
- ✓ Hooke’s law Strain energy

Definitions

Prismatic bar: is a straight structural member having the same cross section throughout its length

Non prismatic member with non uniform stresses

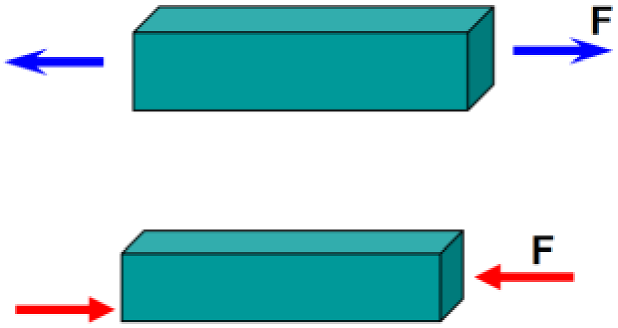




Axial force: A load directed along the axis of the member resulting either tension of compression in the bar.

Examples: members of bridge truss,

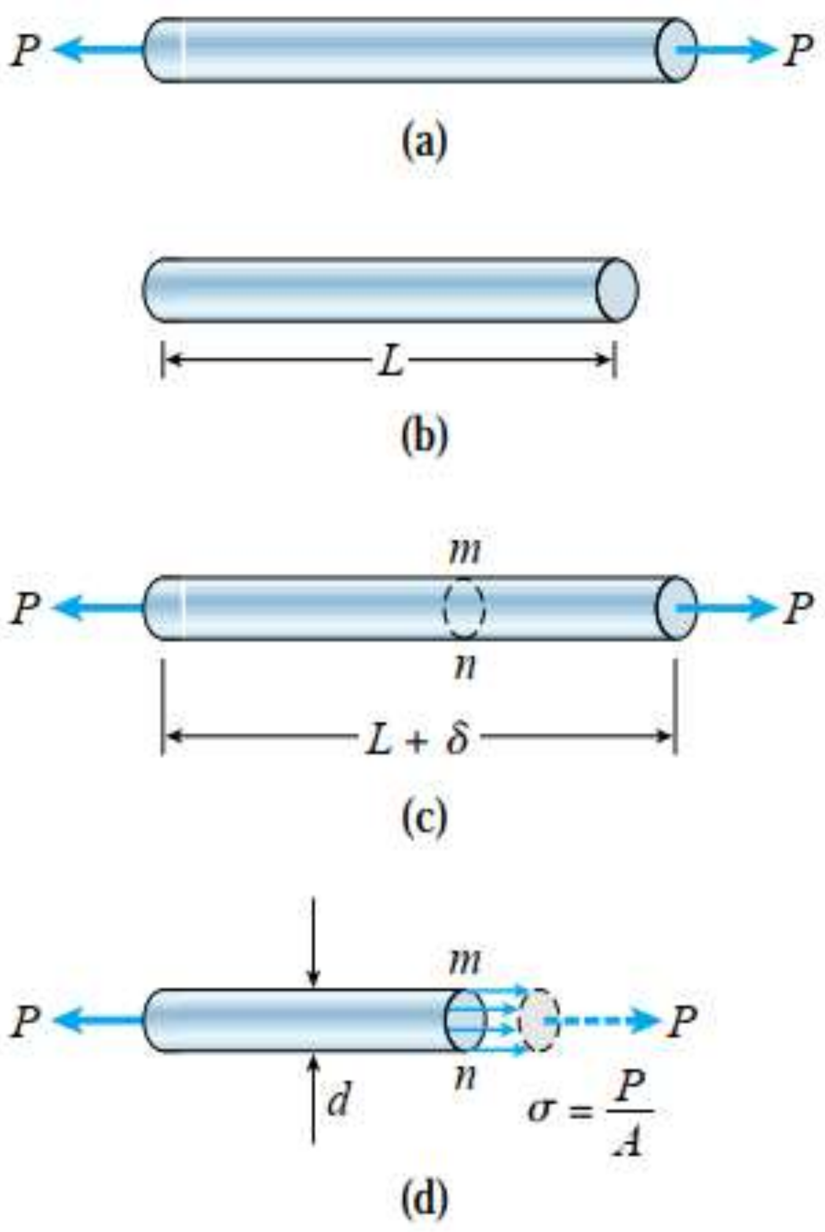
spokes of bicycle wheels, columns in buildings, etc.



These concepts can be illustrated in their most elementary form by considering a prismatic bar subjected to axial forces.

A prismatic bar is a straight structural member having the same **cross section** throughout its length, and an **axial force** is a load directed along the axis Examples are shown in Fig. 1, where the tow bar is a prismatic member in tension.

The internal actions in the bar are exposed if we make an imaginary cut through the bar at section *mn* (Fig. 1-c). Because this section is taken perpendicular to the longitudinal axis of the bar, it is called a **cross section**.



Normal stress and strain

The most fundamental concepts in mechanics of materials are **stress** and **strain**.

Stress is the intensity of the internal forces acting on a specific area. Stress is measured in pascal in SI Unites (1 Pa = 1 N/m2) and in psi (pound per square inch) in US Unites. In general stresses can be classified into:

Normal Stress is the intensity of the force acting normal to the area (σ) sigma, if the normal force or stress pulls on the area then it referred to as tensile stress, and if it pushes on the area it referred to as compressive stress.

Normal Strain As already observed, a straight bar will change in length when loaded axially, becoming longer when in tension and shorter when in compression.

Stress is defined as the distribution of a force acting over an area (stress = force per unit area).

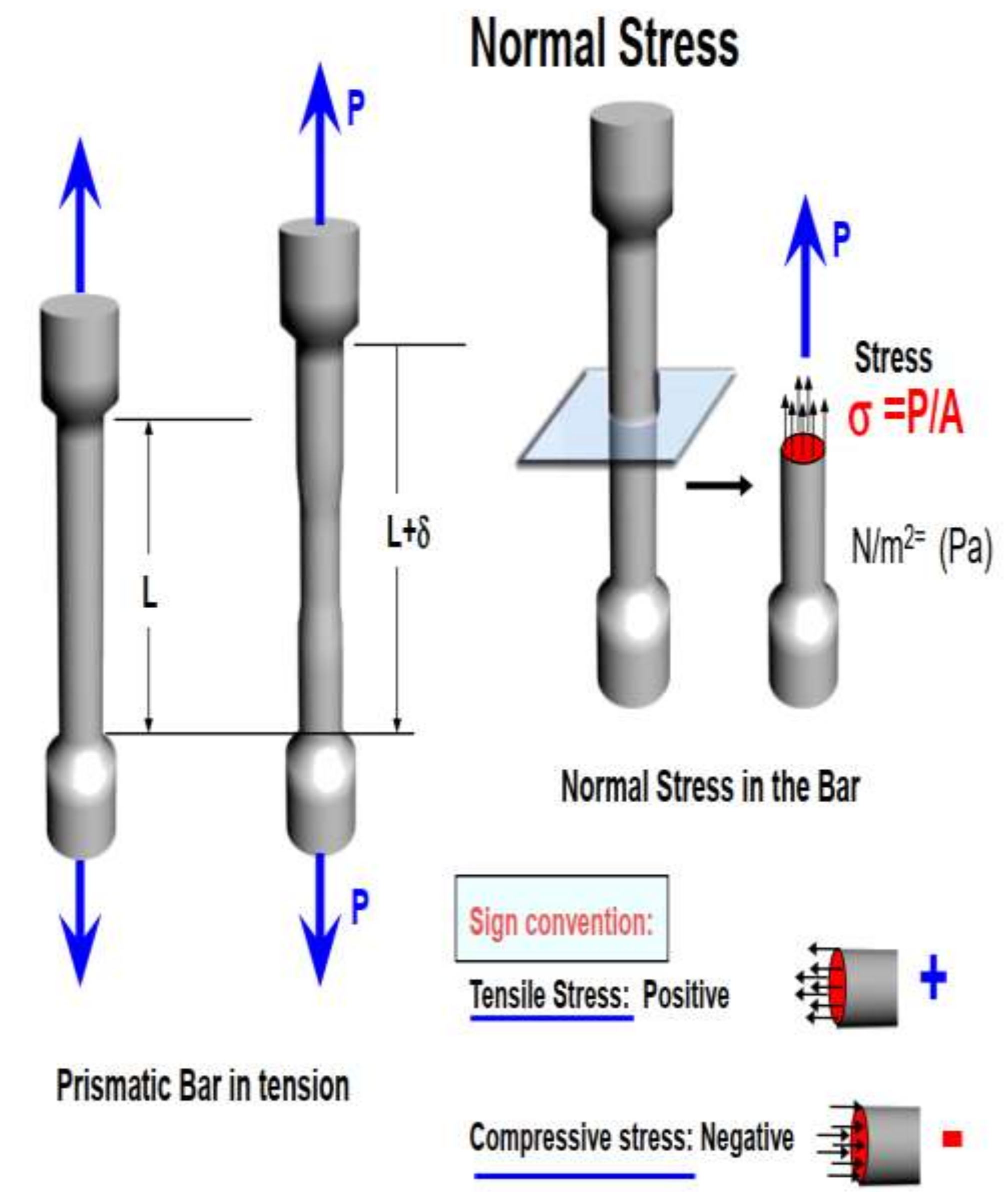
- **Extensional strain** is defined as the elongation/shortening of an element divided by the original length of the element (extensional strain = elongation/shortening per unit length)

