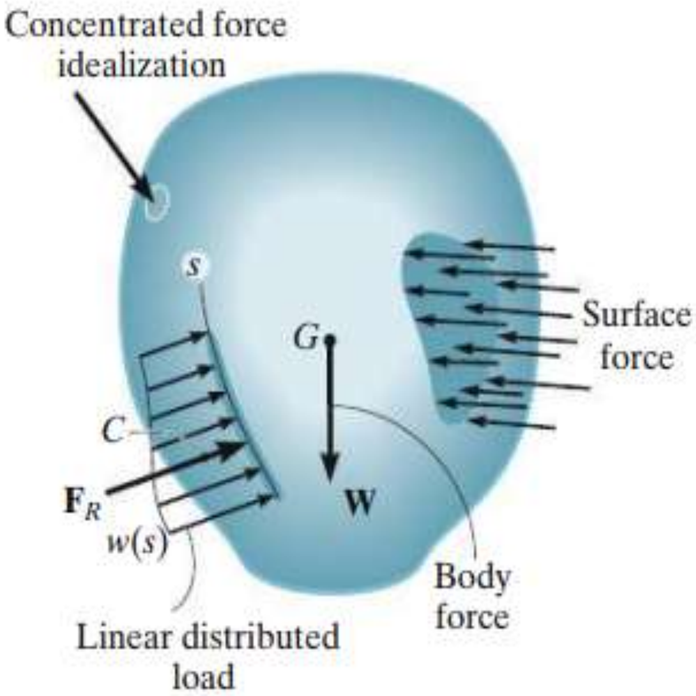
Statics & Strength of Materials

Strength of materials is widely used in mechanical and civil fields. Understanding this subject allows technicians and engineers to better analyze systems and to build more reliable products. Mechanisms are succumb to forces, torques, and loads, exposing them to deformation. The latter (deformation) should be thoroughly studied, since as we have already mentioned how the objects and beams that we’re studying take part in a mechanism, thus, every deformation counts. The slightest mistake in calculation or choice of either material or dimension can lead the system to fail. Reliable systems are demanded and essential for a sustainable future for the industry.

Equilibrium of a Deformable Body

We will review some of the main principles of statics that will be used throughout the text.

External Loads. A body is subjected to only two types of external loads; namely, surface forces and body forces, Fig. 1.



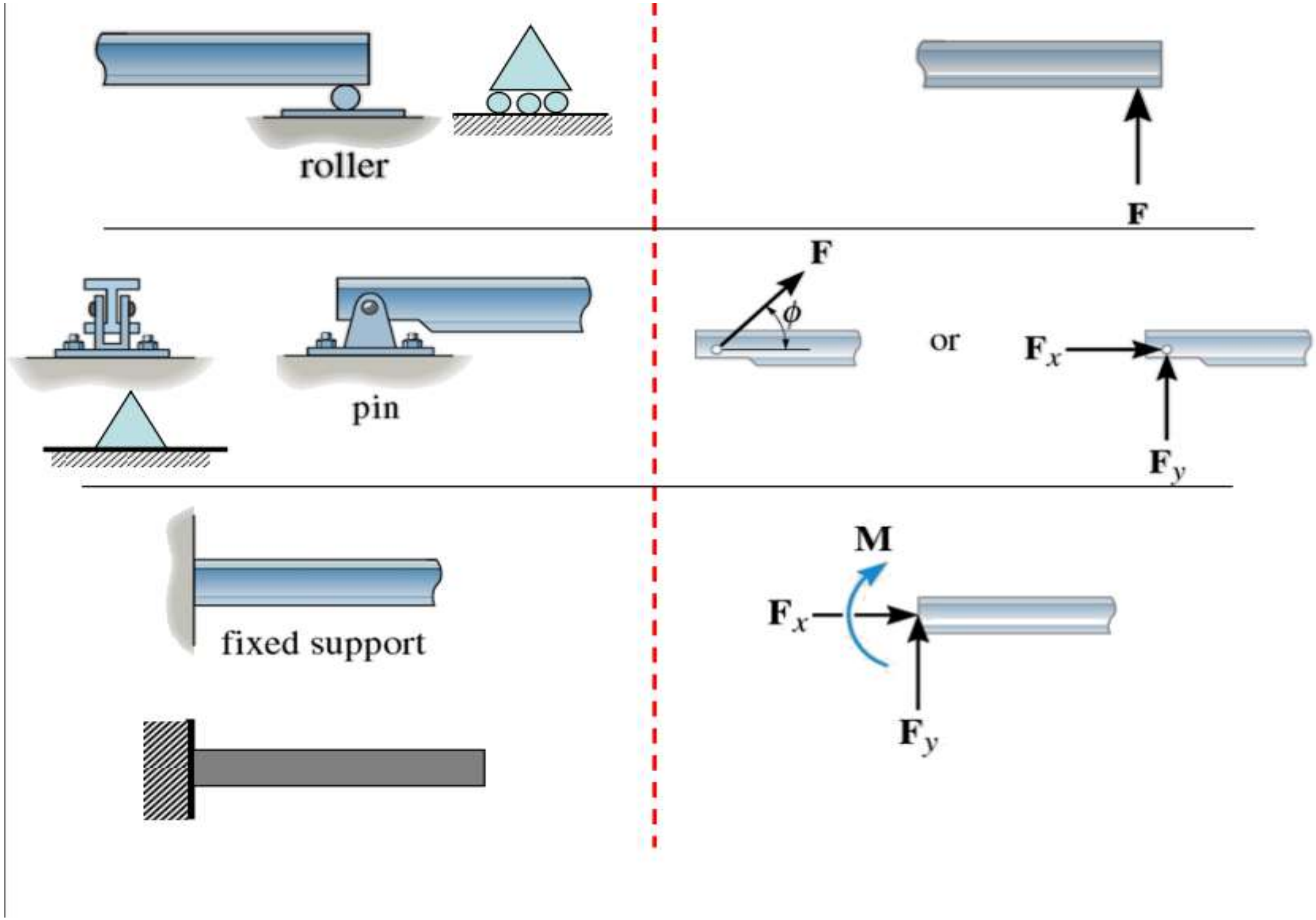
1. **Surface forces** are caused by the direct contact of one body with the surface of another. In all cases the forces are distributed over the area of contact, which results in three types of loads:

