UNIVERSITY CENTER OF MILA
INSTITUTE OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF CIVIL AND HYDRAULIC ENGINEERING

Reinforce concrete 2
(3th year Civil Engineering)

☛ **Responsible:** Pr. GUETTICHE ABDELHEQ

☛ Academic Year 2025- 2026

Course Outline

1.Chapter 01: Simple Bending

Rectangular and T-sections: Ultimate Limit State of Strength (ELU) + Serviceability Limit State (ELS)

2.Chapter 02: Shear Force

- Calculation of transverse reinforcement.
- Concrete strength of the shear strut.
- Support justifications.

3.Chapter 03: Composite Bending

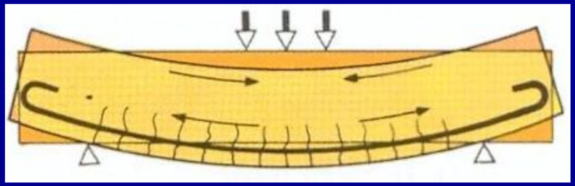
- Calculation of sections at limit states for rectangular and T-sections.
- Buckling of compressed columns.

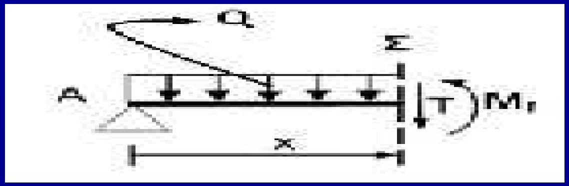
4.Chapter 04: Torsion

- Overview of the torsion phenomenon and justification of concrete and reinforcements (Hollow and solid sections).
- Justification of beams under torsional loading.

Calcul de sections en béton armé soumises à la flexion simple

Definition: An element is subjected to simple bending if, at any cross-section of this element, the solicitations reduce to a flexural moment Mf and a shear force T (the normal force N=0).





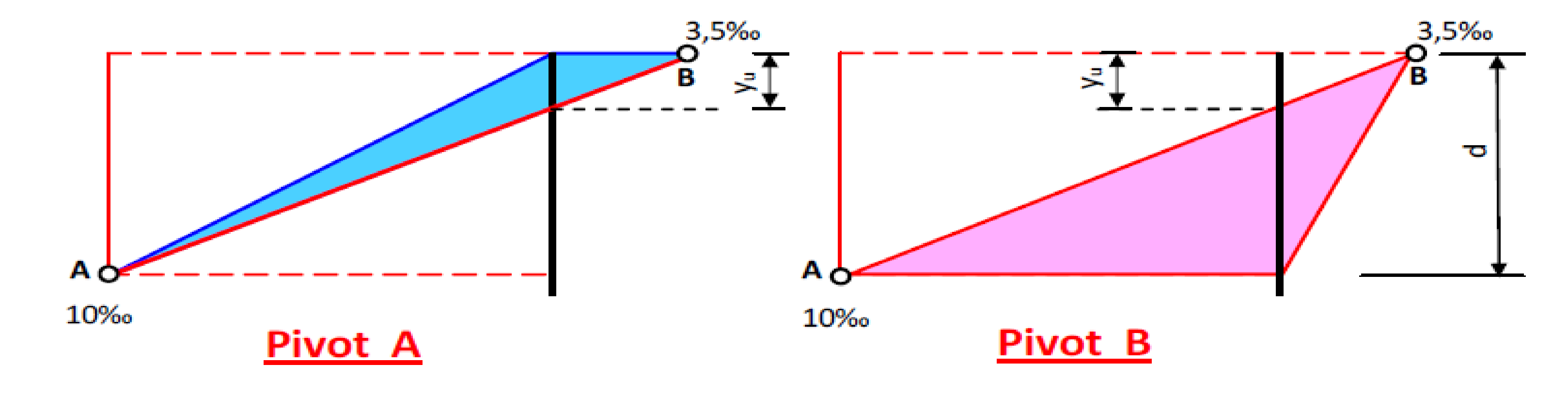
The elements of a structure subjected to simple bending are primarily beams, whether they are statically determinate or continuous. In reinforced concrete, the action of flexural moment leads to the design of longitudinal reinforcement , while the action of shear force guides the design of transverse reinforcement (such as frames, stirrups, or U-bars (Hairpins). These two calculations are conducted separately. In this chapter, we present calculations related to flexural moments, considering both ultimate limit states (ULS) and serviceability limit states (SLS). The study includes rectangular and T-sections, with or without compressed reinforcements.

I. Calculation of Longitudinal Reinforcements at ULS (Ultimate Limit State).

I.1. Calculation Assumptions

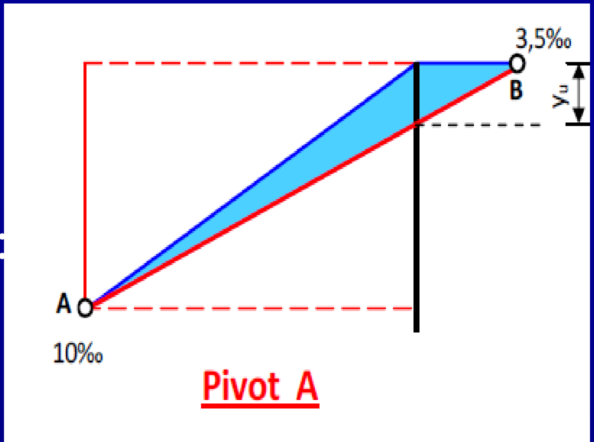
- The tensile strength of concrete is neglected.
- No relative slippage between steel and concrete.
- The steel section is concentrated at its center of gravity.
- Concrete resists compressive stresses, and steel resists tensile and compressive stresses.
- Limiting strains: according to the 'three pivots' method, which requires, in simple bending, reaching one of the pivots A or B.
- \- **Pivot A**: $\varepsilon_{st} = 10$ ‰ et $0 \leq \varepsilon_{bc} \leq 3,5$‰.

\- **Pivot B**: $0 \leq \varepsilon_{st} \leq 10$‰ et $\varepsilon_{bc} = 3,5$‰.



