Abdelhafid Boussouf University — Mila
Faculty of Science and Technology | Common Core Department

CHAPTER: 01

Differential Equations

Separable, Homogeneous, Linear, Bernoulli, Reducible Forms,
Linear Independence, Variation of Parameters & Reduction of Order

Analysis 2 — Academic Year 2025/2026

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Contents

Prerequisites

To effectively follow the course of Analysis 2, students should have mastered:

- Basics of **differentiation** and **integration**.
- Familiarity with **separable** and **homogeneous** first-order differential equations.
- Understanding of **linear algebra** concepts (e.g., roots of polynomials, solving linear systems, determinants).
- Knowledge of **substitution methods** in differential equations.

Preliminary Concepts and Definitions

2.1 What is a Differential Equation?

A **differential equation** relates an independent variable x, an unknown function $y = f(x)$, and its derivatives $y', y'', \dots, y^{(n)}$. It can be written as:

$$F\left(x,\, y,\, \frac{dy}{dx},\, \frac{d^2y}{dx^2},\, \dots,\, \frac{d^ny}{dx^n}\right) = 0$$

When $y = f(x)$ depends on a *single* independent variable, the equation is called an **ordinary differential equation** (ODE).

2.2 Order of a Differential Equation

The **order** of a differential equation is the order of the **highest derivative** present in the equation.

- $y' - 2xy^2 + 5 = 0$ is of **first order**.
- $y'' + k\,y' - b\,y - \sin x = 0$ is of **second order**.

2.3 Solutions of a Differential Equation

A **solution** (or integral) of a differential equation is any function $y = f(x)$ that, when substituted into the equation, satisfies it identically.

Example

For the equation $\dfrac{d^2y}{dx^2} + y = 0$, the following functions are all solutions:

$$y = \sin x, \quad y = 2\cos x, \quad y = 3\sin x - \cos x$$

and more generally, for any constants C_1, C_2:

$$y = C_1 \sin x + C_2 \cos x$$

This last expression is called the **general solution**.

Differential Equations with Separated Variables

3.1 Definition

Let $f : I \to \mathbb{R}$ and $g : J \to \mathbb{R}$ be continuous functions. A **differential equation with separated variables** takes the form:

$$y' = \frac{f(x)}{g(y)} \qquad \text{or equivalently} \qquad y'\, g(y) = f(x)$$

3.2 Solution Method

Separation of Variables

1. Rewrite as $g(y)\, dy = f(x)\, dx$.
2. Integrate both sides:

$$\int g(y)\, dy = \int f(x)\, dx + C$$

3. This gives $G(y) = F(x) + C$, where G and F are antiderivatives of g and f respectively, and C is the constant of integration.

3.3 Examples

3.3.1 Example 1

Solve: $\dfrac{dy}{x} - x = 0$ (i.e. $y' = x^2$)

- Separate: $dy = x^2\, dx$
- Integrate: $y = \dfrac{x^3}{3} + C$

$$y = \frac{x^3}{3} + C$$

3.3.2 Example 2

Solve: $\dfrac{dy}{dx} - y = 0$

- Separate: $\dfrac{dy}{y} = dx$
- Integrate: $\ln|y| = x + C_0 \Rightarrow |y| = e^{C_0} e^{x}$

$$y = C\, e^{x} \quad (C \in \mathbb{R} \setminus \{0\})$$

Homogeneous Differential Equations

4.1 Definition

A first-order ODE is called **homogeneous** (with respect to x and y) if it can be written as:

$$y' = f\left(\frac{y}{x}\right)$$

where f is a continuous function on some interval $I \subset \mathbb{R}$.

Recognition tip: An equation $M(x,y)\,dx + N(x,y)\,dy = 0$ is homogeneous if M and N are homogeneous functions of the same degree, i.e. $M(\lambda x, \lambda y) = \lambda^n M(x,y)$ and similarly for N.

