DW2: Calculation of genetic distances: Hardy-Weinberg equilibrium

EXERCISE 1:

In a population of sheep, the following genotypes are observed:

- 150 individuals AA
- 200 individuals Aa
- 50 individuals aa

Questions:

1. Calculate the frequency of allele A and allele a in this population.
2. Find the expected genotype frequencies at Hardy-Weinberg equilibrium.
3. Verify whether this population is in Hardy-Weinberg equilibrium.

Solution:

1. Calculation of allele frequencies

The total population size is:

N = 150 + 200 + 50 = 400 individuals

The total number of alleles is:

2N = 2 x 400 = 800 alleles

Number of A alleles:

- from AA: 150 x 2 = 300
- from Aa: 200 x 1 = 200

Total A alleles:

A = 300 + 200 = 500

Number of a alleles:

- from aa: 50 x 2 = 100
- from Aa: 200 x 1 = 200

Total a alleles:

a = 100 + 200 = 300

Therefore:

p = frequency of allele A = 500 / 800 = 0.625

q = frequency of allele a = 300 / 800 = 0.375

Check:

p + q = 0.625 + 0.375 = 1.000

2. Calculation of expected genotype frequencies under Hardy-Weinberg equilibrium

At Hardy-Weinberg equilibrium, the expected genotype frequencies are:

$f(AA) = p^2 = (0.625)^2 = 0.390625$

f(Aa) = 2pq = 2 x 0.625 x 0.375 = 0.46875

$f(aa) = q^2 = (0.375)^2 = 0.140625$

The expected genotype numbers are:

E(AA) = 0.390625 x 400 = 156.25

E(Aa) = 0.46875 x 400 = 187.50

E(aa) = 0.140625 x 400 = 56.25

3. Verification of Hardy-Weinberg equilibrium

Observed genotype numbers:

- AA = 150
- Aa = 200
- aa = 50

Expected genotype numbers:

- E(AA) = 156.25
- E(Aa) = 187.50
- E(aa) = 56.25

This population can be considered to be in Hardy-Weinberg equilibrium.

EXERCISE 2:

The coat color of Ardennes horses is determined by a gene with two alleles, C and c, showing incomplete dominance:

- CC = black
- Cc = gray
- cc = white

In a population of this breed, the following genotypes are observed:

- 120 black horses (CC)
- 240 gray horses (Cc)
- 80 white horses (cc)

Questions:

a. What is the genotype frequency for each coat color?

b. What is the frequency of each allele, C and c, in this population?

Solution:

1. Calculation of genotype frequencies

The total number of horses is:

N = 120 + 240 + 80 = 440

Therefore:

f(CC) = 120 / 440 = 0.2727

f(Cc) = 240 / 440 = 0.5455

f(cc) = 80 / 440 = 0.1818

Thus, the genotype frequencies are:

- black (CC) = 0.2727
- gray (Cc) = 0.5455
- white (cc) = 0.1818

2. Calculation of allele frequencies

The total number of alleles is:

2N = 2 x 440 = 880

Number of C alleles:

- from CC: 2 x 120 = 240
- from Cc: 1 x 240 = 240

Total C alleles:

C = 240 + 240 = 480

Number of c alleles:

- from cc: 2 x 80 = 160

- from Cc: 1 x 240 = 240

Total c alleles:
c = 160 + 240 = 400
Therefore:
p = frequency of allele C = 480 / 880 = 0.5455
q = frequency of allele c = 400 / 880 = 0.4545
Check:
p + q = 0.5455 + 0.4545 = 1.000
The genotype frequencies are 0.2727, 0.5455, and 0.1818, and the allele frequencies are 0.5455 for C and 0.4545 for c.

EXERCISE 3:
Consider a B/b locus located on the X chromosome. In a population of rabbits, the following genotypes are observed:

- 50 males XBY
- 10 males XbY
- 60 females XBXB
- 30 females XBXb
- 5 females XbXb

Question:
Calculate the allele frequencies in males, in females, and in the overall population.
Solution:

1. Allele frequencies in females

Total number of females:
Nf = 60 + 30 + 5 = 95
Since females carry two X chromosomes, the total number of female alleles is:
2Nf = 2 x 95 = 190
Number of B alleles in females:

- from XBXB: 2 x 60 = 120
- from XBXb: 1 x 30 = 30

Total B alleles in females:
B(females) = 120 + 30 = 150
Number of b alleles in females:

- from XbXb: 2 x 5 = 10
- from XBXb: 1 x 30 = 30

Total b alleles in females:
b(females) = 10 + 30 = 40
Therefore:
p(females) = frequency of B in females = 150 / 190 = 0.7895
q(females) = frequency of b in females = 40 / 190 = 0.2105

2. Allele frequencies in males

Total number of males:
Nm = 50 + 10 = 60
Since males carry only one X chromosome, the total number of male alleles is:

Nm = 60
Therefore:
p(males) = frequency of B in males = 50 / 60 = 0.8333
q(males) = frequency of b in males = 10 / 60 = 0.1667

3. Allele frequencies in the overall population

Total number of X-linked alleles in the whole population:

- females contribute 190 alleles
- males contribute 60 alleles

Total alleles:
190 + 60 = 250
Total number of B alleles:

- from females: 150
- from males: 50

Total B alleles:
B(total) = 150 + 50 = 200
Total number of b alleles:

- from females: 40
- from males: 10

Total b alleles:
b(total) = 40 + 10 = 50
Therefore:
p(total) = frequency of B in the whole population = 200 / 250 = 0.80
q(total) = frequency of b in the whole population = 50 / 250 = 0.20

- frequency of B in females = 0.7895
- frequency of b in females = 0.2105
- frequency of B in males = 0.8333
- frequency of b in males = 0.1667
- frequency of B in the total population = 0.80
- frequency of b in the total population = 0.20