a) Normal stress resultants

Uni-axial loading of member having a cross-sectional area ***A*** by an axial force ***P***:

Where: σ is the stress, ***P*** is the force applied, ***A*** is the cross-sectional area over which the force is distributed. Stress can be classified into different types, including:

- Tensile stress: When the material is stretched (extended).
- Compressive stress: When the material is compressed.



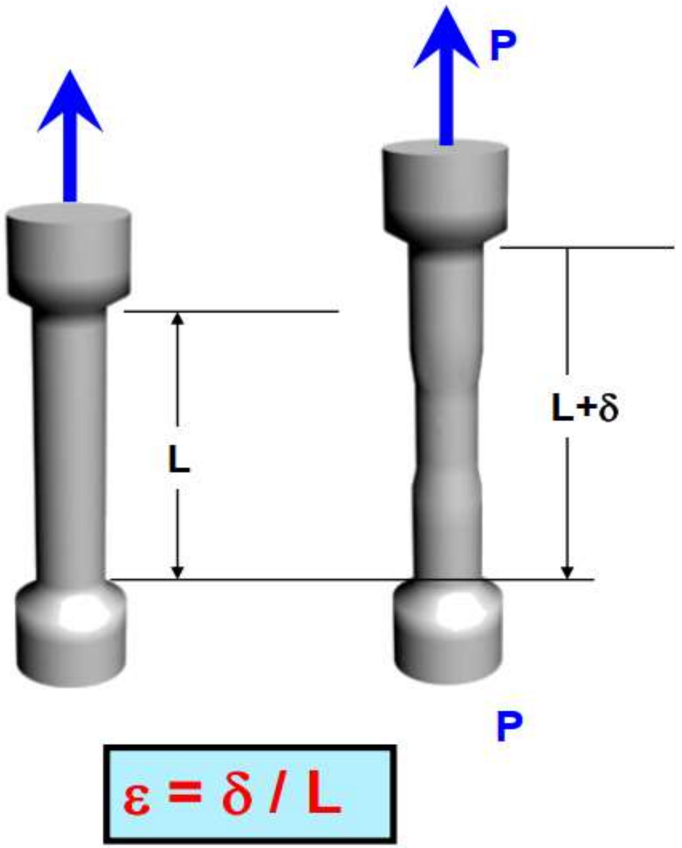
Example.1.

A steel bar having L = 2.0 m and diameter D= 50mm, when loaded in tension with tensile load P= 30kN the bar elongated by 1.4 mm. What is the axial stress and strain?

$$\varepsilon = \delta / L = \frac{1.4\ \text{mm}}{2*1000\text{mm}} = 7\text{x } 10^{-4}$$

$$\sigma = P/A = 30 / |(\pi (0.05)^2/4) = 0.05887$$

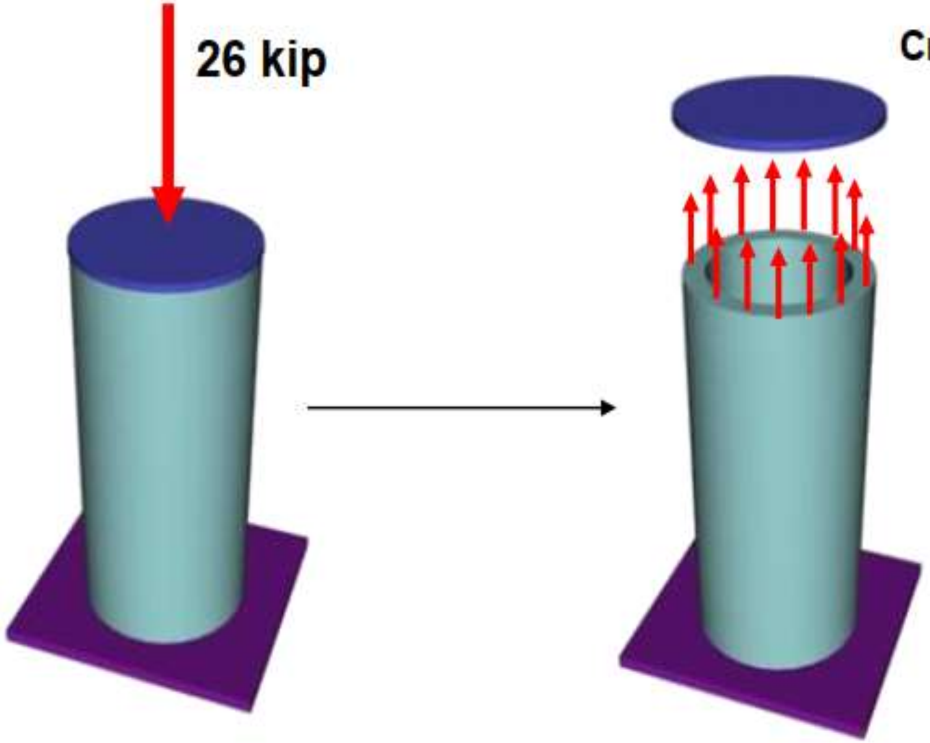
kN/m^2 (kPa)



Example.2.

A short post constructed from a hollow circular tube of Aluminum supports a compressive load of 26 kips (1kips = 4,448 N, 1 inch = 2.54 cm. The inner and outer diameters of the tube are d_1 = 4.0 in and d_2 =4.5 in respectively, and its length is 16 in. the shortening of the post due to the load is 0.012 in.

Determine the compressive stress and strain the post.