4.2 Solution Method

Substitution $t = y/x$

1. Let $t(x) = \dfrac{y}{x}$, so $y = tx$ and $y' = t'x + t$.
2. Substitute into $y' = f(y/x)$:

$$t'x + t = f(t) \implies t' = \frac{f(t) - t}{x}$$

3. Separate variables:

$$\frac{dt}{f(t) - t} = \frac{dx}{x}$$

4. Integrate both sides, then back-substitute $t = \dfrac{y}{x}$.

4.3 Example

Solve: $(x+y)\,dx - x\,dy = 0$

1. Rewrite: $y' = \dfrac{x+y}{x} = 1 + \dfrac{y}{x} \quad \Rightarrow f(t) = 1 + t$
2. Let $t = y/x$: $t'x + t = 1 + t \Rightarrow t'x = 1 \ \Rightarrow t' = \dfrac{1}{x}$
3. Integrate: $t = \ln|x| + C$
4. Back-substitute $t = y/x$:

$$y = x(\ln|x| + C)$$

Specific Objectives

By the end of this lecture, students will be able to:

1. **Define** linear first-order, Bernoulli, and second-order linear differential equations.
2. **Solve** linear first-order DEs using the integrating factor method.
3. **Solve** Bernoulli equations by substitution.
4. **Recognize** equations reducible to homogeneous form and apply the appropriate method.
5. **Determine** linear independence of solutions using the Wronskian and Abel's identity.
6. **Find** particular solutions via variation of parameters.
7. **Apply** the reduction of order method.
8. **Solve** second-order linear DEs with constant coefficients (homogeneous and non-homogeneous).

Linear First-Order Differential Equations

6.1 Definition

A **linear first-order differential equation** takes the form:

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

where $P(x)$ and $Q(x)$ are continuous on $I \subset \mathbb{R}$, and y is the unknown function of x.

Key Points: The equation is “linear” because y and y' appear to the first power; no products of y with itself or its derivatives are present.

6.2 Solution Method: Integrating Factor

Integrating Factor Method

For $\dfrac{dy}{dx} + P(x)\,y = Q(x)$:

1. Compute: $\mu(x) = e^{\int P(x)\,dx}$
2. Multiply through and recognize: $\dfrac{d}{dx}\big[\mu(x)\,y\big] = \mu(x)\,Q(x)$
3. Integrate and solve: $y = \dfrac{1}{\mu(x)}\big[\int \mu(x)\,Q(x)\,dx + C\big]$

6.3 Examples

6.3.1 Example 1

Solve: $\dfrac{dy}{dx} + x\,y = x$

- $\mu(x) = e^{x^2/2}$; $\quad \dfrac{d}{dx}\left(e^{x^2/2}y\right) = xe^{x^2/2} \implies e^{x^2/2}y = e^{x^2/2} + C$

$$y = 1 + C\,e^{-x^2/2}$$

6.3.2 Example 2 — Initial Value Problem

Solve: $\dfrac{dy}{dx} + \dfrac{1}{x}\,y = x, \quad y(2) = \dfrac{1}{3}$

- $\mu(x) = x \;\Rightarrow\; y = \frac{1}{3}x^2 + \frac{C}{x}$; $\quad y(2) = \frac{1}{3} \Rightarrow C = -2$

$$y = \frac{x^2}{3} - \frac{2}{x}$$

Variation of Parameters for First-Order Linear DEs

This section presents the **variation of parameters** approach as an alternative derivation of the solution formula for first-order linear differential equations. It generalises naturally to higher-order equations (see Section 13).

Consider the standard form:

$$\frac{dy}{dx} + P(x)\,y = Q(x).$$

Steps of the Method

1. **Solve the homogeneous equation:**

$$\frac{dy}{dx} + P(x)\,y = 0 \implies y_h = C\,e^{-\int P(x)\,dx}$$

2. **Vary the constant:** Replace C by an unknown function $u(x)$ and assume the particular solution has the form:

$$y_p = u(x)\,e^{-\int P(x)\,dx}$$

3. **Determine $u(x)$:** Differentiate y_p:

$$y_p' = u'(x)\,e^{-\int P\,dx} - u(x)\,P(x)\,e^{-\int P\,dx}$$

Substitute y_p and y_p' into the original equation:

$$\left[u'\,e^{-\int P\,dx} - u\,P\,e^{-\int P\,dx}\right] + P\left[u\,e^{-\int P\,dx}\right] = Q(x)$$

The terms $-u\,P\,e^{-\int P\,dx}$ and $+u\,P\,e^{-\int P\,dx}$ cancel, leaving:

$$u'(x)\,e^{-\int P\,dx} = Q(x) \implies u'(x) = Q(x)\,e^{\int P(x)\,dx}$$

4. **Integrate $u'(x)$:**

$$u(x) = \int Q(x)\,e^{\int P(x)\,dx}\,dx + C$$

5. **General solution:** Substitute $u(x)$ back into y_p:

$$\boxed{y = e^{-\int P\,dx}\left(\int Q(x)\,e^{\int P\,dx}\,dx + C\right)}$$

This formula coincides with the result obtained by the integrating factor method.