I.2. Specific Positions of the Neutral Axis: The assumption of continuity of deformations within the section (no slippage of reinforcements relative to concrete) leads to the following equation:

$$\frac{y_u}{\varepsilon_b} = \frac{d - y_u}{\varepsilon_s}$$



From this relation, we can deduce the following equation:

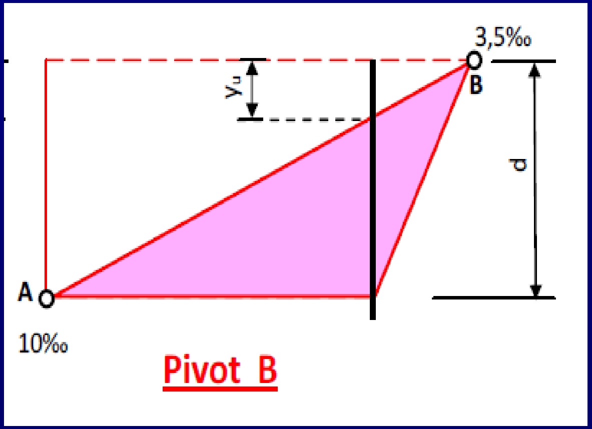
$$\frac{y_u}{d} = \frac{\varepsilon_b}{\varepsilon_b + \varepsilon_s}$$

By introducing:

$$\alpha_u = \frac{y_u}{d}$$

α_u : Relative position of the neutral fiber with respect to the most compressed fiber), we can express that:

$$\alpha_u = \frac{\varepsilon_b}{\varepsilon_b + \varepsilon_s}$$



I.3. Specific Values of αu: : If the deformation line passes through pivots A and B, then:

$$\alpha_u = \alpha_A = \frac{3.5}{3.5+10} = 0.2591$$

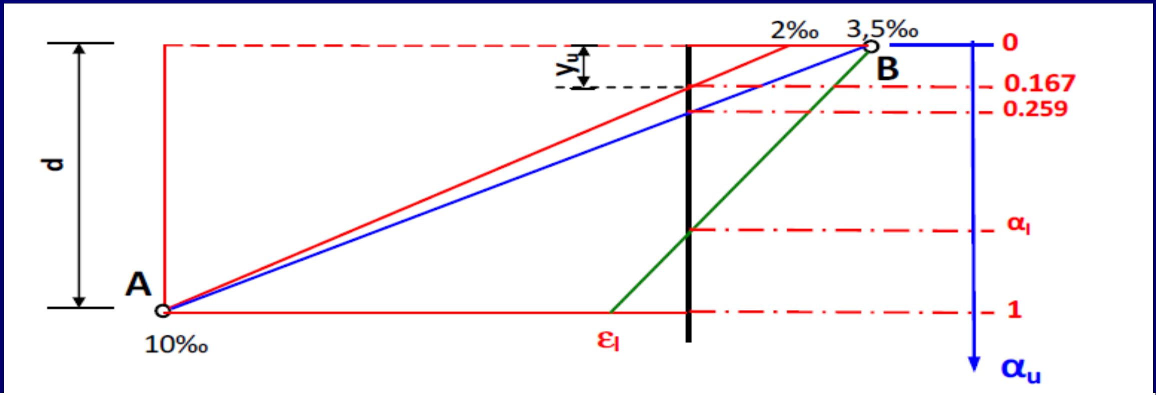
- *if* (ε_{bc} = 2 ‰ et ε_{st} = 10 ‰) **pivot A** ➡ $\alpha_u = 0.167$;
- if (0 ≤ ε_{bc} < 2 ‰ et ε_{st} = 10 ‰) **pivot A** ➡ 0 ≤ α_u < 0.167 : The concrete performs inadequately, and the section is oversized in concrete.
- *Si* (2‰ ≤ ε_{bc} ≤ 3.5 ‰ et ε_{st} = 10 ‰) **pivot A** ; ➡ $0.167 \leq \alpha_u \leq 0.259$
- *Si* (ε_{bc} = 3.5 ‰ et $\varepsilon_l \leq \varepsilon_{st}$ < 10 ‰ **pivot B** ➡ $0.259 \leq \alpha_u \leq \alpha_l$

$$\varepsilon_l = \varepsilon_e = f_e/(\gamma_s \cdot E_s)$$

$$\alpha_l = \frac{3.5}{3.5+(\varepsilon_l \cdot 1000)}$$

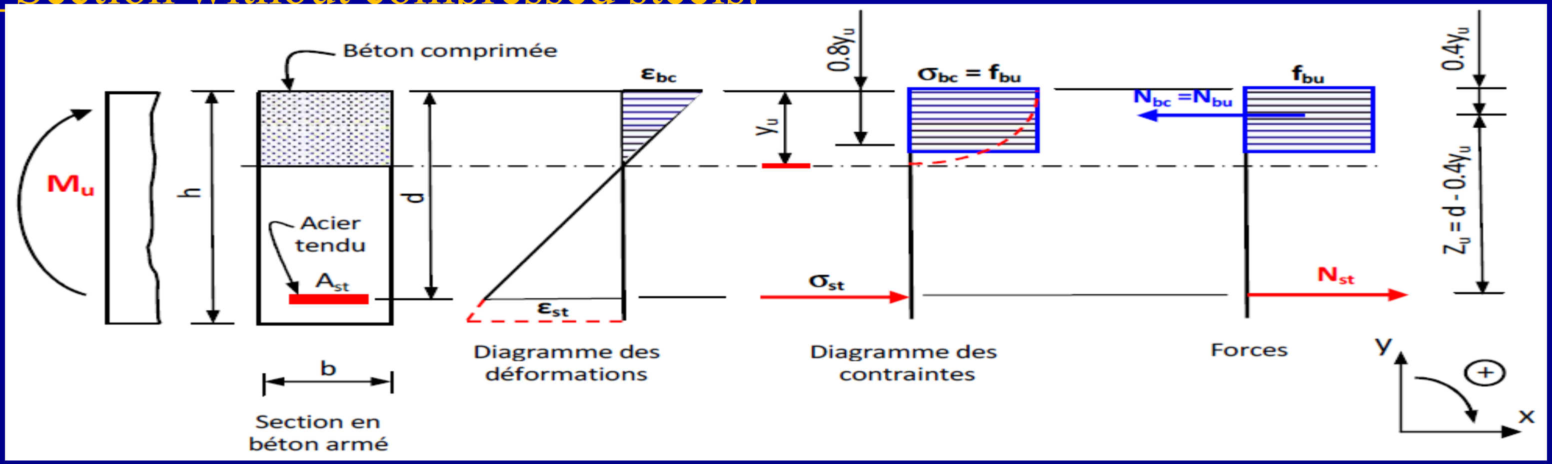
- *if* **pivot B** (ε_{bc}= 3.5 ‰ et $\varepsilon_l \geq \varepsilon_{st} \geq 0$) : ➡ $\alpha_l \leq \alpha_u \leq 1$:The steel works insufficiently, leading to large reinforcement sections.
- **Conclusion** : To optimize the characteristics of both concrete and steel, it is recommended:

$$0.167 \leq \alpha_u \leq \alpha_l$$



I.4. Rectangular Section::

I.4.1. Section without compressed steels:



Equilibrium Equations: $\sum Fx = 0$ ⟹ $N_{st} - N_{bc} = 0$

$N_{bc} = N_{bu} = 0.8\, y_u .b .f_{bu}$ **and** $N_{st} = A_{st} .\sigma_{st}$ **donc :** $0.8\, y_u .b .f_{bu} = A_{st} .\sigma_{st}$

$\sum MA = 0$ (The sum of moments is calculated with respect to the center of gravity of the tensioned steel.)