Cross section Area (A) = $\pi(4.5^2 - 4.0^2)/4 =$

3.338 in^2

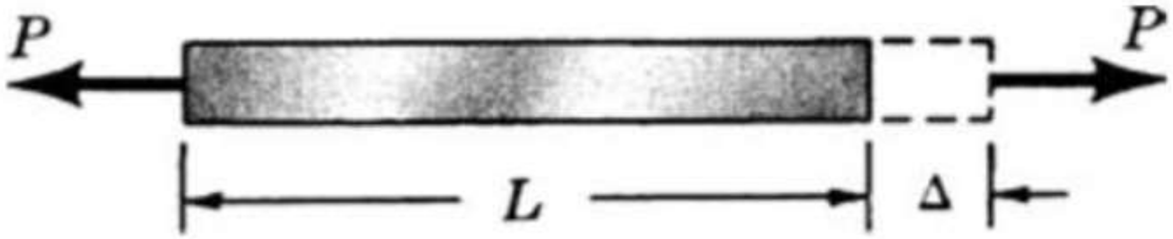
$\sigma = P/A = 26 / 3.338$

$= 7.790 \text{ ksi}$

$\varepsilon = \delta / L = 0.012 / 16$

$= 750 \times 10^{-6} \text{ in/in}$

In Fig, determine an expression for the total elongation of an initially straight bar of length **L**, cross-sectional area **A**, and modulus of elasticity **E** if a tensile load **P** acts on the ends of the bar.



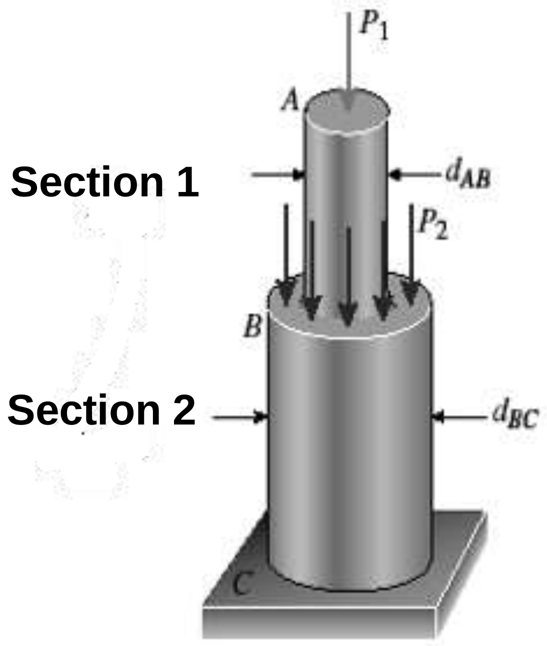
$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta/L} = \frac{P * L}{A * \Delta} \qquad \Delta = \frac{P * L}{A * E}$$

Example.3.

A solid circular post ABC (see figure) supports a load $P_1 = 2500$ lb acting at the top. A second load P_2 is uniformly distributed around the shelf at B. The diameters of the upper and lower parts of the post are $d_{AB} = 1.25$ in. and $d_{BC} = 2.25$ in., respectively.

(a) Calculate the normal stress σ_{AB} in the upper part of the post.

(b) If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load P_2?



Section 1

(a) Normal stress in part AB

$$\sigma_{AB} = \frac{P_1}{A_{AB}} = \frac{2500 \text{ lb}}{\frac{\pi}{4}(1.25 \text{ in.})^2} = 2040 \text{ psi} \quad \longleftarrow$$

Section 2

(b) Load P_2 for equal stresses

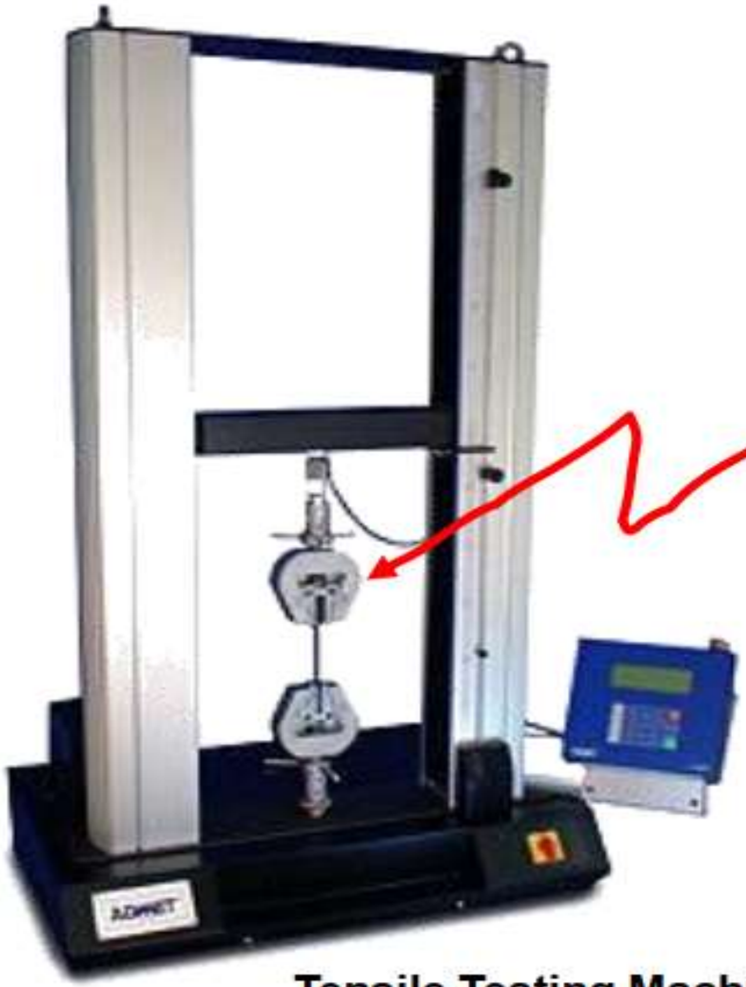
$$\sigma_{BC} = \frac{P_1 + P_2}{A_{BC}} = \frac{2500 \text{ lb} + P_2}{\frac{\pi}{4}(2.25 \text{ in.})^2}$$

$$= \sigma_{AB} = 2040 \text{ psi}$$

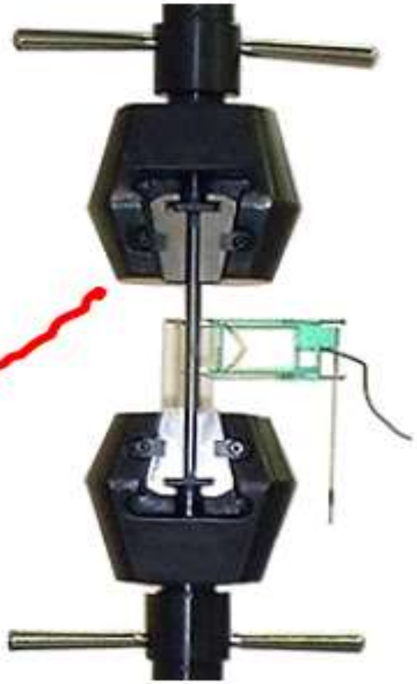
Solve for P_2: $\quad P_2 = 5600$ lb $\longleftarrow$

Mechanical Properties of Materials

The design of machines and structures so that they will function properly requires that we understand the *mechanical behavior of the materials* being used.