Connection: The integrating factor $\mu(x) = e^{\int P\,dx}$ is precisely the inverse of the exponential factor $e^{-\int P\,dx}$ appearing in y_h. Variation of parameters (Variation of Parameters) makes this relationship explicit by treating the integration constant as a function.

7.1 Exemples résolus par la méthode de variation du paramètre (Variation of Parameters (Variation du Parametre))

7.1.1 Exemple 1

Solve using **variation of parameters** (Variation of Parameters):
$\dfrac{dy}{dx} + x\,y = x$

Step 1 — Homogeneous equation:

$$\frac{dy}{dx} + xy = 0 \implies \frac{dy}{y} = -x\,dx \implies y_h = C\,e^{-x^2/2}$$

Step 2 — Vary the constant:

$$y_p = u(x)\,e^{-x^2/2}$$

Step 3 — Find $u(x)$:

$$y_p' = u'\,e^{-x^2/2} - u\,x\,e^{-x^2/2}$$

Substitute into $y' + xy = x$:

$$u'\,e^{-x^2/2}\underbrace{-u\,x\,e^{-x^2/2} + x \cdot u\,e^{-x^2/2}}_{=0} = x \implies u'(x) = x\,e^{x^2/2}$$

Step 4 — Integrate:

$$u(x) = \int x\,e^{x^2/2}\,dx = e^{x^2/2} + C$$

Step 5 — General solution:

$$y = \left(e^{x^2/2} + C\right)e^{-x^2/2}$$

$$y = 1 + C\,e^{-x^2/2}$$

Identical to the integrating factor result. ✓

7.1.2 Exemple 2 — Initial Value Problem

Solve using **variation of parameters** (Variation of Parameters):
$\dfrac{dy}{dx} + \dfrac{1}{x}\,y = x, \quad y(2) = \dfrac{1}{3}$

Step 1 — Homogeneous equation:

$$\frac{dy}{dx} + \frac{y}{x} = 0 \implies \frac{dy}{y} = -\frac{dx}{x} \implies y_h = \frac{C}{x}$$

Step 2 — Vary the constant:

$$y_p = \frac{u(x)}{x}$$

Step 3 — Find $u(x)$:

$$y_p' = \frac{u'x - u}{x^2}$$

Substitute into $y' + \frac{y}{x} = x$:

$$\frac{u'x - u}{x^2} + \frac{u}{x^2} = x \implies \frac{u'}{x} = x \implies u'(x) = x^2$$

Step 4 — Integrate:

$$u(x) = \frac{x^3}{3} + C$$

Step 5 — General solution:

$$y = \frac{1}{x}\left(\frac{x^3}{3} + C\right) = \frac{x^2}{3} + \frac{C}{x}$$

Apply initial condition $y(2) = \frac{1}{3}$:

$$\frac{4}{3} + \frac{C}{2} = \frac{1}{3} \implies \frac{C}{2} = -1 \implies C = -2$$

$$y = \frac{x^2}{3} - \frac{2}{x}$$

Identical to the integrating factor result. ✓

Bernoulli Differential Equations

8.1 Definition

A **Bernoulli differential equation** takes the form:

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n, \qquad n \neq 0,\, 1$$

It is nonlinear due to the y^n term; the substitution $z = y^{1-n}$ transforms it into a linear equation.

8.2 Solution Method

Substitution $z = y^{1-n}$

1. Divide by y^n: $y^{-n}\dfrac{dy}{dx} + P(x)\,y^{1-n} = Q(x)$
2. Let $z = y^{1-n}$, so $\dfrac{dz}{dx} = (1-n)y^{-n}\dfrac{dy}{dx}$; the equation becomes: $\dfrac{1}{1-n}\dfrac{dz}{dx} + P(x)\,z = Q(x)$
3. Solve (linear in z), then back-substitute.

