

$$\frac{(E, +_E, \cdot_E)}{VS} \quad | \quad (K, +_{\mathbb{K}}, \cdot_{\mathbb{K}})$$

1) $(E, +_E)$ Abelian group

2)

a) $\forall \alpha \in K, \forall x, y \in E : \alpha(x+y) = \alpha x + \alpha y$

b) $\forall \alpha, \beta \in K, \forall x \in E : (\alpha + \beta) \cdot x = \alpha x + \beta x$

c) $\forall \alpha, \beta \in K, \forall x \in E : (\alpha \cdot \beta) \cdot x = \alpha \cdot (\beta \cdot x)$

d) $1_{\mathbb{K}} \cdot x = x$

EXERCISE 1

$$\forall (a, b), (c, d) \in \mathbb{R}^2 : (a, b) + (c, d) = (a + c, b + d)$$

$(\mathbb{R}^2, +)$ is an abelian group

$$\begin{aligned} (a, b) + (c, d) &= (a + c, b + d) = (c + a, d + b) \\ &= (c, d) + (a, b) \end{aligned}$$

$$\begin{aligned} [(a, b) + (c, d)] + (e, f) &= (a, b) + [(c, d) + (e, f)] \\ \underline{(0, 0)} + (a, b) &= (a, b) \end{aligned}$$

$$(a, b) + (-a, -b) = (0, 0)$$

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$$a) \forall \alpha \in \mathbb{R}, (a, b), (c, d) \in \mathbb{R}^2$$

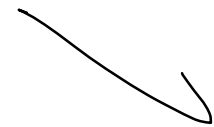
$$\alpha \cdot [(a, b) + (c, d)] = \alpha (a+c, b+d)$$

$$= (\alpha(a+c), b+d)$$

$$= (\alpha a + \alpha c, b+d)$$

$$= (\alpha a, b) + (\alpha c, d)$$

$$= \alpha \cdot (a, b) + \alpha (c, d)$$



$$b) \text{ Let } \alpha, \beta \in \mathbb{R}, (a, b) \in \mathbb{R}^2$$

$$(\alpha + \beta)(a, b) = ((\alpha + \beta)a, b)$$

$$\begin{aligned} \alpha(a, b) + \beta(a, b) &= (\alpha a, b) + (\beta a, b) \\ &= (\alpha a + \beta a, 2b) \end{aligned}$$

then $(\alpha + \beta)(a, b) \neq \alpha(a, b) + \beta(a, b)$

then $(\mathbb{R}^2, +, \cdot)$ is not vector space.

2) ~~1)~~ $(\mathbb{R}^2, +)$ is an abelian group

②

~~Let~~ $\alpha \in \mathbb{R}, (a, b), (c, d) \in \mathbb{R}^2$

$$\begin{aligned}\alpha [(a, b) + (c, d)] &= \alpha (a + c, b + d) \\ &= (\alpha^2(a + c), \alpha^2(b + d)) \\ &= (\alpha^2 a + \alpha^2 c, \alpha^2 b + \alpha^2 d) \\ &= (\alpha^2 a, \alpha^2 b) + (\alpha^2 c, \alpha^2 d) \\ &= \alpha (a, b) + \alpha (c, d) \quad \checkmark\end{aligned}$$

$$\alpha, \beta \in \mathbb{R}, (a, b) \in \mathbb{R}^2$$

$$(\alpha + \beta)(a, b) = ((\alpha + \beta)^2 a, (\alpha + \beta)^2 b)$$

$$\begin{aligned} \alpha(a, b) + \beta(a, b) &= (\alpha^2 a, \alpha^2 b) + (\beta^2 a, \beta^2 b) \\ &= ((\alpha^2 + \beta^2) a, (\alpha^2 + \beta^2) b) \end{aligned}$$

$$(\alpha + \beta)^2 \neq \alpha^2 + \beta^2$$

then $(\mathbb{R}^2, +, \cdot)$ is not a vector space

$$\textcircled{c} \quad (a, b) + (c, d) = (a + c, b + d)$$

$$\lambda (a, b) = (\lambda a, \lambda b)$$

$(\mathbb{R}^2, +, \cdot)$ is a \mathbb{R} -VS

$$(a, b) + (c, d) = (a + d, c)$$

$$\lambda (a, b) = (\lambda a, \lambda b)$$

$(\mathbb{R}^2, +)$ is an abelian group \checkmark

~~$(a, b) + (c, d) = (a + d, c)$~~

~~$(c, d) + (a, b) = (c + b, a)$~~

$$(1, 2) + (3, 4) = (5, 3)$$

$$(3, 4) + (1, 2) = (5, 1)$$

$(\mathbb{R}^2, +, \cdot)$ is not a v

$$(a, b, c) + (a', b', c') = (a + a', b + b', c + c')$$

$$\lambda (a, b, c) = (\lambda a, \lambda b, \lambda c)$$

$(\mathbb{R}^3, +, \cdot)$ is V S

$(\mathbb{R}^n, +, \cdot)$ is V S

$$(x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n) = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

$$\lambda (x_1, x_2, \dots, x_n) = (\lambda x_1, \lambda x_2, \dots, \lambda x_n)$$