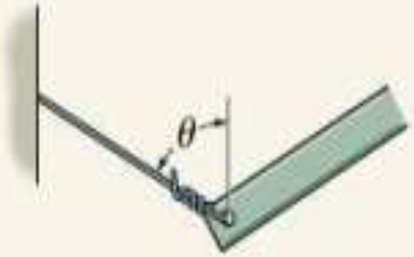
Concentrated force when the area of contact is too small compared to the total surface area (Ex. Force of ground on a wheel of a bicycle),

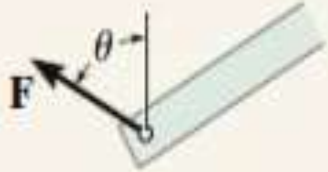
Linear distributed load when the contact area is a narrow strip of the body (Ex. Load along the length of a beam),

1. **Body Force** is developed when one body exerts a force on another body without a direct physical contact (Ex. Gravity force, weight)

Support Reactions: (as shown in Table 1)



<table>
<tr><th>Type of connection</th><th>Reaction</th><th>Type of connection</th><th>Reaction</th></tr>
<tr>
<td>

Cable</td>
<td>

One unknown: F</td>
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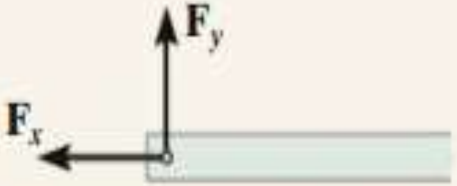
External pin</td>
<td>

Two unknowns: F_x, F_y</td>
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<tr>
<td>
Roller</td>
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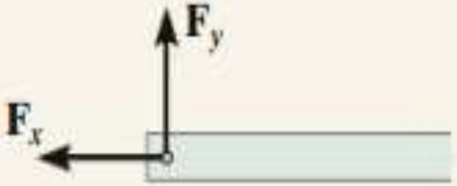
One unknown: F</td>
<td>
Internal pin</td>
<td>

Two unknowns: F_x, F_y</td>
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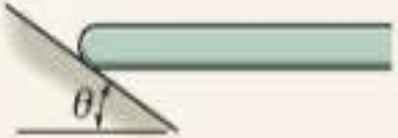
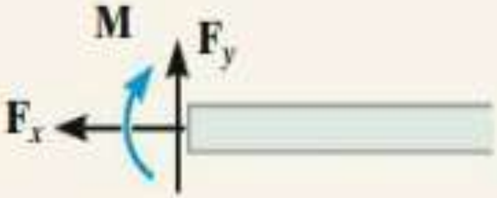
<td>

Smooth support</td>
<td>

One unknown: F</td>
<td>
Fixed support</td>
<td>

Three unknowns: F_x, F_y, M</td>
</tr>
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Equation of Equilibrium

Equilibrium of a body requires both a **balance of forces** (to prevent movement) and a **balance of moments** (to prevent rotation). Mathematically, these two conditions can be expressed as: (Summation of all forces acting on the body) (Summation of the moments of all forces about any point)