8.3 Example

Solve: $\dfrac{dy}{dx} + y = x\,y^3$

- $z = y^{-2} \Rightarrow \dfrac{dz}{dx} - 2z = -2x$; $\quad \mu = e^{-2x} \Rightarrow z = x + \frac{1}{2} + Ce^{2x}$

$$y = \pm\frac{1}{\sqrt{x + \dfrac{1}{2} + Ce^{2x}}}$$

Equations Reducible to Homogeneous Form

9.1 Introduction

Consider:

$$\frac{dy}{dx} = \frac{ax + by + c}{dx + ey + f}, \qquad a, b, c, d, e, f \in \mathbb{R}$$

If $c = f = 0$ the equation is already **homogeneous**. Otherwise two cases arise depending on whether the lines $ax + by + c = 0$ and $dx + ey + f = 0$ are **intersecting** or **parallel**.

9.2 Case 1: Intersecting Lines ($ae - bd \neq 0$)

Translation Method

1. Solve $\begin{cases} ax_0 + by_0 + c = 0 \\ dx_0 + ey_0 + f = 0 \end{cases}$ for (x_0, y_0).
2. Translate: $X = x - x_0$, $Y = y - y_0$.

3. The equation becomes homogeneous: $\dfrac{dY}{dX} = \dfrac{aX + bY}{dX + eY}$
4. Solve via $Y = vX$ (separable), then back-substitute.

9.2.1 *Example*

Solve: $\dfrac{dy}{dx} = \dfrac{2x + 3y + 1}{x + y - 1}$

1. System gives $(x_0, y_0) = (4, -3)$.
2. $X = x - 4$, $Y = y + 3$: $\dfrac{dY}{dX} = \dfrac{2X + 3Y}{X + Y}$
3. Set $Y = vX$ and solve the separable equation in v.

9.3 Case 2: Parallel Lines ($ae - bd = 0$)

Substitution Method

1. Let $u = ax + by \Rightarrow \dfrac{du}{dx} = a + b\dfrac{dy}{dx}$
2. Substitute into the ODE to get a **separable** or **Bernoulli** equation in u.
3. Solve, then back-substitute.

9.3.1 *Example*

Solve: $\dfrac{dy}{dx} = \dfrac{2x + 4y + 1}{x + 2y + 3}$

1. $ae - bd = (2)(2) - (4)(1) = 0 \Rightarrow$ parallel.
2. $u = x + 2y \Rightarrow \dfrac{du}{dx} = \dfrac{3u + 5}{u + 3}$
3. Separate variables, integrate, back-substitute $u = x + 2y$.

9.4 Summary of Cases

Case	Condition	Method
Intersecting	$ae - bd \neq 0$	Translate → Homogeneous → Separable
Parallel	$ae - bd = 0$	$u = ax + by$ → Separable / Bernoulli

Second-Order Linear Differential Equations

10.1 Definition

A **second-order linear DE** has the form:

$$y'' + a(x)\,y' + b(x)\,y = f(x)$$

where $a(x),\ b(x),\ f(x)$ are continuous on $I \subset \mathbb{R}$.

Key Concepts: Homogeneous if $f(x) = 0$; constant coefficients if a, b are constants; superposition principle applies.

10.2 Homogeneous Case — Characteristic Equation

For $y'' + ay' + by = 0$, solve $r^2 + ar + b = 0$:

Case	Roots	General Solution
Distinct real	$r_1 \neq r_2$	$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$
Repeated real	$r_1 = r_2$	$y = (C_1 + C_2 x)e^{r_1 x}$
Complex	$\alpha \pm \beta i$	$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

10.3 Non-Homogeneous Case

General Solution: $y = y_h + y_p$

Common guesses for y_p (undetermined coefficients):

- $f(x) = e^{kx}$: try $y_p = Ae^{kx}$
- $f(x) = \sin(ax)$ or $\cos(ax)$: try $y_p = A\cos(ax) + B\sin(ax)$
- $f(x) =$ polynomial of degree n: try same-degree polynomial
- **Resonance** (overlap with y_h): multiply guess by x