Tensile Testing Machine



Gripping Devices

Tensile Testing

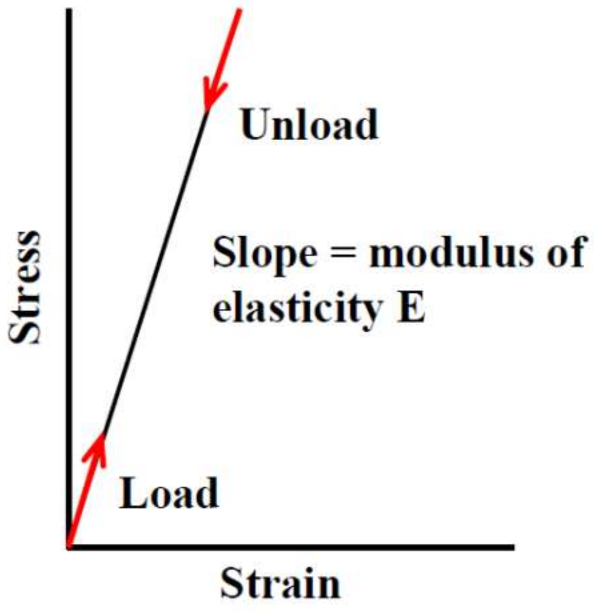
Tensile tests are carried out by gripping the ends of a suitably prepared standardised test piece in a tensile testing machine and then applying a continually increasing uni-axial load until such time as failure occurs

Linear elasticity & Hooke's law

Stress-Strain Behavior: Elastic deformation in tensile tests, if the deformation is elastic, the stress strain relationship is called Hooke's law:

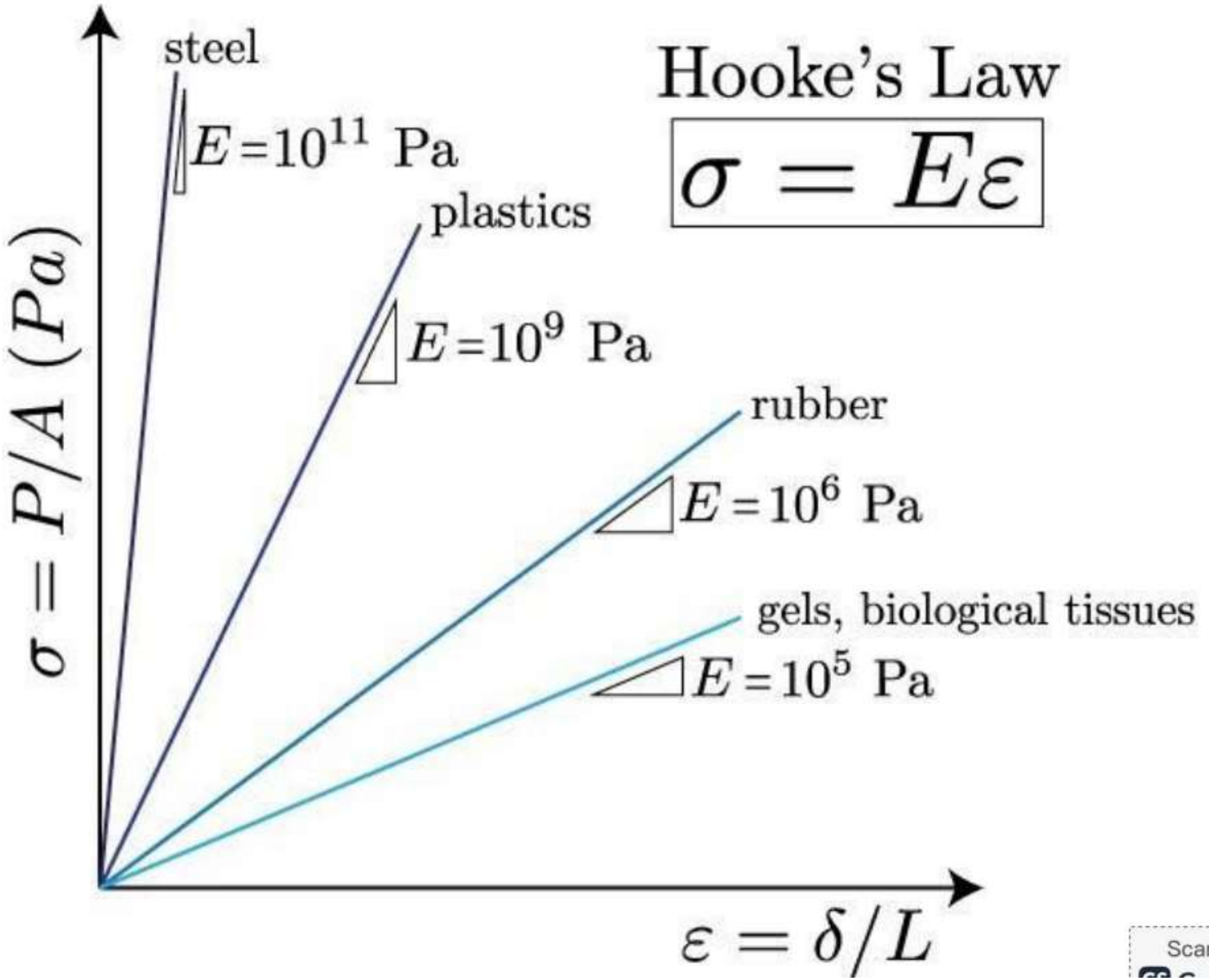
$$\sigma = E\,\varepsilon$$

E is Young's modulus or modulus of elasticity, has the same units as σ, N/m2 or Pa.



Higher E → higher "stiffness"

Simple Strain



Example 4

A 80 m long wire of 5mm diameter is made of a steel with E=200 Gpa and

an ultimate tensile strenght of 400 Mpa. If a factor of safety of 3.2 is desired,

determine.

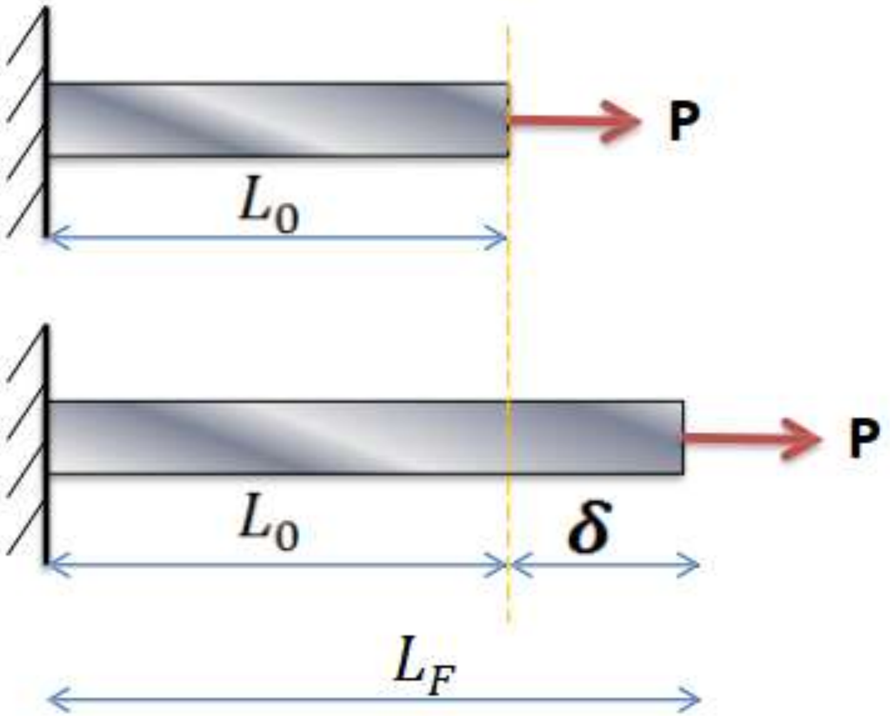
a-The largest allowable tension in the wire;

b-The corresponding elongation of the wire.