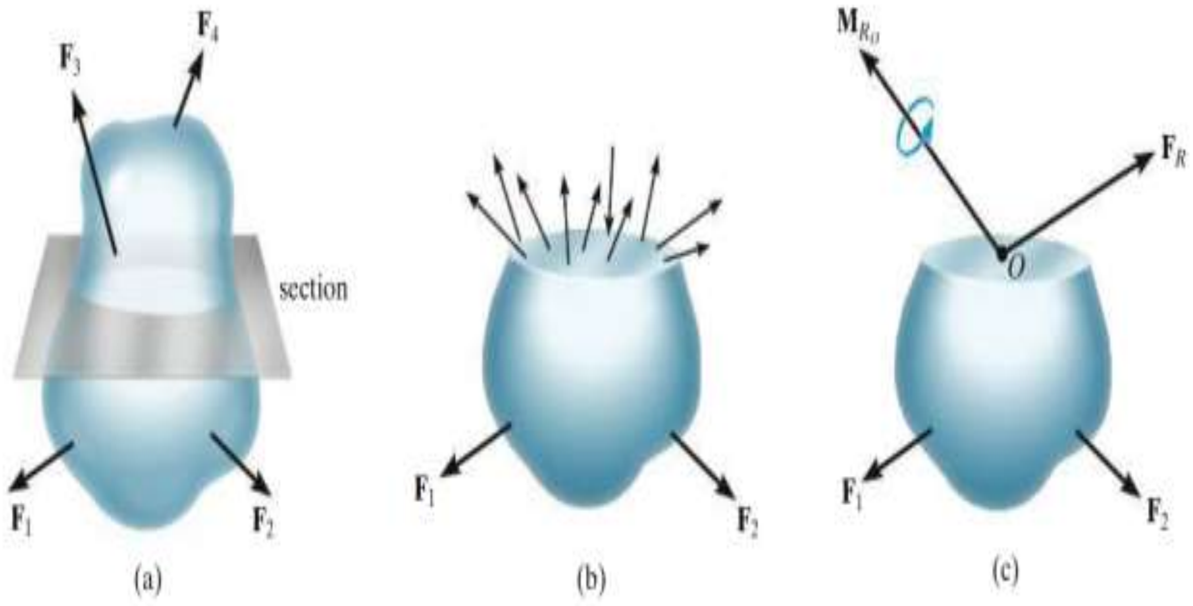
$$\sum \vec{F} = 0$$
$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0,$$
$$\sum \vec{M}_O = 0$$
$$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$$

When the (x, y, z) coordinate of the system are established, the forces and moment vectors can be resolve into components along each coordinate axis, as below:

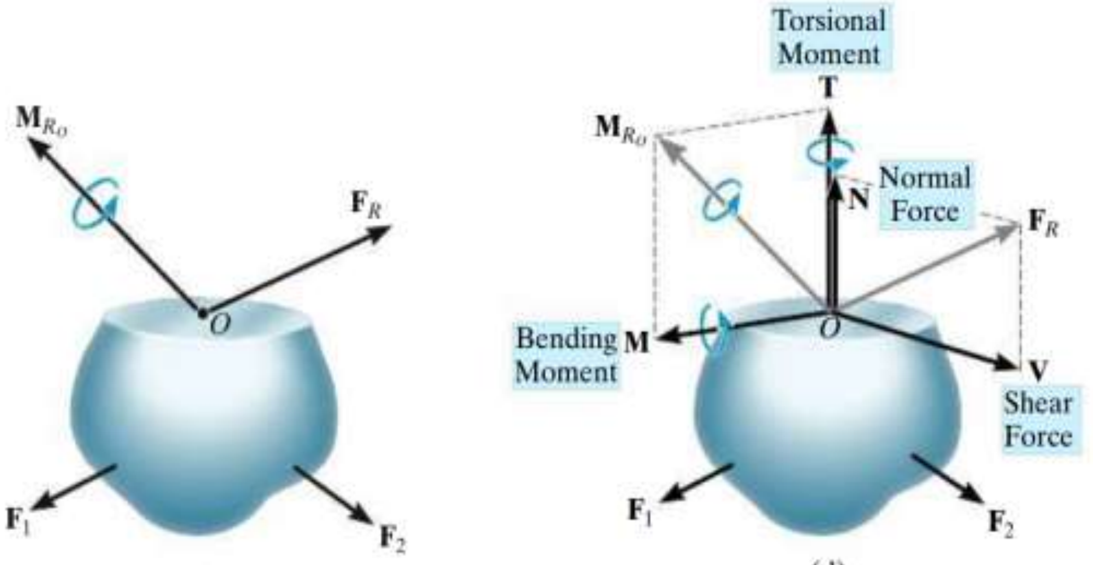
The best way to account for all these forces is draw the free-body diagram of the system.

Internal Resultant Loadings

To determine the Resultant loading that acts within a body, we use the principles of statics by taking an imaginary cut on the specific region within the body. The exact distribution of the internal loading may be unknown; we can use the principles of equilibrium to find the resultants and the components.



The four different types components resultant loading can be defined as:



(c) (d)

