Management of Agricultural Operations and Agri-Food Enterprises

Third-Year Agronomy Course — License Level

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Chapter 06

Introduction to Agricultural Farm Modeling

Introduction

Every season, farmers face complex decisions: What crops to plant? How to allocate land, water, labor, and capital? How to maximize profit with limited resources? Mathematical modelling gives clear, rigorous answers to these questions.

Opening Example
Mr. Ahmed owns a 10-hectare farm in Algeria.
He can grow wheat (profit: 25,000 DA/ha) or potatoes (profit: 60,000 DA/ha).
But potatoes need more water and labour.
Question: What crop mix gives him the highest total income?
→ This type of problem is solved using Linear Programming.

1. Linear and Non-Linear Programming

1.1. Linear Programming (LP)

Linear Programming is a mathematical optimization technique used to maximize or minimize a linear objective function, subject to a set of linear constraints.

Mathematical Structure of an LP Model
Maximize (or Minimize): $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$
Subject to constraints:
$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$
…
$x_1, x_2, \ldots, x_n \geq 0$ (non-negativity constraints)

The Four Components of an LP Model

Component	What it means
Decision variables (x_i)	The quantities we want to find (e.g., hectares of wheat, maize…)
Objective function (Z)	What we want to optimize (e.g., maximize income or minimize costs)
Constraints	Limits on resources (land, water, labour, capital, equipment…)
Non-negativity	Variables cannot be negative ($x_i \geq 0$)

Complete LP Example (Crop Allocation)

Problem 1 (Crop Allocation)
A farm has: 50 ha of land | 800 hours of labour | 120,000 DA of capital
Crop 1: Durum wheat → Income: 20,000 DA/ha | Labour: 10 h/ha | Capital: 1,800 DA/ha
Crop 2: Corn → Income: 35,000 DA/ha | Labour: 18 h/ha | Capital: 2,500 DA/ha
Crop 3: Tomato → Income: 80,000 DA/ha | Labour: 25 h/ha | Capital: 4,000 DA/ha

Mathematical Formulation
Variables: x_1 = area of wheat (ha), x_2 = area of maize (ha), x_3 = area of tomato (ha)
Maximize $Z = 20{,}000x_1 + 35{,}000x_2 + 80{,}000x_3$
Subject to:
(Land) $x_1 + x_2 + x_3 \leq 50$
(Labour) $10x_1 + 18x_2 + 25x_3 \leq 800$
(Capital) $1{,}800x_1 + 2{,}500x_2 + 4{,}000x_3 \leq 120{,}000$
(Non-neg) $x_1, x_2, x_3 \geq 0$

Graphical Solution (2 variables)