1) $M_u - N_{bu} .Z_u = 0$ ou : 2) $Z_u = d - 0.4y_u$ (Lever arm (bras de levier))

3) $M_u - (0.8\, y_u . b . f_{bu}). (d - 0.4\, y_u) = 0$

4) $M_u = (0.8\, y_u . b . f_{bu}). (d - 0.4\, y_u)$ so

5) $M_u = A_{st} .\sigma_{st} Z_u$ ⟹ $A_{st} = M_u / (\sigma_{st}.Z_u)$

Recall that in the previous paragraph, we established:$\alpha = \frac{y_u}{d}$ so : $Zu = d(1 – 0.4\ \alpha u)$. Soit $\beta u = 1 – 0.4\ \alpha u$ (bras de levier réduit)

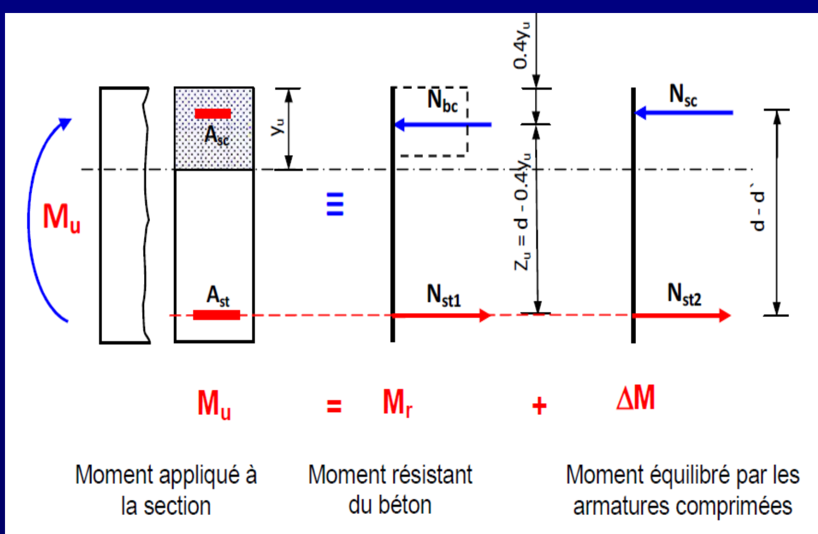
The equation (4) becomes:: $Mu = 0.8\ \alpha u\ .\ (1 - 0.4\ \alpha u\ )\ .\ b.\ d^2.\ Fbu$

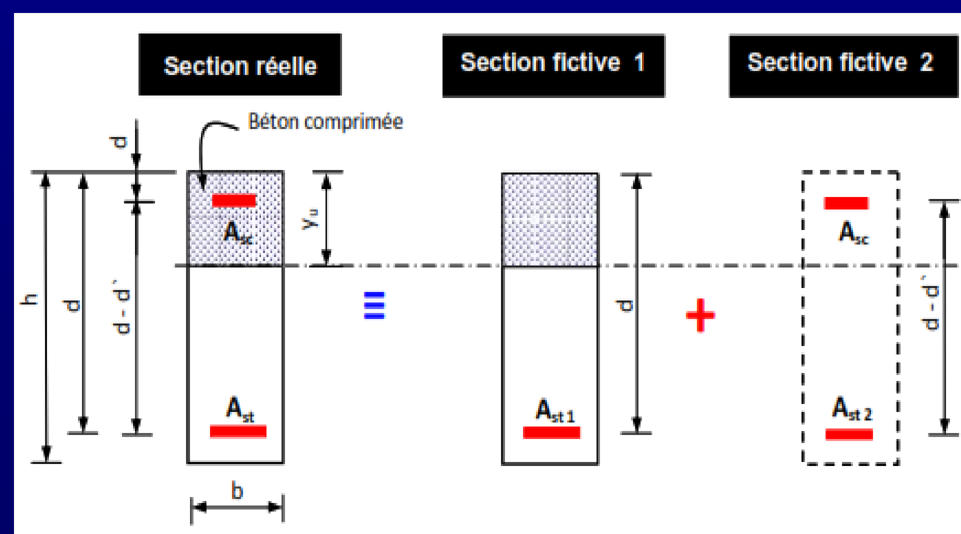
$$A_{st} = M_u / (\sigma_{st}.\beta_u\ .\ d)$$

We put : $\mu_{bu} = M_u / (b.\ d^2.\ f_{bu})$ μbu : Reduced ultimate moment)

We have : $\mu_{bu} = 0.8\ \alpha_u\ .\ (1 - 0.4\ \alpha_u)$ and $\alpha_u = 1.25.\ (1 - \sqrt{1 - 2\ .\mu_{bu}}\ )$

I.4.2. Section with Compressed Steels: When, in a rectangular section with prescribed dimensions, it is determined that $\mu bu > \mu l$, the moment Mu can be balanced by reinforcing the compressed part of the section with compression reinforcement of section Asc. Therefore, Ast (tensile reinforcement) = **Ast1 +Ast2**





- ✓ **Mr (he moment resistance of concrete)** : is the ultimate moment that the section can balance without adding compressed steels.
- ✓ ΔM (The residual moment) : is the difference between the ultimate moment (Mu) demanding the section and the moment resistance of concrete (Mr).
- ✓ **Calculation of concrete moment resistance Mr :**

$$N_{st} = N_{st\,1} + N_{st\,2}$$

With μ_l : the ultimate reduced limit moment

$$\mu_l = M_r / (b.\,d^2.\,f_{bu})$$

Calculation of the residual moment ΔM :

$$M_r = \mu_l\,.\,b.\,d^2.\,f_{bu}$$

so:

$$\Delta M = M_u - M_r$$

Calculation of the tensile reinforcements for the fictitious Section 1(A_{st1}):