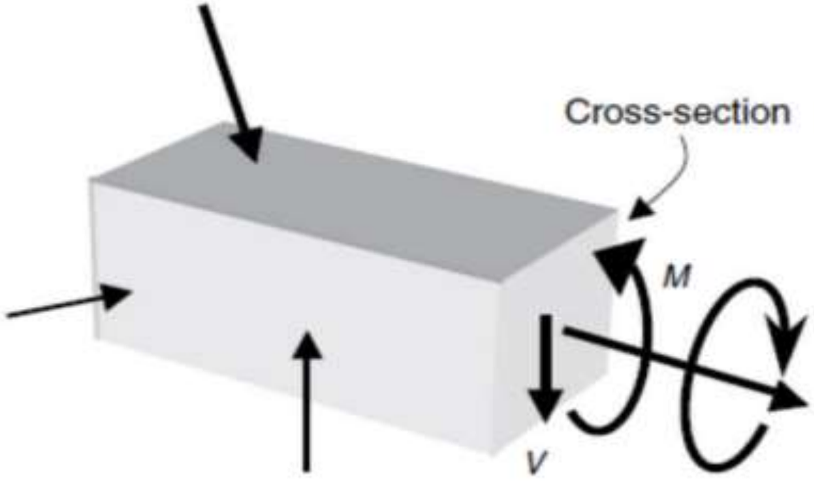
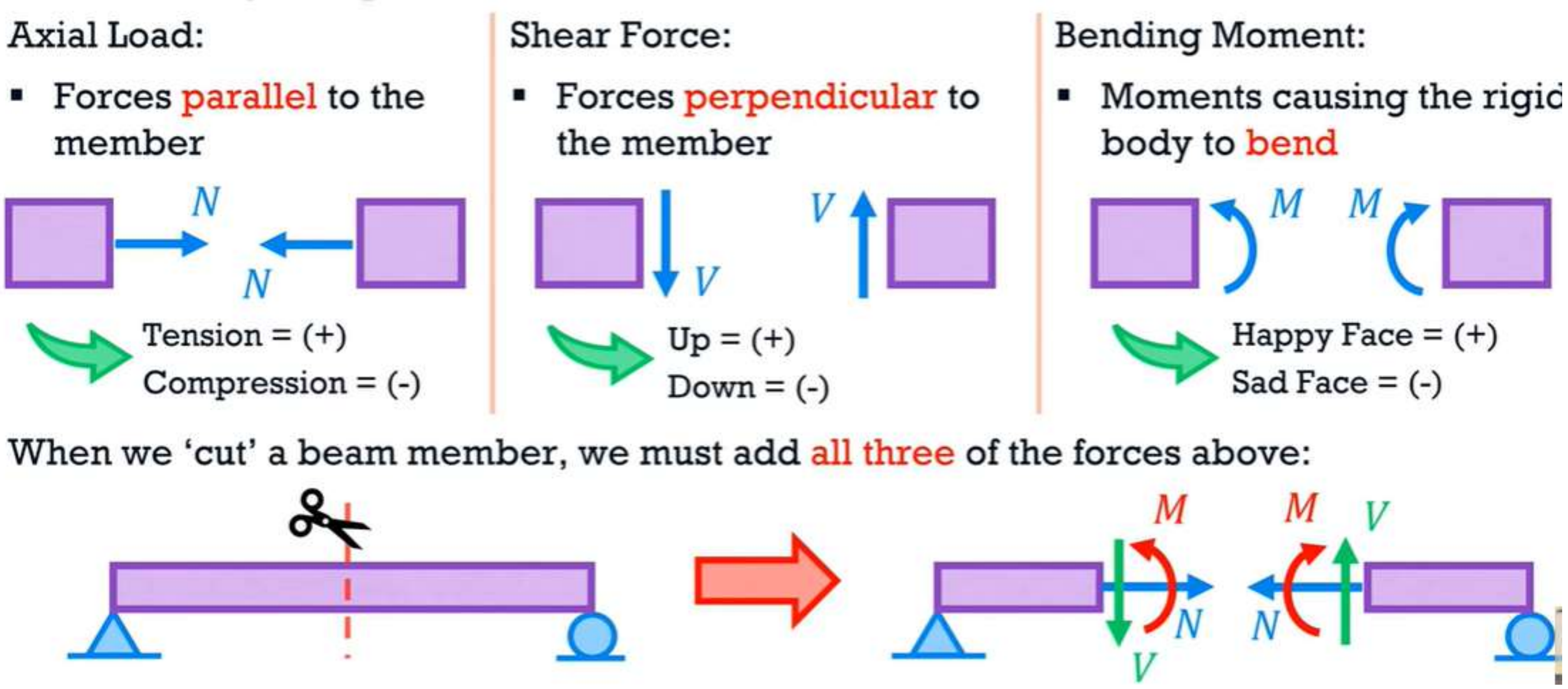


Figure 1.12: Internal force

Internal forces in Beams.

Beams members carry multiple forces.



The weight of this sign and the wind loadings acting on it will cause normal and shear forces and bending and torsional moments in the supporting column.

Normal force, N. it is a force perpendicular to the area and is developed when the external loads tend to push or pull on the two segments of the body.