Also known as unit deformation, strain is the ratio of the change in length caused by the applied force, to the original length.

$$\epsilon = \frac{\delta}{L_0} = \frac{L_F - L_0}{L_0}$$

(Unit less)



Hooke's Law

$$\epsilon = \frac{\sigma}{E} = \frac{P/A}{E}$$

Deformation law

$$\frac{\delta}{L_0} = \frac{P/A}{E}$$

$$\delta = \frac{PL}{EA}$$

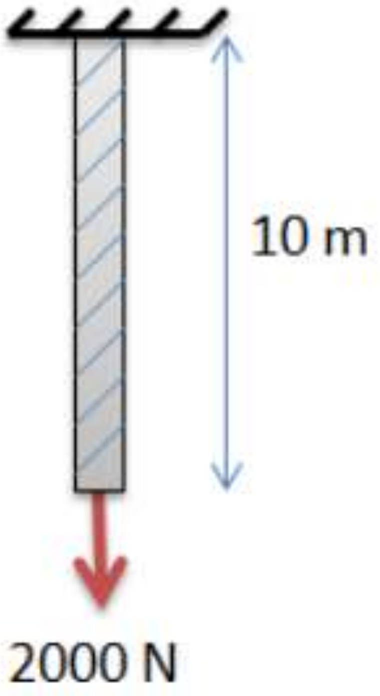
Unit :
mm

Q/ A steel wire 10m long, hanging vertically supports a tensile load of 2000 N. Neglecting the weight of the wire, Determine the required diameter if the stress is not to exceed 140Mpa and the total elongation is not o exceed 5mm. Assume E=200 Gpa?

Sol:

$$\sigma \leq 140Mpa$$

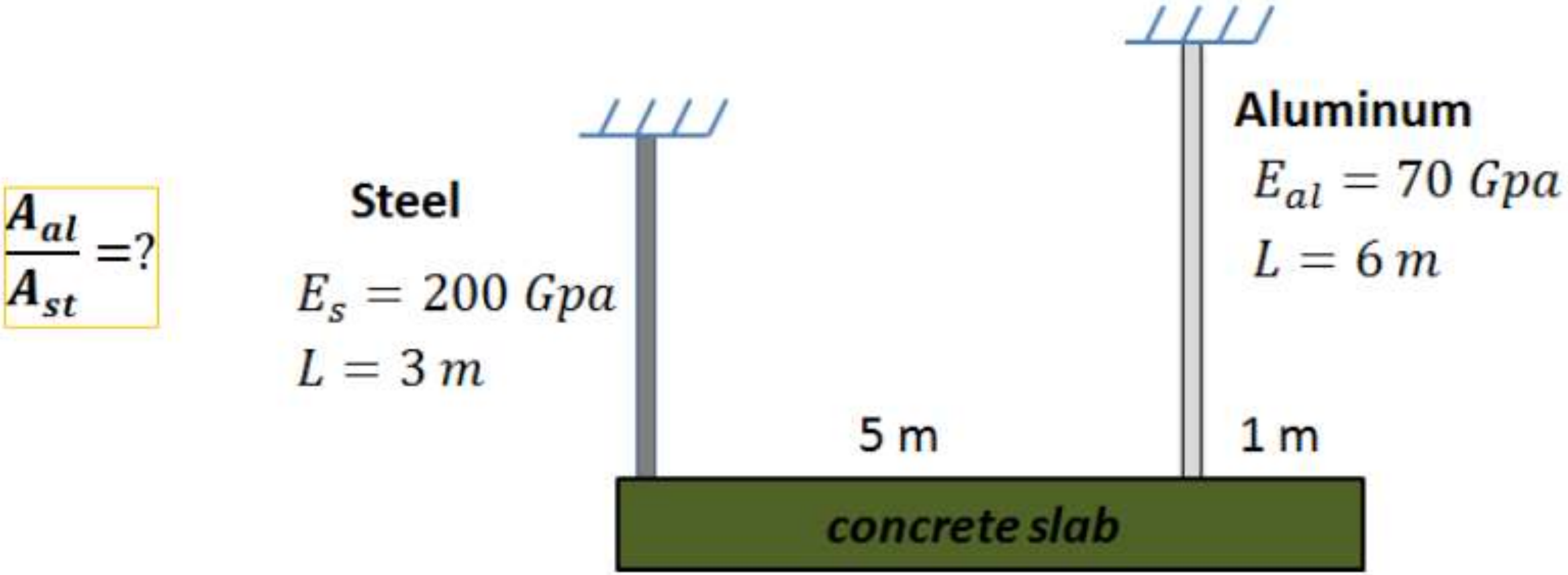
$\delta = 5\ mm$
P= 2000N
L= 10m
E= 200Gpa
d=?



$$\sigma = \frac{P}{A} = \frac{2000}{\frac{\pi}{4}(5.05)^2} = 99.90Mpa$$

$$99.90Mpa \leq 140Mpa$$

Q/ A uniform concrete slab of mass M is to be attached, as shown in Fig. to two rods whose lower ends are initially at the same level. Determine the ratio of the areas of the rods so that the slab will remain level after it is attached to the rods?



Stress- Strain Diagram

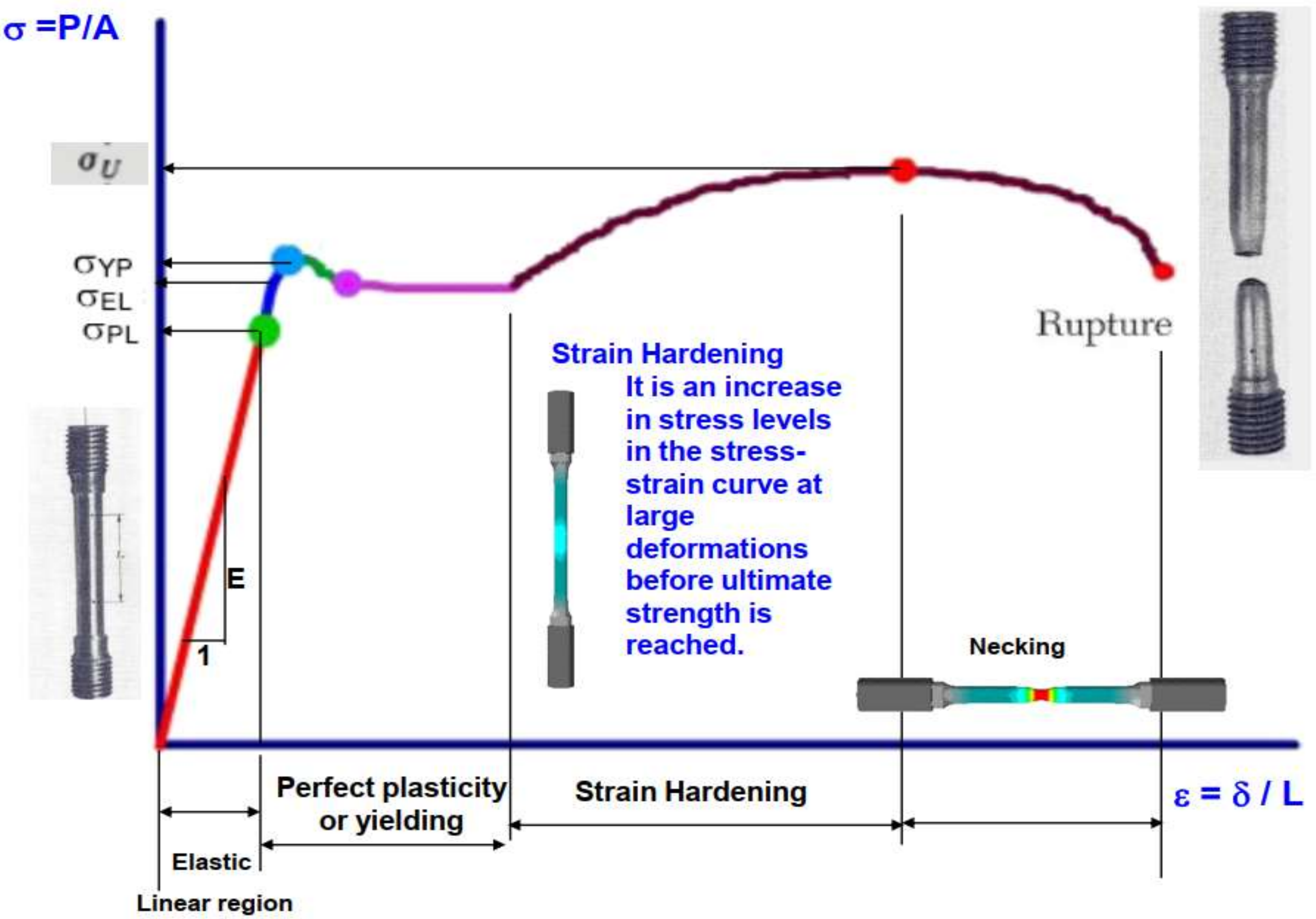
E . Modulus of Elasticity (Young’s Modulus) - Slope of the initial linear portion of the stress-strain diagram. The modulus of elasticity may also be characterized as the “stiffness” or ability of a material to resist deformation within the linear range.