10.4 Worked Examples

10.4.1 Example 1 — Real Distinct Roots

Solve: $y'' - 3y' + 2y = 0$

- $r^2 - 3r + 2 = 0 \ \Rightarrow\ r_1 = 1,\ r_2 = 2$

$$y = C_1 e^{x} + C_2 e^{2x}$$

10.4.2 Example 2 — Repeated Root

Solve: $y'' - 4y' + 4y = 0$

- $r^2 - 4r + 4 = 0 \;\Rightarrow\; r_1 = r_2 = 2$

$$y = (C_1 + C_2 x)e^{2x}$$

10.4.3 Example 3 — Complex Roots

Solve: $y'' + 4y = 0$

- $r = \pm 2i; \quad \alpha = 0, \beta = 2$

$$y = C_1 \cos(2x) + C_2 \sin(2x)$$

10.4.4 Example 4 — Non-Homogeneous (Exponential)

Solve: $y'' + 2y' + 2y = e^x$

- $r = -1 \pm i; \quad y_h = e^{-x}(C_1 \cos x + C_2 \sin x)$
- $y_p = Ae^x$: $5A = 1 \;\Rightarrow\; A = \frac{1}{5}$

$$y = e^{-x}(C_1 \cos x + C_2 \sin x) + \tfrac{1}{5}e^x$$

10.4.5 Example 5 — Non-Homogeneous (Polynomial)

Solve: $y'' - y' - 2y = 4x^2$

- $r = -1, 2; \quad y_h = C_1 e^{-x} + C_2 e^{2x}$
- $y_p = Ax^2 + Bx + C$: $A = -2$, $B = 2$, $C = -3$

$$y = C_1 e^{-x} + C_2 e^{2x} - 2x^2 + 2x - 3$$

10.4.6 *Example 6 — Non-Homogeneous (Resonance)*

Solve: $y'' + y = 2\sin x$

- $y_h = C_1 \cos x + C_2 \sin x;$ resonance: $y_p = x(A\cos x + B\sin x)$
- $A = -1,\ B = 0$

$$y = C_1 \cos x + C_2 \sin x - x\cos x$$

Summary of Solution Types

Type	Condition	General Solution
Distinct real roots	$r_1 \neq r_2$	$C_1 e^{r_1 x} + C_2 e^{r_2 x}$
Repeated real root	$r_1 = r_2$	$(C_1 + C_2 x)e^{r_1 x}$
Complex roots	$\alpha \pm \beta i$	$e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$
Non-homogeneous	$f(x) \neq 0$	$y = y_h + y_p$

Linear Independence and the Wronskian

12.1 Linear Independence of Solutions

Two solutions y_1 and y_2 of $y'' + a_1 y' + a_2 y = 0$ are **linearly independent** on $[a, b]$ if

$$\frac{y_1}{y_2} \neq \text{constant}$$

Otherwise they are **linearly dependent**.

For $y'' - y = 0$, the solutions e^x, e^{-x}, $3e^x$, $5e^{-x}$ satisfy:

- $\dfrac{3e^x}{e^x} = 3 = \text{const} \Rightarrow$ $3e^x$ and e^x are **linearly dependent**.
- $\dfrac{e^x}{e^{-x}} = e^{2x} \neq \text{const} \Rightarrow$ e^x and e^{-x} are **linearly independent**.

12.2 The Wronskian Determinant

For two functions $y_1,\ y_2$:

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2$$

For n solutions the Wronskian generalises to:

$$W = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$

Theorem:

- If y_1, y_2 are linearly dependent then $W(y_1, y_2) = 0$.
- If $W \neq 0$ then the solutions are linearly independent.

12.3 Abel's Identity

Abel's Identity

Let y_1, y_2 be solutions of $y'' + P(x)y' + Q(x)y = 0$. The Wronskian satisfies:

$$W(x) = C\, e^{-\int P(x)\, dx}$$

where C is a constant determined by initial conditions.

Method of Variation of Parameters

This method finds the particular solution y_p of a non-homogeneous second-order equation when undetermined coefficients is not applicable.