$A_{st1} = M_r / (\sigma_{st}.\beta_l\,.\,d)$

We have: $\varepsilon_{st} = \varepsilon_l$ → $\sigma_{st} = f_{su}$ so :

$$A_{st\,1} = M_r / (f_{su}\,.\,\beta_l\,.\,d)$$

Calculation of the tensile reinforcements for the fictitious Section 2($A_{st\,2}$):

$$\Delta M = N_{st\,2}\,.\,(d - d')$$

et

$$N_{st\,2} = f_{su}\,.\,A_{st\,2}$$

→ $\Delta M = f_{su}\,.\,A_{st2}.\,(d - d') \Rightarrow A_{st\,2} = \Delta M / (f_{su}\,.(d - d'))$

Calculation of the compressed reinforcements for the fictitious Section 2(A_{sc}):

$N_{sc} = N_{st2}$ and $\Delta M = N_{sc}\,.\,(d - d')$ → $A_{sc} = \Delta M / (\sigma_{sc}\,.(d - d'))$

$N_{sc} = \sigma_{sc}\,.\,A_{sc}$ so $\Delta M = \sigma_{sc}\,.A_{sc}.\,(d - d')$

□ Determining the stress in the compressed steel (σsc) :

Based on the principle of similar triangles, we have:

$$\frac{\varepsilon_{sc}}{3.5\,.\,10^{-3}} = \frac{y_u - d`}{y_u}$$

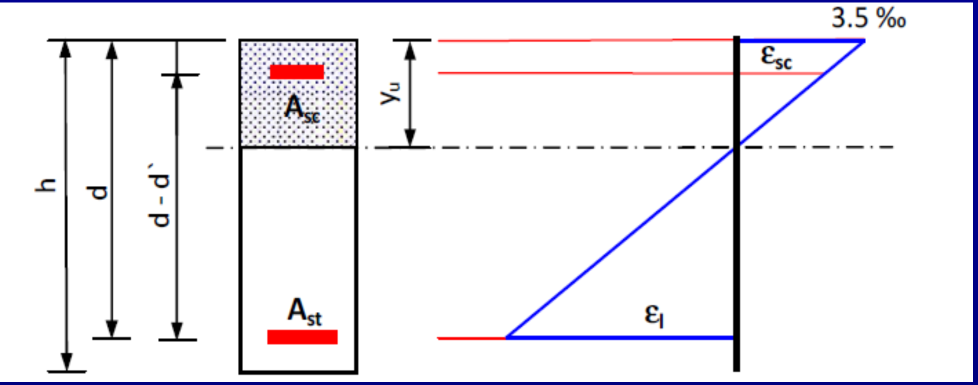
$$\Rightarrow \quad \varepsilon_{sc} = 3.5\,.\,10^{-3}\,.\,\left(\frac{\alpha_l\,.\,d - d`}{\alpha_l\,.\,d}\right)$$

- Given we know ε_{sc} we can :

determine σ_{sc} :

if $\varepsilon_{sc} < \varepsilon_l$ so: $\sigma_{sc} = \varepsilon_{sc}\,.\,E_{sc}$

if $\varepsilon_{sc} > \varepsilon_l$ so : $\sigma_{sc} = f_{su}$



Finally, we substitute the value of **σsc** Finally, we substitute the value of **Asc**.

➢ The total area of tension reinforcement Ast:

$$A_{st} = A_{st\,1} + A_{st_2}$$

Application:

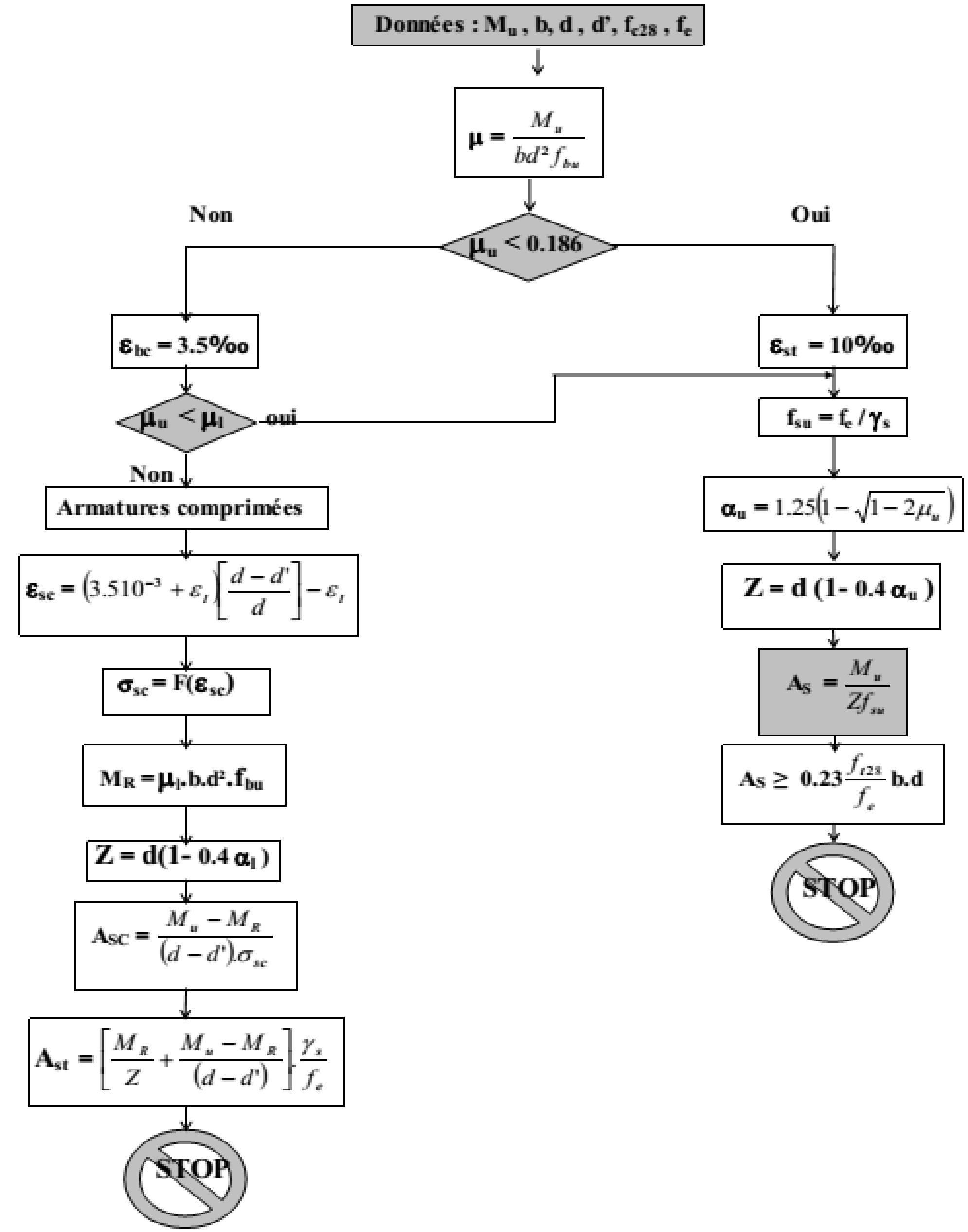
a (25×50) section subjected to a bending moment Mu = 0.153MN.m with fc28 = 25MPA and FeE400 reinforcement steel.