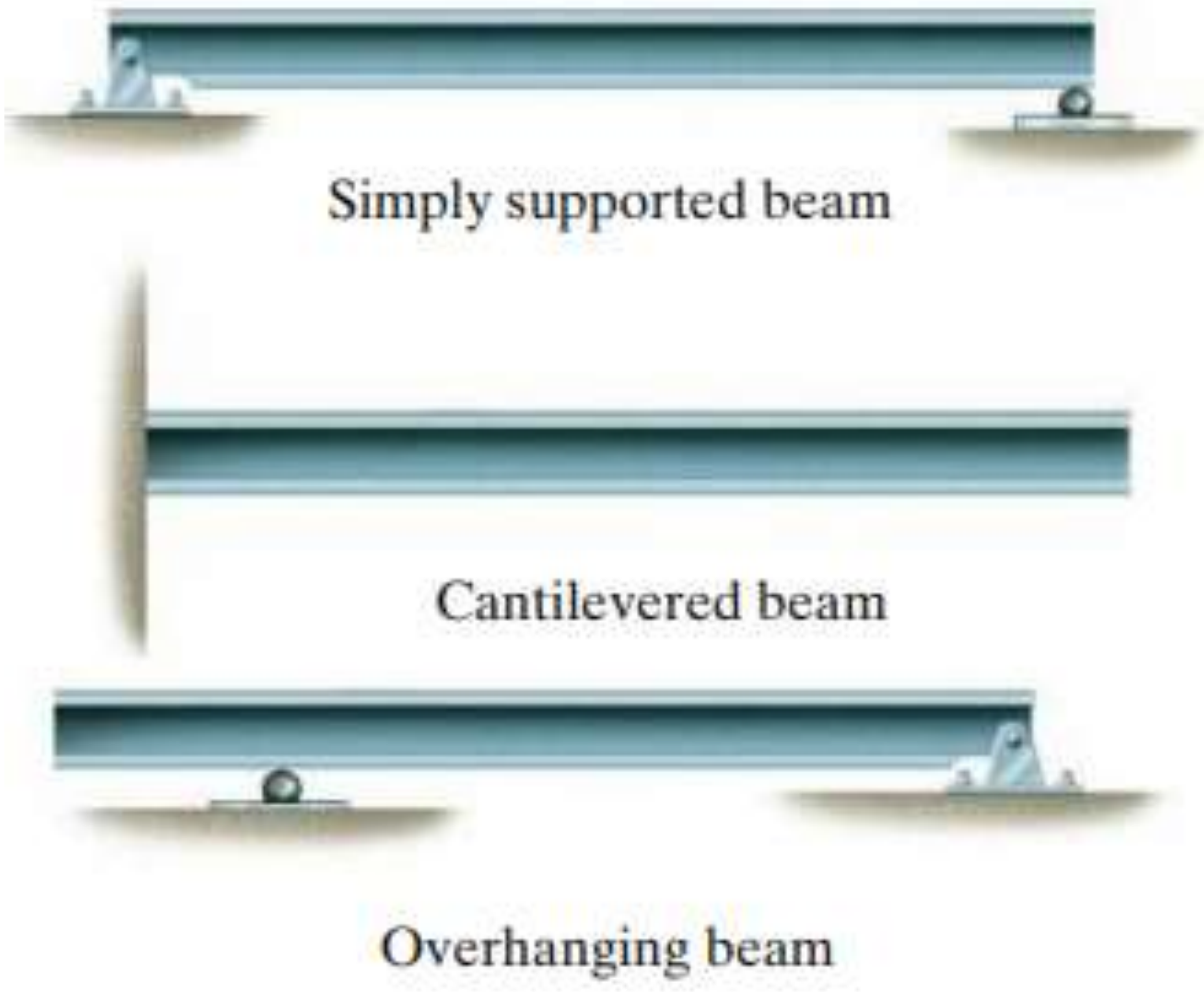
Shear force, V. the shear force lies in the plane of the area and it is developed when the external loads tend to cause the two segments of the body to side over one another.

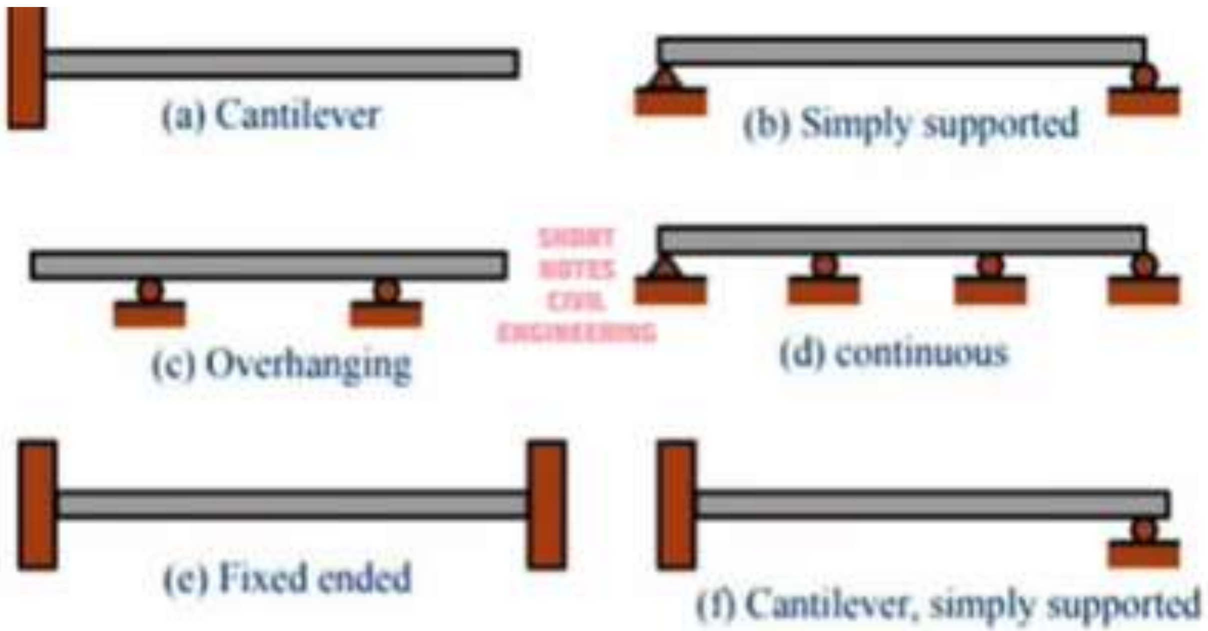
Torsional moment or Torque, T. This effect is developed when external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