Variation of Parameters

1. Solve the homogeneous equation: $y_h = C_1 y_1 + C_2 y_2$
2. Compute the Wronskian: $W = y_1 y_2' - y_2 y_1'$
3. Replace constants by functions: $y_p = u_1(x)\, y_1 + u_2(x)\, y_2$
4. Compute:
$$u_1 = \int \frac{-y_2\, f(x)}{W}\, dx, \qquad u_2 = \int \frac{y_1\, f(x)}{W}\, dx$$
5. General solution: $y = y_h + y_p$

13.1 Example

Solve: $y'' + y = \dfrac{1}{\cos x}$

1. $r^2 + 1 = 0 \Rightarrow y_1 = \cos x,\ y_2 = \sin x$
2. $W = \cos^2 x + \sin^2 x = 1$
3. $u_1 = \displaystyle\int \frac{-\sin x}{\cos x}\,dx = \ln|\cos x|; \quad u_2 = \int 1\ dx = x$
4. $y_p = \ln|\cos x| \cdot \cos x + x \sin x$

$$y = C_1 \cos x + C_2 \sin x + \ln|\cos x| \cos x + x \sin x$$

Reduction of Order Method

Used when the equation is missing either y or x.

14.1 Case 1 — y is Missing

The equation contains x, y', y'' but not y.

Substitution $p = y'$

Let $p = y' \Rightarrow y'' = \dfrac{dp}{dx}$. The equation reduces in order from 2 to 1 in p. Solve for p, then integrate to find y.

14.2 Case 2 — x is Missing

The equation contains y, y', y'' but not x.

Substitution $p = y'$, chain rule

Let $p = y' \Rightarrow y'' = p\dfrac{dp}{dy}$. The equation reduces to a first-order ODE in $p(y)$. Solve for p, then integrate $\dfrac{dy}{dx} = p$ to find y.

14.3 Example

Solve: $y'' = y \quad$ (x is missing)

1. Let $p = y' \Rightarrow p\dfrac{dp}{dy} = y$
2. Separate and integrate: $\frac{1}{2}p^2 = \frac{1}{2}y^2 + C_1 \Rightarrow \ p = \dfrac{dy}{dx} = \sqrt{y^2 + C_1}$
3. Integrate: $\displaystyle\int \frac{dy}{\sqrt{y^2 + C_1}} = \int dx \ \Rightarrow\ \sinh^{-1}\!\left(\frac{y}{\sqrt{C_1}}\right) = x + C_2$

Exercises

15.1 Set A — First-Order Equations

1. **(Bernoulli)** Solve: $\frac{dy}{dx} + 2y = y^2 e^x$
2. **(Bernoulli)** Solve: $\frac{dy}{dx} - y = xy^4$
3. **(Reducible, Parallel)** Solve: $\frac{dy}{dx} = \frac{x + 2y - 1}{2x + 4y + 5}$
4. **(Reducible, Parallel)** Solve: $\frac{dy}{dx} = \frac{3x - y + 2}{6x - 2y + 1}$
5. **(Reducible, Intersecting, IVP)** Solve with $y(0) = 1$: $\frac{dy}{dx} = \frac{x - y + 2}{x + y - 4}$

15.2 Set B — Second-Order Equations

1. **(Undetermined coefficients)** Solve: $y'' + y' + y = \cos x$
2. **(Resonance)** Solve: $y'' + 4y = \sin 2x$
3. **(Variation of parameters)** Solve: $y'' + y = \tan x$
4. **(Reduction of order, x missing)** Solve: $yy'' = (y')^2$

Additional Challenge: Show that the substitution $u = ax + by + c$ can sometimes reduce the equation directly to a separable form. When does this work?

Conclusion and Recap

This lecture covered the following solution methods:

- **Linear first-order DEs** — integrating factor method.
- **Bernoulli equations** — substitution $z = y^{1-n}$.
- **Equations reducible to homogeneous form** — translation (intersecting lines) or substitution (parallel lines).
- **Linear independence** — Wronskian test and Abel's identity.
- **Variation of parameters** — particular solution for any $f(x)$.
- **Reduction of order** — lowers degree when y or x is absent.
- **Second-order DEs** — characteristic equation, undetermined coefficients, resonance.

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