Calculate the area of reinforcement AS for ULS

	Nuance	fe (MPa)	$1000\xi_l$	α_L	μ_L	β_L
Ronds lisses	fe E215	215	0.935	0.789	0.432	0.684
	fe E235	235	1.022	0.774	0.427	0.690
Barres HA	fe E400	400	1.739	0.668	0.392	0.733
	fe E500	500	2.174	0.617	0.372	0.753

FLEXION SIMPLE (E.L.U)

SECTION RECTANGULAIRE



II. Verification for Serviceability Limit States (SLS): Reinforced concrete elements subjected to simple bending moment are generally designed for serviceability limit states for the following cases: non-damaging cracking and highly-damaging cracking. The checks to be performed concerning serviceability limit states with respect to the durability of the structure lead to ensuring that the limiting design stresses at SLS are not exceeded:

- Concrete compressive stress
- Steel tensile stress depending on the case of cracking considered (crack opening limit state).

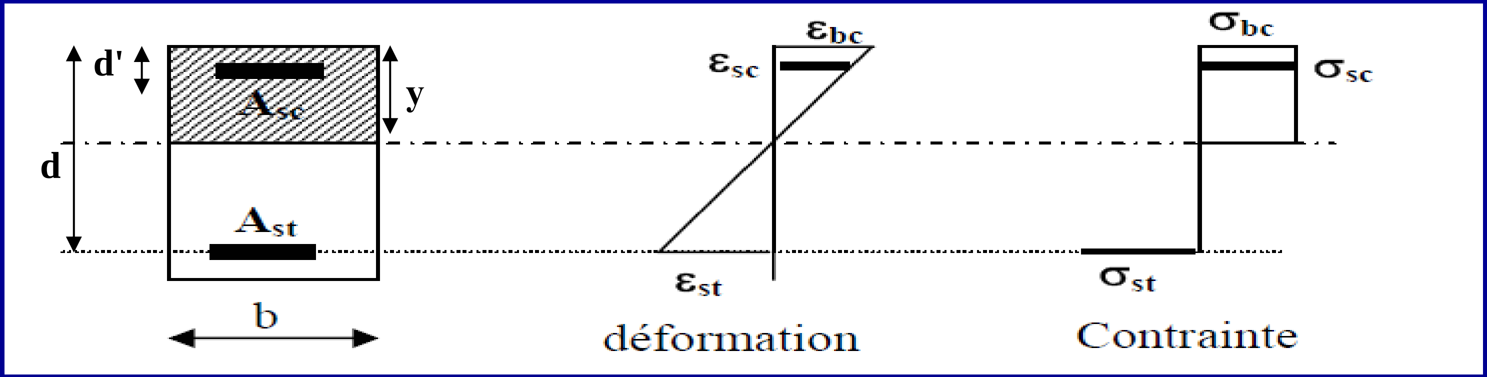
II.1- Détermination des contraintes :

- The concrete compressive stress: $\sigma_{bc} \leq \bar{\sigma}_{bc}$ où: $\overline{\sigma}_{bc} = 0{,}6 \cdot f_{c28}$
- The steel tensile stress: $\sigma_{st} \leq \bar{\sigma}_{st}$:

detrimental cracking $\overline{\sigma}_{st} \leq \min\left(\frac{2}{3}.fe \; ; \; 110\sqrt{\eta.f_{t28}}\right)$

Very detrimental cracking : $\overline{\sigma}_{st} \leq \min\left(\frac{1}{2}.fe \; ; \; 90\sqrt{\eta.f_{t28}}\right)$

The two fundamental unknowns that must be determined for verification are y and I.



II.2- determination of the neutral axis.

By the equilibrium of static moments ∑M/C.d.G

$F_b + F_{sc} - F_{st} = 0$ and $\sum {}^{M}/_{Aciers\ Tendus} = M - F_b Z - F_{sc}(d - d') = 0$

The stress in concrete and steel

$$\sigma_{bc} = \frac{M_{ser} \times y}{I_{GZ}}) \qquad \sigma_{st} = \frac{n . M_{Ser} (d - y)}{I_{GZ}}$$

In addition, according to the homogeneity condition:

$$\sigma_s = \sigma_b . \frac{E_s}{E_b} \qquad \frac{E_s}{E_b} = n = 15$$

The position of the neutral axis.

$$b . y . \frac{y}{2} + n . A_{sc} . (y - d') - n . A_{st} (d - y) = 0$$

If there is no compressed reinforcement, Asc=0

Determining the moment of inertia I: Moment of inertia of compressed concrete + moment of inertia of tension reinforcement + moment of inertia of compressed reinforcement.

Compressed concrete: $$I_b = \frac{by^3}{12} + by\left(\frac{y}{2}\right)^2 = \frac{by^3}{3}$$

Tension reinforcement: $$I_{st} = nbre\ de\ barres.\frac{\pi D_{st}^4}{64} + nA_{st}(d-y)^2 \approx nA_{st}(d-y)^2$$

Compressed reinforcement: $$I_{sc} = nbre\ de\ barres.\frac{\pi D_{sc}^4}{64} + nA_{sc}(y-d')^2 \approx nA_{sc}(y-d')^2$$