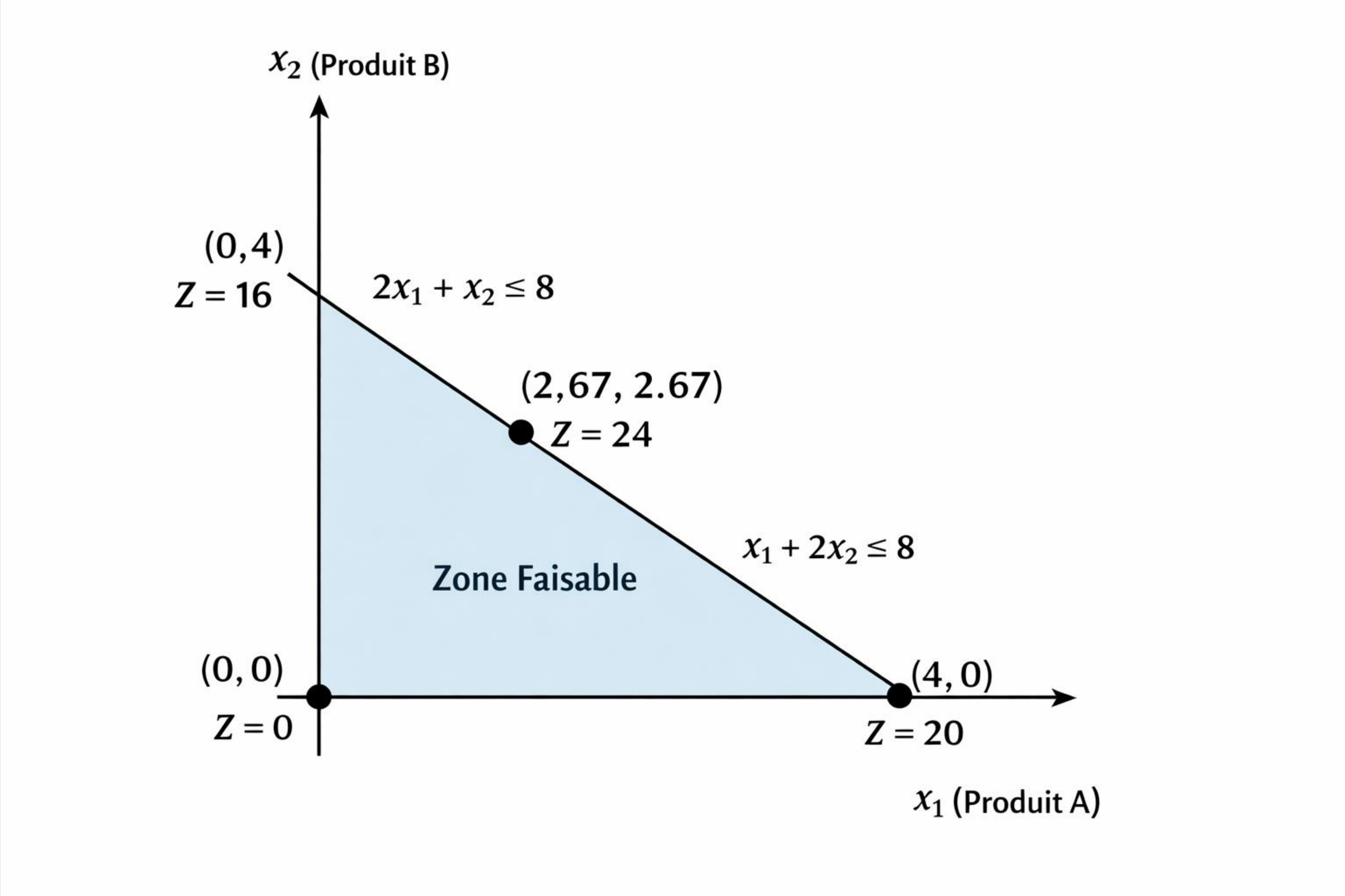
When there are only 2 decision variables, you can solve the problem graphically. Draw each constraint as a line and find the feasible region (the shaded area that satisfies all constraints). The optimal solution is always at one of the corner points of this feasible region.

Corner-Point Method (Steps)
1. Draw each constraint as a line on a graph with axes x_1 and x_2
2. Find the feasible region (the area that satisfies ALL constraints at once)
3. Calculate the value of Z at each corner point of the region
4. The corner with the highest (or lowest) Z value is the optimal solution

The Simplex Method

For problems with more than 2 variables, we use the Simplex Method (an iterative algorithm). You do not need to master the full algorithm (you need to know how to apply it using software).

- Start at an initial corner point of the feasible region
- Move to a neighbouring corner that improves Z
- Stop when no neighbouring corner can improve Z further


x_2 (Produit B)
(0,4)
Z = 16
$2x_1 + x_2 \leq 8$
(2,67, 2.67)
Z = 24
$x_1 + 2x_2 \leq 8$
Zone Faisable
(0, 0)
Z = 0
(4, 0)
Z = 20
x_1 (Produit A)

1.2. Non-Linear Programming (NLP)

When do we use NLP?