Proportional limit : is the maximum value of the stress from the stress-strain diagram, where the stress and strain are proportional

Elastic Limit : is the maximum stress for a material to behave elastically, - the specimen will return to its original undeformed shape if the load is removed so long as the stress is below the elastic limit.

Yield Point: This defined as the maximum stress on stress-strain curve, where there is an appreciable increase in strain with no increase in stress. It is generally easier to determine than the proportional limit or elastic

Some materials do not exhibit a distinct yield point


σ =P/A
σU
σYP
σEL
σPL
E
1
Strain Hardening
It is an increase in stress levels in the stress-strain curve at large deformations before ultimate strength is reached.
Rupture
Necking
ε = δ / L
Elastic
Linear region
Perfect plasticity or yielding
Strain Hardening

Stress- Strain Diagram

σ = normal stress on a plane perpendicular to the longitudinal axis of the specimen

P = applied load

A = original cross sectional area

ε = normal strain in the longitudinal direction

δ = change in the specimen's gage length

L = original gage length

- **Engineering stress**

$\sigma = P/A_0$

- **True stress**

$\sigma = P/A$

- **Engineering strain**

$\varepsilon = (l - l_0)/l_0$

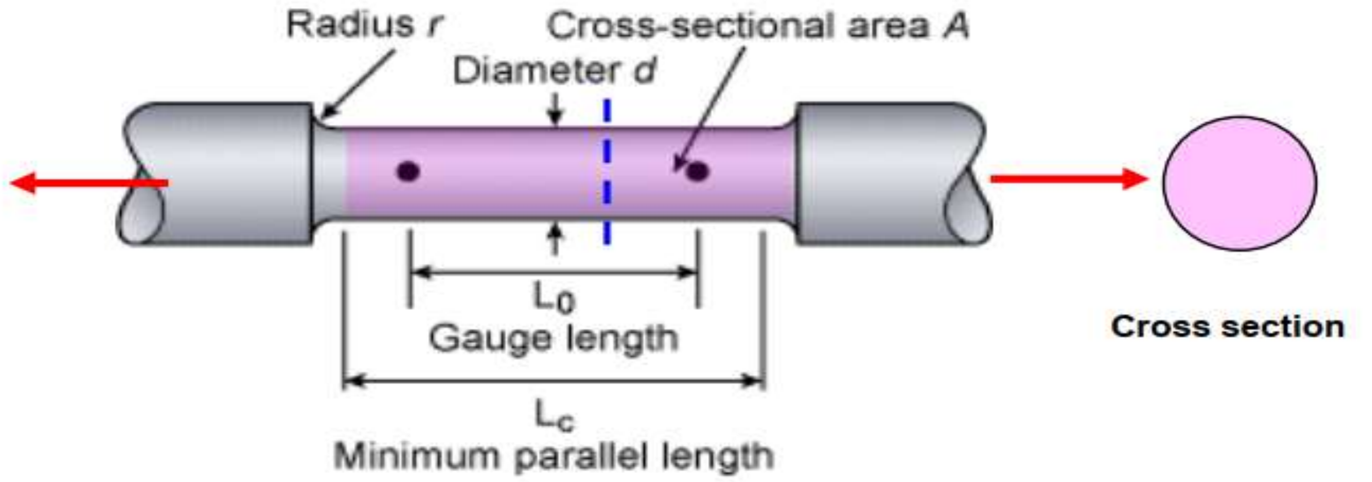
- **True strain (Logarithmic strain)**

$\varepsilon = \ln(l/l_0) = \ln(A/A_0)$

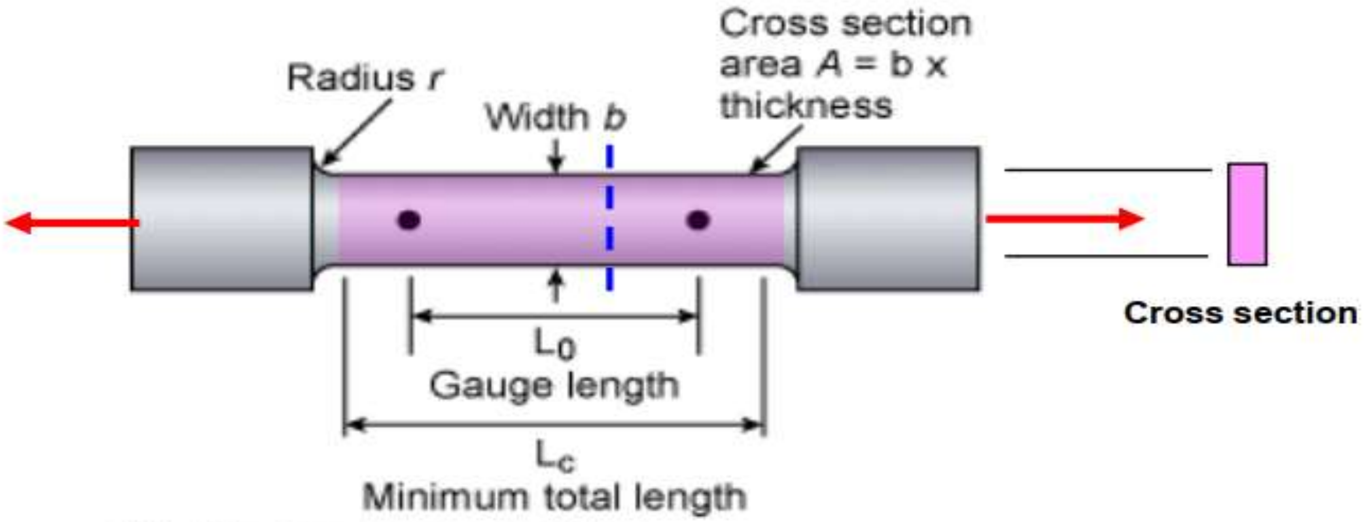
- **Volume must be the same $Al = A_0 l_0$**

stress-strain curve or diagram gives a direct indication of the material properties.