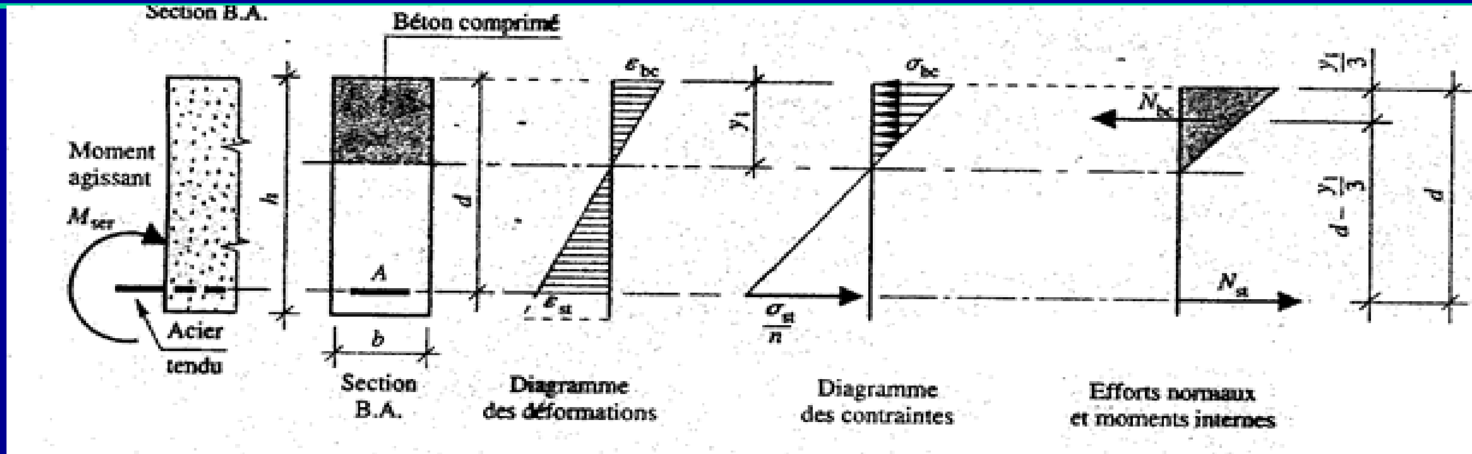
It follows then. $$I = \frac{by^3}{3} + nA_{st}(d-y)^2 + nA_{sc}(y-d')^2$$

II.3- Determination of reinforcements:

a- Rectangular section without compressed reinforcements:

Consider a common rectangular section of a beam subjected to a simple bending moment. Under the action of forces, the section deforms until reaching a state of equilibrium of forces: ∑ compression forces of concrete = ∑ tension forces of steel.

$N_{bc} = N_{st}$



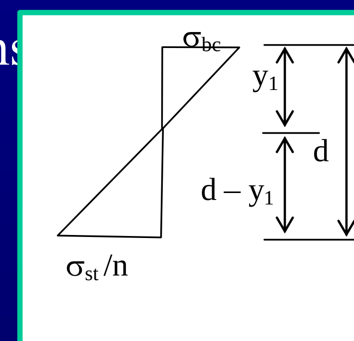
so: $$N_{ser} = \frac{1}{2} b y_1 \sigma_{bc_{\max}} - A_s \sigma_{st} = 0 \quad (1)$$

Moment résistant du béton : M_{rsb} : This is the maximum service moment that a section can balance without adding compressed reinforcements. The materials have then reached their allowable stresses.

Equilibrium of moments with respect to the centroid of the tens

Bending moment = compression force × lever arm.

$M_{ser} = N_{bc} \times Z_1$

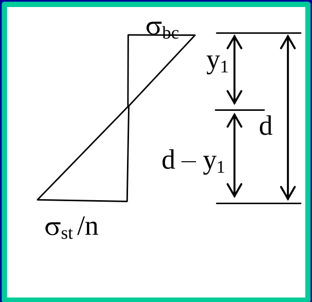


lever arm. (symbole Z1): $$z_1 = d - \frac{y_1}{3} \quad (2)$$

$$M_{ser} = \frac{1}{2} b y_1 \sigma_{bc_{\max}} \left(d - \frac{y_1}{3}\right) \quad (3)$$

Expression of Mser: with (1) and (2), we obtain:(4) $$M_{scr} = A_{scr} \cdot \sigma_{st} \cdot z_1$$

elation in the stress diagram of the homogenized section



$$\frac{\sigma_{bc}}{\frac{\sigma_{st}}{n}} = \frac{y_1}{d - y_1}$$ so $$\frac{y_1}{d} = \frac{n \cdot \sigma_{bc}}{n \cdot \sigma_{bc} + \sigma_{st}}$$

$\alpha = y_1 / d$ so: $$\alpha = \frac{n\sigma_{bc}}{n\sigma_{bc} + \sigma_{st}}$$

Note: When the Ultimate Limit State (ULS) is reached, the stresses are then equal to their allowable values. $\sigma_{bc} = \overline{\sigma_{bc}}$ et $\sigma_{st} = \overline{\sigma_{st}}$

In this case, we can calculate:

$$\overline{\alpha} = \frac{n\overline{\sigma_{bc}}}{n\overline{\sigma_{bc}} + \overline{\sigma_{st}}}$$

when $M_{rsb} = ½\, b\, y_1\, \sigma_{bc}.Z$

The comparison of this resisting moment with the service moment should enable us to choose between a single reinforcement system or a double reinforcement system.

$M_{ser} \leq M_{rsb}$: double reinforcement : In this case, we can establish : $\alpha = \overline{\alpha}$

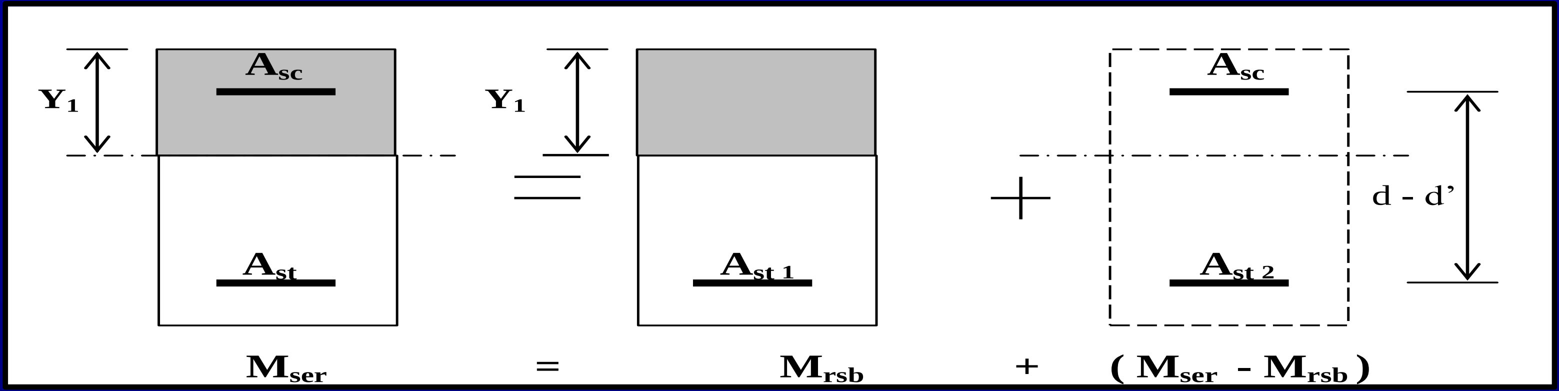
We obtain satisfactory approximate results. $Z = d\,(1 - \overline{\alpha}\,/\,3)$

so : $$A_{ser} = \frac{M_{ser}}{z_1 \cdot \overline{\sigma_{st}}}$$

Note: Ensure compliance with the non-fragility condition. $\mathbf{A_{ser} \geq A_{min}}$

b- Rectangular section with compressed reinforcements:

$M_{ser} > M_{rsb}$: double reinforcement: In this case, we determine a tension steel section Ast1 capable of balancing the resisting moment of the concrete, followed by a tension steel section Ast2 and a compressed steel section Asc capable of balancing the complementary moment to achieve. M_{ser}.