LP assumes all relationships are perfectly linear. In reality, many agricultural relationships are non-linear, for example:

- Crop yield as a function of fertilizer dose follows a curve (Mitscherlich's law)
- Selling price may fall as production quantity increases (market saturation)
- Water consumption can be quadratic relative to irrigated area

Definition (NLP)
A model is Non-Linear when the objective function OR at least one constraint
contains non-linear terms: squares (x^2), products ($x_1 \cdot x_2$), exponentials, logarithms...
Example: Maximize $Z = 50x - 0.5x^2$ (decreasing returns)
→ There is a maximum at the top of the parabola:
$dZ/dx = 50 - x = 0 \rightarrow x^* = 50$ units

Linear Programming (LP)	Non-Linear Programming (NLP)
All relationships are linear	At least one relationship is non-linear
Guaranteed global solution	May find only a local optimum
Simplex method applies	Methods: gradient, Newton, metaheuristics
Simpler to solve	More complex, requires advanced software
Example: crop area allocation	Example: optimal fertilizer dose, variable prices

2. Learning a Software Tool

2.1. Why Use Software?

Manual solving is only possible for very small problems (2–3 variables). As soon as a farm has many crops, fields, and constraints, software becomes essential.

2.2. Available Software

Software	Key Features
Microsoft Excel (Solver)	Built into Excel, easy to use, good for small models
LINDO / LINGO	Specialized optimization tool, widely used in operations research
WinQSB	Educational, simple interface for students
GAMS / AMPL	Professional tools for large farm models
Python (PuLP / SciPy)	Open source, modern, very flexible

2.3. Using Excel Solver

We will solve Problem 1 using Excel Solver. Follow each step carefully.

STEP 1 (Set Up the Excel Spreadsheet)

Cell Organisation
Cell B2: value of x_1 (wheat) → Enter 0 to start
Cell C2: value of x_2 (maize) → Enter 0 to start
Cell D2: value of x_3 (tomato) → Enter 0 to start
Cell E2 (Total Income / Objective function):
= 20000*B2 + 35000*C2 + 80000*D2
Cell E4 (Land constraint): = B2 + C2 + D2
Cell E5 (Labour constraint): = 10*B2 + 18*C2 + 25*D2
Cell E6 (Capital constraint): = 1800*B2 + 2500*C2 + 4000*D2

STEP 2 (Open the Solver)

- Go to the Data tab and click Solver
- If not visible: File → Options → Add-ins → Solver → Enable

STEP 3 (Configure the Solver)

Parameter	Value to Enter	Explanation
Objective cell	$E$2	The cell containing Z = total income
Objective type	Max	We want to maximize income
Variable cells	$B$2:$D$2	The decision variables x_1, x_2, x_3
Constraint 1	$E$4 <= 50	Total area ≤ 50 ha
Constraint 2	$E$5 <= 800	Labour hours ≤ 800
Constraint 3	$E$6 <= 120000	Capital ≤ 120,000 DA
Non-negativity	Tick the checkbox	Variables ≥ 0 (always tick this)
Method	Simplex LP	For a linear problem

STEP 4 (Run the Solver and Read the Results)

- Click Solve
- Excel shows the optimal values in cells B2, C2, D2
- Request the Sensitivity Report to analyse margins

2.4. Reading the Sensitivity Report

The Sensitivity Report (also called post-optimal analysis) answers the question: what happens if the data changes slightly?

Indicator	Agricultural Meaning
Reduced cost	By how much must a crop's profit increase before it becomes worth growing?
Ranging (allowable range)	How much can a profit figure change without changing the optimal solution?
Shadow price	How much would Z increase if we had one more unit of a scarce resource?
Constraint ranging	How much can a resource's available quantity change while keeping the same solution?

3. Building a Simple Model from a Real Case

3.1. Model-Building Methodology

Building a model follows a clear, step-by-step process:

Step	Action	Key Questions to Ask
1. Define the problem	Identify the decision to make	What do we want to optimize? What resources are limited?
2. Identify the variables	Name the decision variables	What quantities do we need to determine?
3. Write the objective	Write the function Z	Maximize income? Minimize costs?
4. Write the constraints	List all limitations	Land, water, labour, capital, crop rotations...
5. Solve	Use software	Is the solution technically feasible?
6. Validate	Compare with reality	Can the farmer actually apply this plan?
7. Interpret	Analyse the results	Which resource is the main limiting factor?