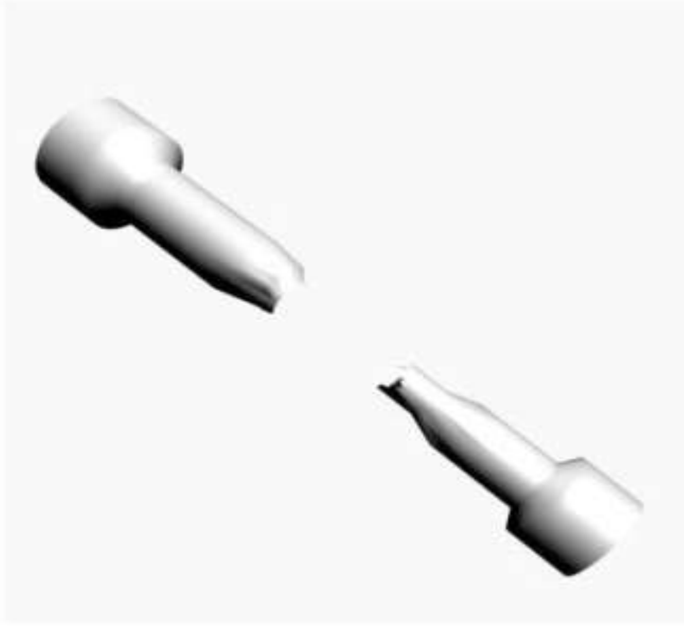
Tensile Test Specimen

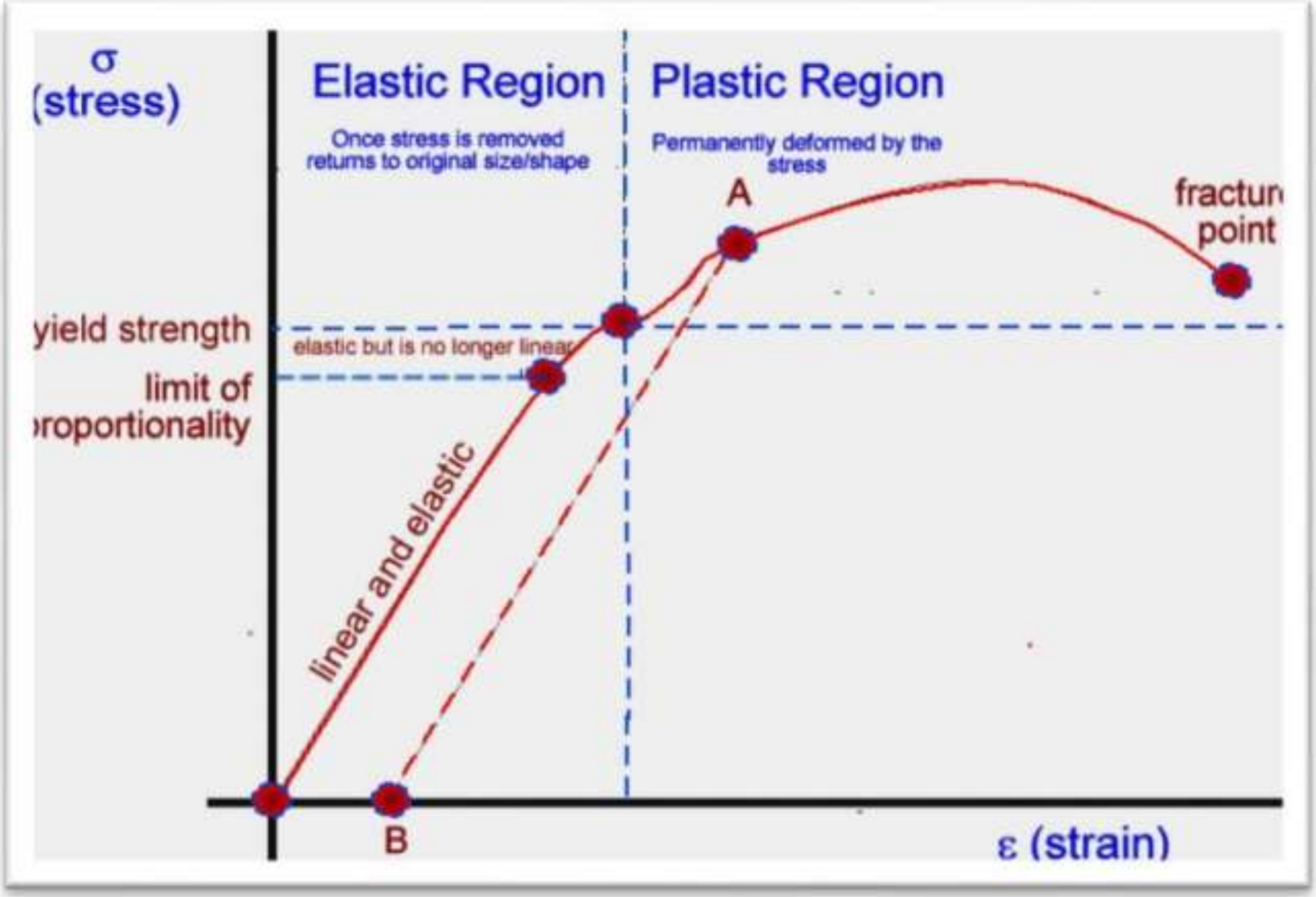


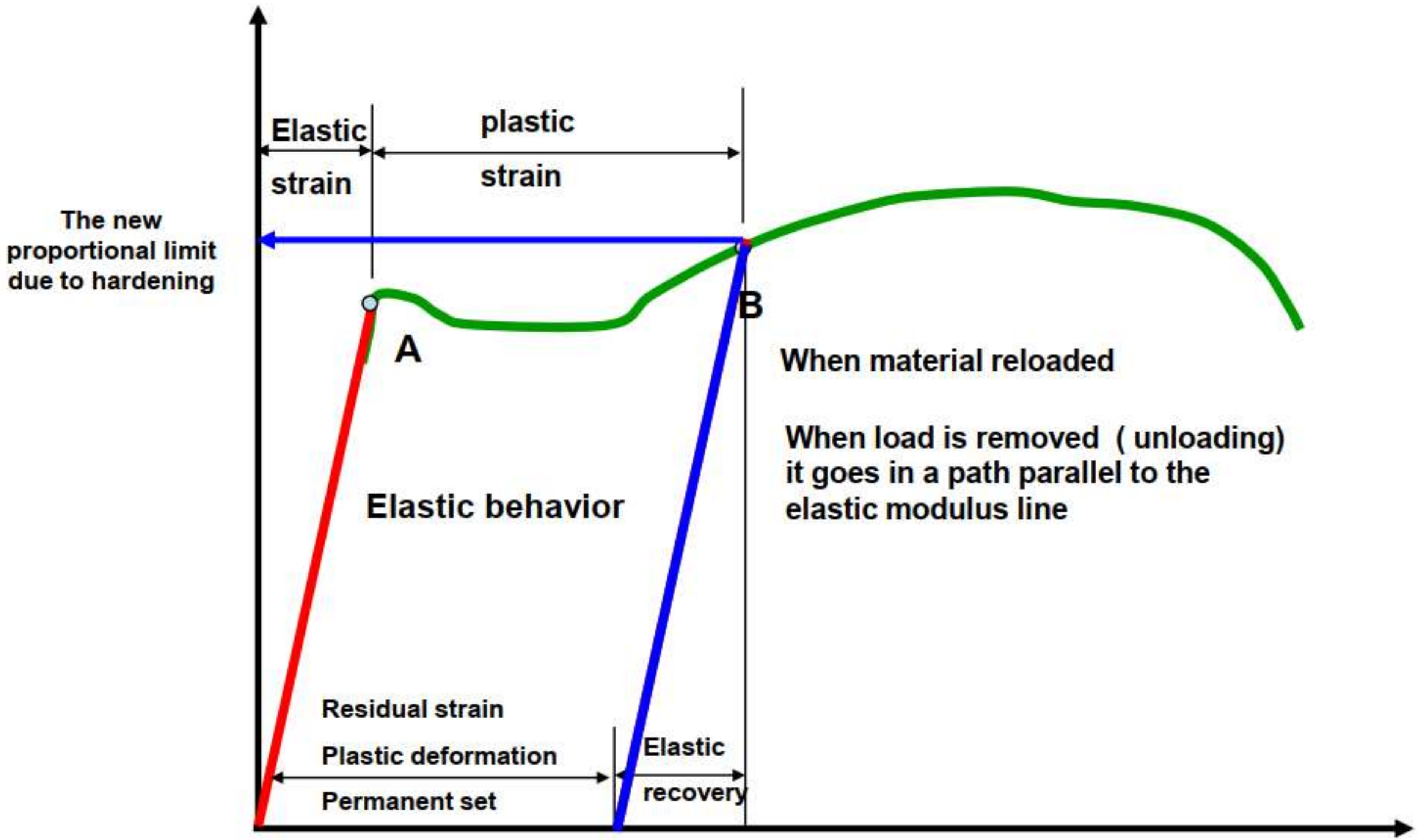
(a) Round cross section



(b) Square cross section




σ
(stress)
Elastic Region
Plastic Region
Once stress is removed
returns to original size/shape
Permanently deformed by the
stress
A
fractur
point
yield strength
elastic but is no longer linear
limit of
roportionality
linear and elastic
B
ε (strain)



Linear relationship between stress and strain

Strain is temporary, meaning that all strain is fully recovered upon removal of the stress

The slope of this is called the elastic modulus

Definition

Allowable strength design is a structural design methodology that ensures a structure's components can safely carry expected loads without exceeding their material strength limits. This approach uses a factor of safety to establish allowable stress levels, which are based on the material properties and the anticipated load conditions. The main goal is to prevent structural failures by keeping stresses within safe limits while also optimizing the material use for efficiency.

Design methods for strength

1- ASD Method:

In this method, the engineer uses the Allowable Stress Design ASD load combinations (below) to determine the required strength of a member and arranges for the allowable strength to satisfy this equation:

$$R_a \leq \frac{R_n}{\Omega}$$

Where:

R_a = required strength,

R_n = nominal strength,

Ω = safety factor,

R_n/Ω = allowable strength.

Example.4.

The 60-kg flowerpot is suspended from wires *AB* and *BC*. If the wires have a failure normal stress of $\sigma_{fail} = 380$ MPa, **determine** the minimum diameter of each wire. Use a factor of safety of 2.

SOLUTION

Internal Loading: The normal force developed in cables *AB* and *BC* can be determined by considering the equilibrium of joint *B*, Fig *a*.

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC}\cos 45^\circ - F_{AB}\cos 60^\circ = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\sin 45^\circ + F_{AB}\sin 60^\circ - 60(9.81) = 0 \qquad (2)$$

Solving Eqs. (1) and (2),

$$F_{AB} = 430.89 \text{ N} \qquad F_{BC} = 304.68 \text{ N}$$

Allowable Normal Stress: Using the F.S. = 2,