Tension steel section:

$$A_{st1} = \frac{M_{rsb}}{Z \times \bar{\sigma}_{st}}$$

We know:

$$\bar{\alpha} = \frac{n\bar{\sigma}_{bc}}{n\bar{\sigma}_{bc} + \bar{\sigma}_{st}}$$

and

$$y_1 = \alpha . d$$

$$Z = d (1 - \bar{\alpha} / 3)$$

$A_{st\,2}$:balance a moment. **(M_{ser} - M_{rsb})** In this section, the lever arm is **(d – d')**

$$A_{st2} = \frac{M_{ser} - M_{rsb}}{(d-d') \times \bar{\sigma}_{st}}$$

where : $A_{st} = \frac{1}{\bar{\sigma}_{st}} \left[\frac{M_{rsb}}{Z} + \frac{M_{ser} - M_{rsb}}{(d-d')} \right]$

Compressed steel section:

A_{sc} balance a moment **(M_{ser} - M_{rsb})** the lever arm is **(d – d')**

From where : $A_{sc} = \frac{M_{ser} - M_{rsb}}{(d-d')\sigma_{sc}}$

σ_{sc}:is the working stress of the compressed steel section. It depends on the position of the reinforcements within the section.

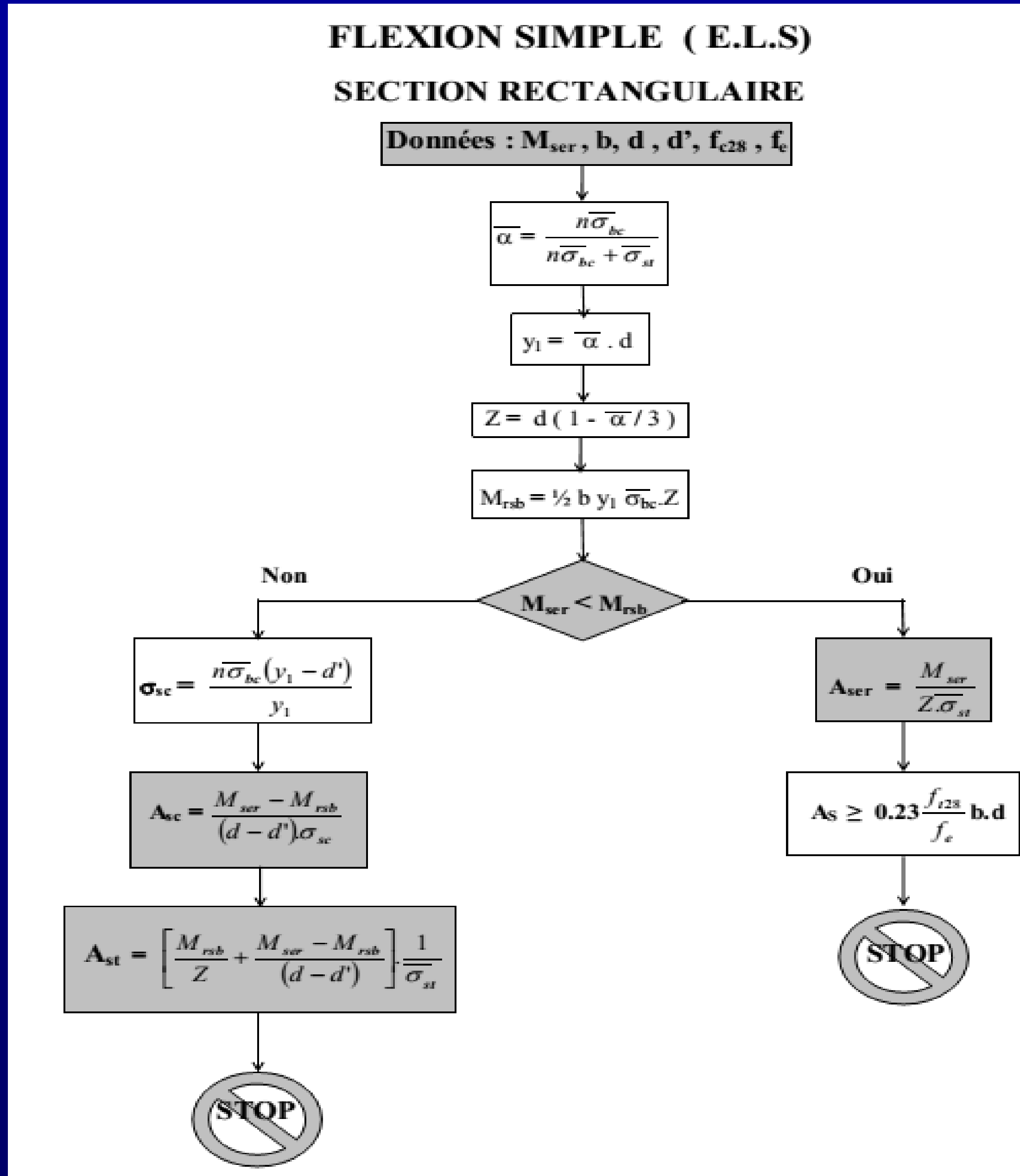
$$\sigma_{sc} = \frac{n\bar{\sigma}_{bc}(y_1 - d')}{y_1}$$

d' : top cover

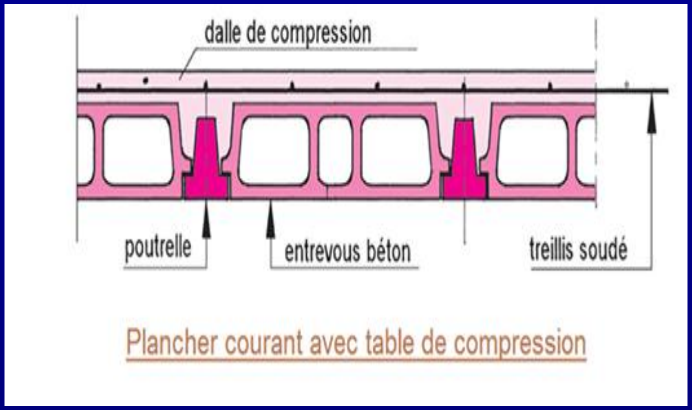
with : $y_1 = \bar{\alpha} \cdot d$

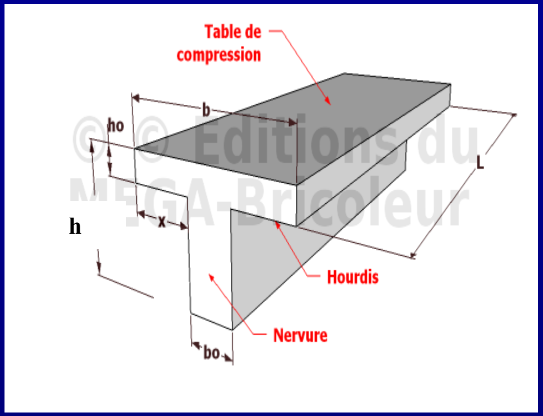
Application :