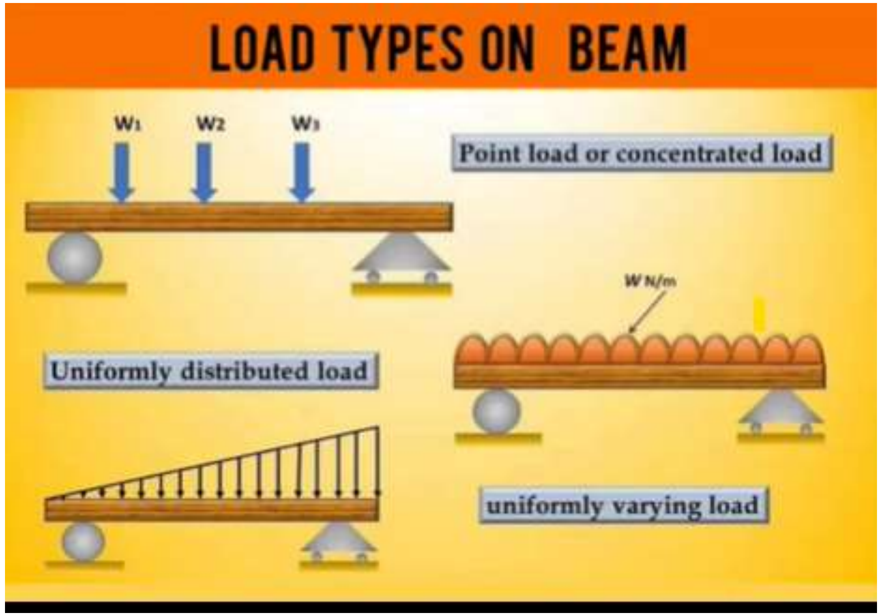
Bending moment, M. it is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

Twisting moment or Torsional moment or torque, *T*.(torque),This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