$$\sigma_{allow} = \frac{\sigma_{fail}}{\text{F.S.}} = \frac{380 \text{ MPa}}{2} = 190 \text{ MPa}$$

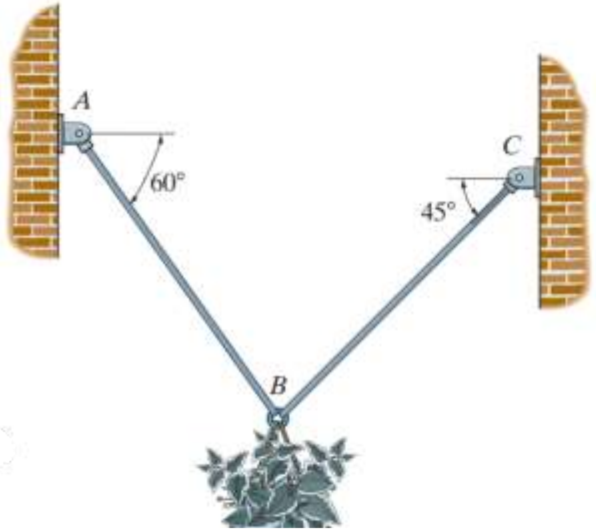
Then,

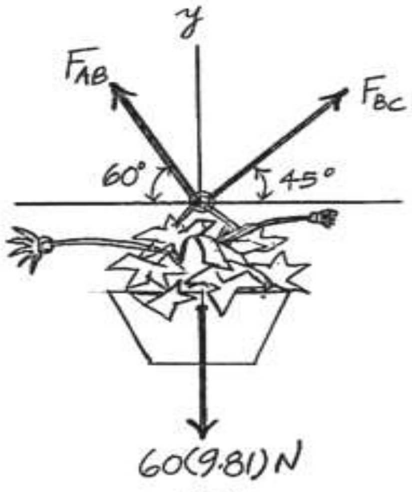
$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 190(10^6) = \frac{430.89}{\frac{\pi}{4}d_{AB}^2}$$

$$d_{AB} = 1.6993(10^{-3})\text{m} = 1.70 \text{ mm} \qquad \textbf{Ans.}$$

$$\sigma_{allow} = \frac{F_{BC}}{A_{BC}}; \quad 190(10^6) = \frac{304.68}{\frac{\pi}{4}d_{BC}^2}$$

$$d_{BC} = 1.4289(10^{-3})\text{m} = 1.43 \text{ mm} \qquad \textbf{Ans.}$$





(a)

2- LRFD Method:

In this method, the engineer uses the Load and Resistance Factor Design (LRFD) load combinations (below) to determine the required strength of a member and arranges for the allowable strength to satisfy this equation:

$$R_u \leq \phi * R_n$$

Where:

R_u = Required strength.

R_n = Nominal strength.

ϕ = Resistance factor.

$\phi \cdot R_n$ = Design strength.

Important Note: ASD versus LRFD: As per the AISC SCM, 14th ed., either

The two design methods are related through the Ω factor of ASD and the ϕ factor of LRFD. While these factors have different uses, they are always related by the following expression:

$$\Omega = \frac{1.5}{\phi}$$

The fundamental requirement of structural design is that the required strength not exceed the available strength; that is,

$$\text{Required strength} \leq \text{available strength}$$

In *allowable strength design* (ASD), a member is selected that has cross-sectional properties such as area and moment of inertia that are large enough to prevent the maximum applied axial force, shear, or bending moment from exceeding an allowable, or permissible, value. This allowable value is obtained by dividing the nominal, or theoretical, strength by a factor of safety. This can be expressed as

$$\text{Required strength} \leq \text{allowable strength} \tag{2.1}$$

where

$$\text{Allowable strength} = \frac{\text{nominal strength}}{\text{safety factor}}$$

Strength can be an axial force strength (as in tension or compression members), a flexural strength (moment strength), or a shear strength.

If stresses are used instead of forces or moments, the relationship of Equation 2.1 becomes

$$\text{Maximum applied stress} \leq \text{allowable stress} \tag{2.2}$$