$\cancel{(E, +_E, \cdot_E)}$, $(\mathbb{K}, +_{\mathbb{K}}, \cdot_{\mathbb{K}})$

VS

1) $(E, +_E)$ Abelien groupe

2)

a) $\forall \alpha \in \mathbb{K}, \forall x, y \in E : \alpha(x+y) = \alpha x + \alpha y$

b) $\forall \alpha, \beta \in \mathbb{K}, \forall x \in E : (\alpha + \beta) x = \alpha x + \beta x$

c) $\forall \alpha, \beta \in \mathbb{K}, \forall x \in E : (\alpha \cdot \beta) \cdot x = \alpha \cdot (\beta \cdot x)$

d) $1_{\mathbb{K}} \cdot x = x$

Exo N°1

1) $\forall (a,b),(c,d) \in \mathbb{R}^2 : (a,b)+(c,d) = (a+b, b+$

$(\mathbb{R}^2, +)$ is abelian group

$$(a,b)+(c,d) = (a+c, b+d) = (c+a, d+b)$$
$$= (c,d)+(a,b)$$

$$[(a,b)+(c,d)]+(e,f) = (a,b)+[(c,d)+(e,f)]$$

$$\underline{(0,0)}+(a,b) = (a,b)$$

$$(a,b)+(-a,-b)=(0,0)$$

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a) Let $\alpha \in \mathbb{R}$, $(a,b),(c,d) \in \mathbb{R}^2$

$$\alpha \cdot [(a,b)+(c,d)] = \alpha(a+c,\ b+d)$$
$$= (\alpha(a+c),\ b+d)$$
$$= (\alpha a + \alpha c,\ b+d)$$
$$= (\alpha a,\ b) + (\alpha c,\ d)$$
$$= \alpha \cdot (a,b) + \alpha (c,d)$$

✓

b) Let $\alpha, \beta \in \mathbb{R}$, $(a, b) \in \mathbb{R}^2$

$(\alpha+\beta)(a, b) = ((\alpha+\beta)a, b)$

$\alpha(a, b) + \beta(a, b) = (\alpha a, b) + (\beta a, b)$

$= (\alpha a + \beta a, 2b)$

then $(\alpha+\beta)\cdot(a, b) \neq \alpha\cdot(a, b) + \beta(a, b)$

then $(\mathbb{R}^2, +, \cdot)$ is not vector space.

2) ~~a)~~ $(\mathbb{R}^2, +)$ is an abelian group

2

Let $\alpha \in \mathbb{R}$, $(a,b), (c,d) \in \mathbb{R}^2$

$$\alpha\left[(a,b)+(c,d)\right] = \alpha(a+c, b+d)$$

$$= \left(\alpha^2(a+c), \alpha^2(b+d)\right)$$

$$= \left(\alpha^2 a + \alpha^2 c, \alpha^2 b + \alpha^2 d\right)$$

$$= (\alpha^2 a, \alpha^2 b) + (\alpha^2 c, \alpha^2 d)$$

$$= \alpha(a,b) + \alpha(c,d)$$

Let $\alpha, \beta \in \mathbb{R}$, $(a,b) \in \mathbb{R}^2$

$$(\alpha+\beta)(a,b) = \left((\alpha+\beta)^2 a, (\alpha+\beta)^2 b\right)$$

$$\alpha(a,b) + \beta(a,b) = (\alpha^2 a, \alpha^2 b) + (\beta^2 a, \beta^2 b)$$

$$= \left((\alpha^2+\beta^2) a, (\alpha^2+\beta^2) b\right)$$

$$(\alpha+\beta)^2 \neq \alpha^2 + \beta^2$$

Then $(\mathbb{R}^2, +, \cdot)$ is not a vector space a $(a,b)+(c,d)=(a+c,\ b+d)$

$\lambda(a,b)=(\lambda a,\ \lambda b)$

$(\mathbb{R}^2, +, \cdot)$ is V.S

$$(a,b)+(c,d) = (a+d, c)$$

$$\lambda(a,b) = (\lambda a, \lambda b)$$

$(\mathbb{R}^2, +)$ is an abelian group No

$$* \ (a,b)+(c,d) = (a+d, c)$$

$$* \ (c,d)+(a,b) = (c+b, a)$$

$$(1,2)+(3,4) = (5,3)$$

$$\neq$$

$$(3,4)+(1,2) = (5,1)$$

$(\mathbb{R}^2, +, \cdot)$ is not V

$$(a, b, c) + (a', b', c') = (a + a', b + b', c + c')$$

$$\lambda (a, b, c) = (\lambda a, \lambda b, \lambda c)$$

$(\mathbb{R}^3, +, \cdot)$ is VS

$(\mathbb{R}^n, +, \cdot)$ is VS

$$(x_1, x_2, \ldots x_n) + (y_1, y_2, \ldots y_n) = (x_1 + y_1, x_2 + y_2$$

$$\lambda (x_1, x_2, \ldots x_n) = (\lambda x_1, \lambda x_2, \ldots, \lambda x$$