Beams – Types

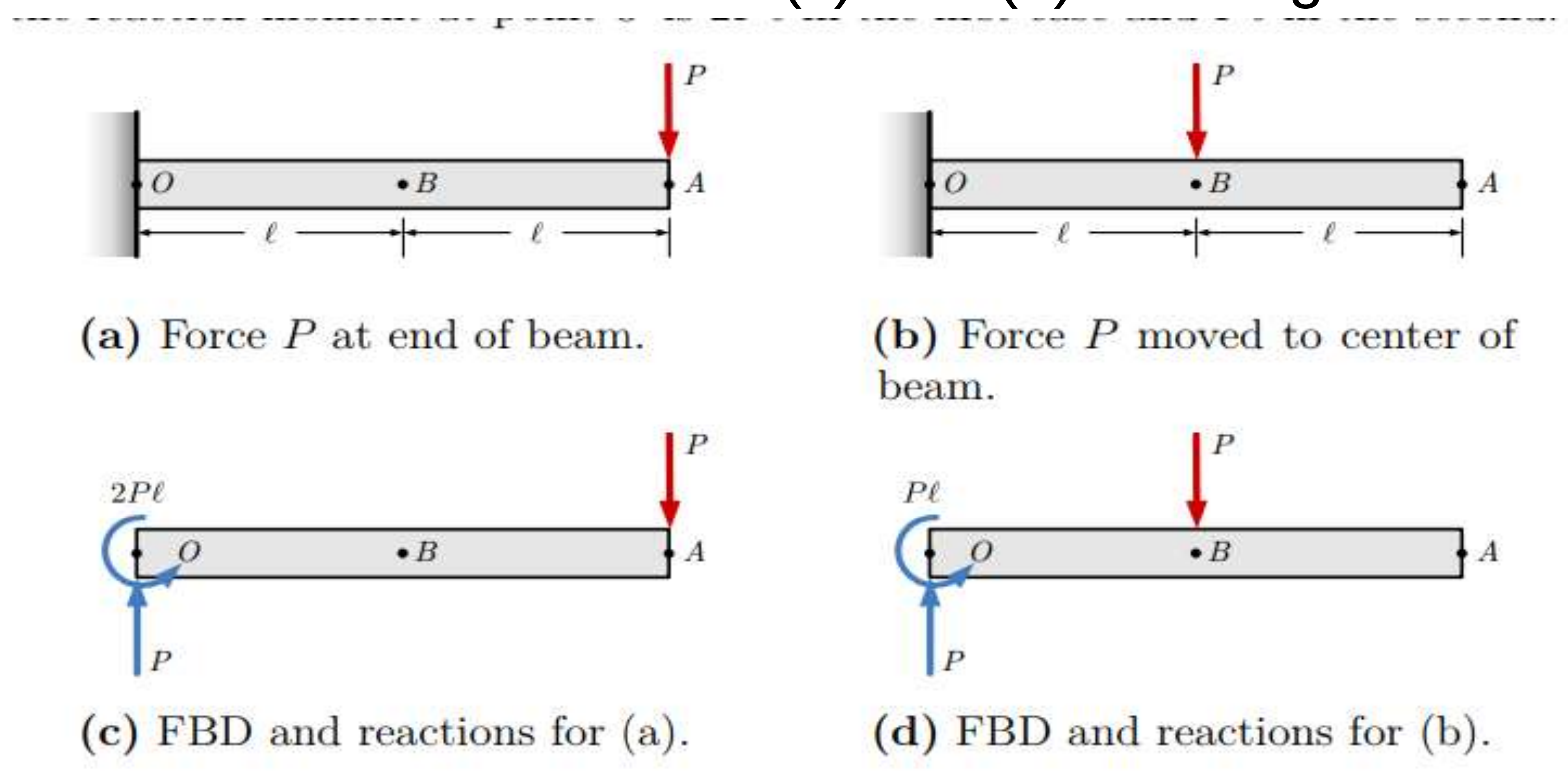

Simply supported beam
Cantilevered beam
Overhanging beam


(a) Cantilever
(b) Simply supported
(c) Overhanging
(d) continuous
(e) Fixed ended
(f) Cantilever, simply supported


LOAD TYPES ON BEAM
W1
W2
W3
Point load or concentrated load
W N/m
Uniformly distributed load
uniformly varying load

Force-Couple Systems. One transformation you might want to make is to move a force to another location. While *sliding* a force along its line of action is fine, *moving* a force to another point changes its line of action and thus its rotational effect on the object, so moving a force to a new line of action is not an equivalent transformation. Consider the cantilever beam below.

In diagram (a), the load P is at the end of the beam, and in (b) it has been moved to the center. The external effects are shown in (c) and (d). Although the vertical reaction force is the P the first case and $P\ell$ in the sec



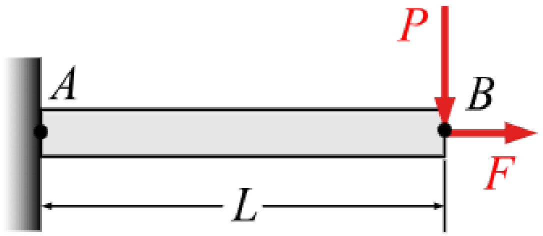
(a) Force P at end of beam.

(b) Force P moved to center of beam.

(c) FBD and reactions for (a).

(d) FBD and reactions for (b).

Example 1. Internal forces in a cantilever beam.



Consider a cantilever beam which is supported by a fixed connection at A,

and loaded by a vertical force P and horizontal force

at F the free end B. Determine the internal forces at a point a distance *a* from the left end.

This workflow typically includes:

- Establishing a horizontal and vertical coordinate system.
- Taking a cut at the point of interest.
- Assuming that the internal forces act in the positive direction and drawing a free-body diagram accordingly

