

## Exercise Series No. 02

### Exercise 1

A student estimates his chances of passing his statistics course at 65%, his chances of passing his chemistry course at 80%, and his chances of passing both subjects at 50%.

- (1) What is the probability that the student passes Statistics but not Chemistry?
- (2) What is the probability that he passes Chemistry but not Statistics?
- (3) What is the probability that the student passes Statistics or Chemistry?
- (4) What is the probability that he passes neither Chemistry nor Statistics?
- (5) What is the probability that he passes Statistics given that he has passed Chemistry?

### Exercise 2

Consider the random experiment: “a six-faced die is rolled and the result is observed.”  
We consider the following game:

- If the result is even, you win 2 DA
- If the result is 1, you win 3 DA
- If the result is 3 or 5, you lose 4 DA

We define a random variable  $X$  representing the gain in this game.

- (1) Give the sample space  $\Omega$ .
- (2) Determine  $X(\Omega)$ .
- (3) Determine the probability distribution of  $X$ .
- (4) Determine the cumulative distribution function of  $X$ .
- (5) Compute the mathematical expectation  $E(X)$ , the variance  $V(X)$ , and the standard deviation  $\delta_x$ .

### Exercise 3

Consider the following random experiment: a pair of fair dice numbered from 1 to 6 is thrown. The random variable  $X$  is defined by:

$$X : \Omega \longrightarrow \mathbb{R}$$
$$(a, b) \longmapsto X(a, b) = \max(a, b)$$

- (1) Give the sample space  $\Omega$ .
- (2) Determine  $X(\Omega)$ .
- (3) Determine the probability distribution of  $X$ .
- (4) Determine the cumulative distribution function of  $X$ .
- (5) Compute the mathematical expectation  $E(X)$ , the variance  $V(X)$ , and the standard deviation  $\delta_x$ .

### Exercise 4

Consider the function defined by

$$f(x) = \begin{cases} \frac{1}{2}(2-x) & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (1) Show that  $f$  is a probability density function of a random variable  $X$ .
- (2) Determine its cumulative distribution function.
- (3) Compute the mathematical expectation of  $X$ .
- (4) Compute the variance and the standard deviation of  $X$ .

### Exercise 5

Let  $X$  be a random variable with the associated function:

$$f(x) = \begin{cases} ax(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (1) Determine  $a$  so that  $f$  is a probability density function.
- (2) Determine its cumulative distribution function.
- (3) Compute  $P(0 \leq X \leq \frac{1}{2})$ .