3.2. Full Case Study (Mitidja Plain Farm)

Farm Description
Family farm in the Mitidja plain (Algeria)
Available land: 30 ha of good irrigated soil
Family labour: 600 hours per season
Irrigation water available: 1,200 m^3
Budget: 500,000 DA
Crops considered:
- Durum wheat
- Potatoes
- Vegetables (courgettes / green beans)

Technical and Economic Data

Item	Durum Wheat	Potatoes	Vegetables
Net income (DA/ha)	18,000	55,000	90,000
Labour (h/ha)	8	20	30
Water (m^3/ha)	25	60	45
Production cost (DA/ha)	12,000	22,000	35,000
Minimum required area (ha)	5	0	0

Step 1 (Identify the Decision Variables)

- x_1 = area allocated to durum wheat (in hectares)
- x_2 = area allocated to potatoes (in hectares)
- x_3 = area allocated to vegetables (in hectares)

Step 2 (Objective Function)

Maximize Total Net Income:
$Z = 18{,}000\,x_1 + 55{,}000\,x_2 + 90{,}000\,x_3$
(We use NET income = gross revenue minus production cost)

Step 3 (Constraints)

Full List of Constraints:
Land (ha): $x_1 + x_2 + x_3 \leq 30$
Labour (h): $8x_1 + 20x_2 + 30x_3 \leq 600$
Water (m^3): $25x_1 + 60x_2 + 45x_3 \leq 1{,}200$
Budget (DA): $12{,}000x_1 + 22{,}000x_2 + 35{,}000x_3 \leq 500{,}000$
Agronomic (wheat): $x_1 \geq 5$ (crop rotation requirement)
Non-negativity: $x_1, x_2, x_3 \geq 0$

Step 4: Solution (Excel Solver)

Optimal Solution
x_1* (Durum wheat) = 5.0 ha (minimum constraint is binding)
x_2* (Potatoes) = 8.5 ha
x_3* (Vegetables) = 16.5 ha
Total area used: 30 ha (all land is fully used)
Z* (Optimal net income) = 18,000×5 + 55,000×8.5 + 90,000×16.5
= 90,000 + 467,500 + 1,485,000
= 2,042,500 DA per season

Step 5 (Analysis and Interpretation)

Constraint	Status and Meaning
Land (30 ha)	BINDING → All land is used. Acquiring more land would increase Z.
Labour (600 h)	BINDING → Labour is the main limiting factor.
Water (1,200 m^3)	NOT BINDING → There is water left over (safety margin).
Budget (500,000 DA)	NOT BINDING → Not all available capital is spent.

Practical Recommendations for the Farmer
1. Hire seasonal workers (labour is the main limiting factor)
2. Increase the vegetable area if possible (it is the most profitable crop)
3. Wheat is only grown to respect the crop rotation rule, not for profit
4. Water is NOT a limiting factor (no need to invest in extra irrigation)
5. More capital can be invested without financial risk

3.3. Limitations and Extensions of the Model

The LP model is based on simplifying assumptions. It is important to know them: These limitations have led to more advanced models: Integer Programming (integer variables), Stochastic Programming (uncertainty), Multi-Objective Programming (MOOP), and integrated bio-economic models.

Model Assumption	Real-World Limitation in Agriculture
All relationships are linear	In reality: diminishing returns, variable prices
All data is certain	Crop yields and farm prices are uncertain
Variables are perfectly divisible	In practice, you cannot plant 0.3 ha of vines
Single objective (Z)	Farmers may aim for both security AND income
Single-season horizon	Multi-year planning needs dynamic models

Exercise: Full Model with Software (Advanced)

Problem Statement			
Orchard farm (Tlemcen region): 40 ha available			
Labour: 900 h/year | Water: 2,000 m³ | Budget: 800,000 DA			
Crop:	Olive	Vine	Fig
Net income (DA/ha):	30,000	70,000	50,000
Labour (h/ha):	12	28	20
Water (m³/ha):	40	60	30
Cost (DA/ha):	15,000	25,000	18,000
Min area (ha):	10	0	0
Tasks:			
1. Write the LP model			
2. Solve using Excel Solver			
3. Identify the binding constraints			
4. Interpret the Sensitivity Report			

End of Chapter 06