Equations of Equilibrium. Applying the equations of equilibrium we have

$\sum F_x = 0$ **N= F**

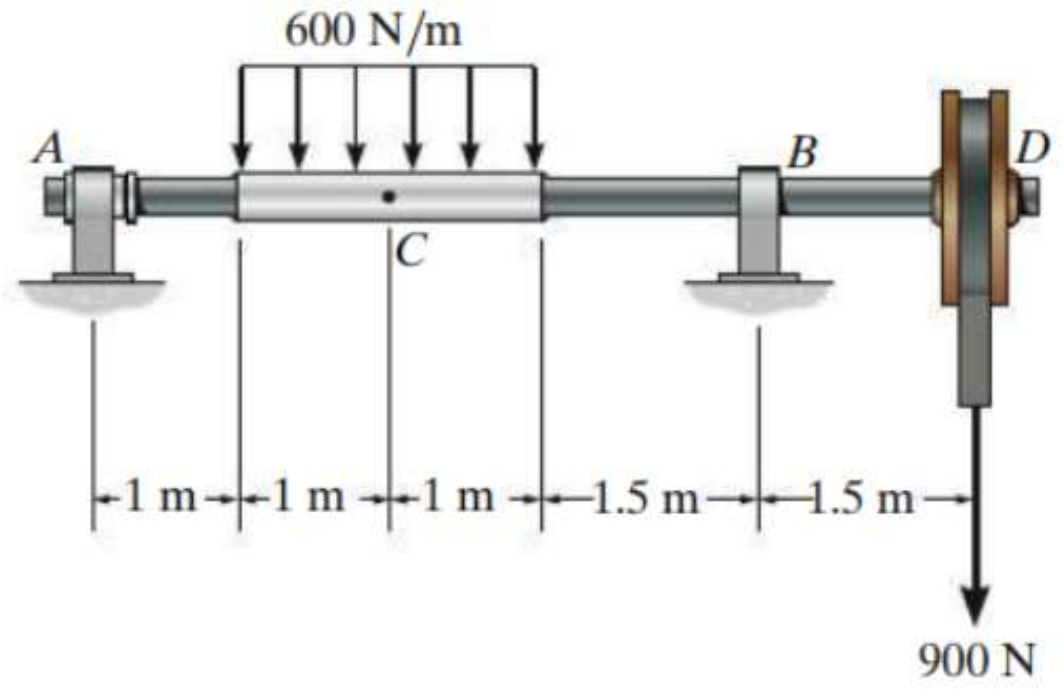
$\sum F_y = 0$ **V= P**

$\sum \vec{M}_O = 0$ $M = -Pl + Pa = -P(L - a) = -Pb$

Example 2

The shaft is supported by a smooth thrust bearing at ***A*** and a smooth journal bearing at ***B.***

Determine the resultant internal loadings acting on the cross section at ***C.***



Steps to Determine Internal Loadings at C

1. **Free-Body Diagram (FBD) of Entire Shaft:** Draw the entire shaft, including all applied forces (pulleys/loads).

2. **Support Reactions:**

    1. **Thrust Bearing (A):** Provides reactive forces in all directions (axial A_x and radial A_y, A_z).

    2. **Journal Bearing (B):** Provides only radial forces (B_y, B_z).

    3. Calculate reactions A_x, A_y, A_z, B_y, B_z using $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0, \sum M_y = 0, \sum M_z = 0.$

3. **Method of Sections:**

    1. Cut the shaft at section C.

    2. Draw the FBD of either the left or right segment (left is often simpler).

    3. Include the external reactions from Step 2 acting on the segment.

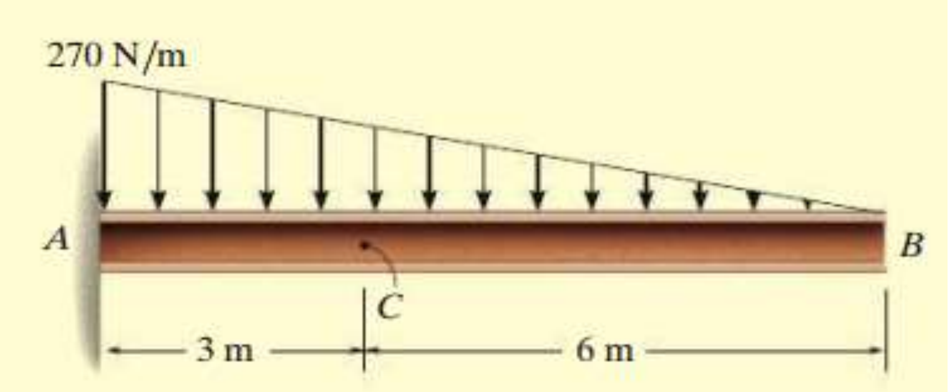
4. **Equilibrium Equations at C:**
    1. **Normal Force (N_C):** $\sum F_{\text{axial}} = 0$
    2. **Shear Force (V_C):** $\sum F_{\text{radial}} = 0$
    3. **Bending Moment (M_C):** $\sum M_C = 0$

Common Results (Based on Typical Hibbeler Problems)

- **Axial Load (N_C):** Often zero if no axial forces are applied between the thrust bearing and the section.
- **Shear Force (V_C):** Sum of transverse loads on one side of the section.
- **Bending Moment (M_C):** Sum of moments caused by radial forces and applied torques/forces about section C.

Example 3

Determine the resultant internal loadings acting on the cross section at *C* of the cantilevered beam shown in Fi



Solution

Support Reactions. The support reactions at *A* do not have to be determined if segment *CB* is considered.

Free-Body Diagram. The free-body diagram of segment *CB* is shown in Figure

It is important to keep the distributed loading on the segment until *after* the section is made.

Only then should this loading be replaced by a single resultant force.

Notice that the intensity of the distributed loading at *C* is

w /6 m = (270 N/m)/9 m, *w* = 180 N/m.

The magnitude of the resultant of the distributed load is equal to the area under the loading

curve (triangle) and acts through the centroid of this area.

Thus, F = 12 (180 N/m)(6 m) = 540 N, which acts (6 m) /3= 2 m from C as shown in Figure.

Equations of Equilibrium. Applying the equations of equilibrium we have

$$\sum F_x = 0$$

$-N_c = 0$, $N_c = 0$

$$\sum F_y = 0$$

V_c - 540 N = 0, V_c= 540 N

$$\sum \vec{M}_O = 0$$

$-M_c$ - 540 N(2 m) = 0

M_c = -1080 N/ m