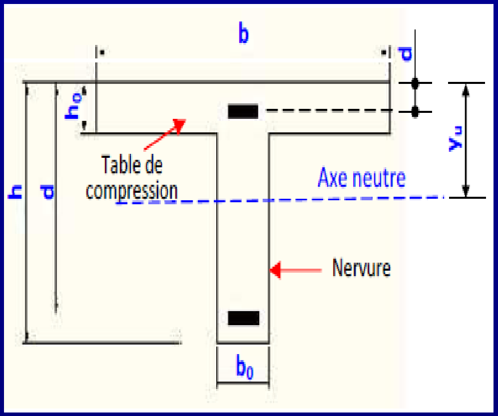
Let's consider a beam with a cross-section of 30x60, having a cover of 5 cm. It is subjected to a moment Mser=0.2 MNm·. The concrete has a characteristic strength fc28=20 Mpa and the steel has a yield strength fe=400 Mpa The cracking is detrimental. Calculate the reinforcements at the S.L.S.


FLEXION SIMPLE (E.L.S)
SECTION RECTANGULAIRE
Données : M_{ser} , b, d , d', f_{c28} , f_e
$\overline{\alpha} = \frac{n\overline{\sigma}_{bc}}{n\overline{\sigma}_{bc} + \overline{\sigma}_{st}}$
$y_1 = \overline{\alpha} . d$
$Z = d (1 - \overline{\alpha} / 3)$
$M_{rsb} = ½ b y_1 \overline{\sigma}_{bc}.Z$
Non
$M_{ser} < M_{rsb}$
Oui
$\sigma_{sc} = \frac{n\overline{\sigma}_{bc}(y_1 - d')}{y_1}$
$A_{sc} = \frac{M_{ser} - M_{rsb}}{(d - d')\sigma_{sc}}$
$A_{st} = \left[\frac{M_{rsb}}{Z} + \frac{M_{ser} - M_{rsb}}{(d - d')}\right]\frac{1}{\overline{\sigma}_{st}}$
STOP
$A_{ser} = \frac{M_{ser}}{Z.\overline{\sigma}_{st}}$
$A_s \geq 0.23\frac{f_{t28}}{f_e}$ b.d
STOP

III. T-section: Reinforced concrete beams in a building or a bridge often support slabs. The BAEL code allows considering that a certain width of the slab is an integral part of the beams. In this case, the cross-section of the beam takes the form of a T.







III.1. Widths of T-Beam Compression Flanges:

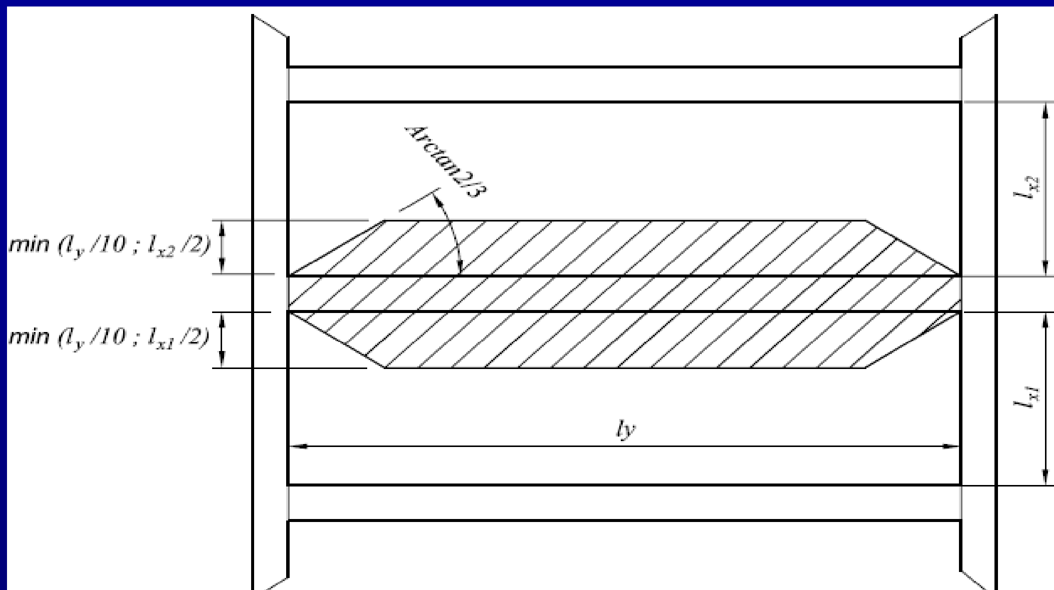
The width of the slab to be considered on each side of a rib, measured from its surface, should not exceed the smaller of the following values

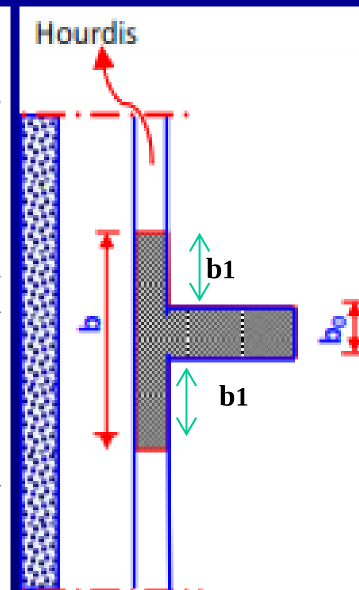
$$b = b_0 + 2b_1$$

With b1 equal to : $b_1 = min\left\{\frac{L_x}{2};\frac{L_y}{10}\right\}$ $b = b_0 + 2b_1$

- **Half** of the distance between the adjacent faces of two consecutive ribs.(Lx/2)

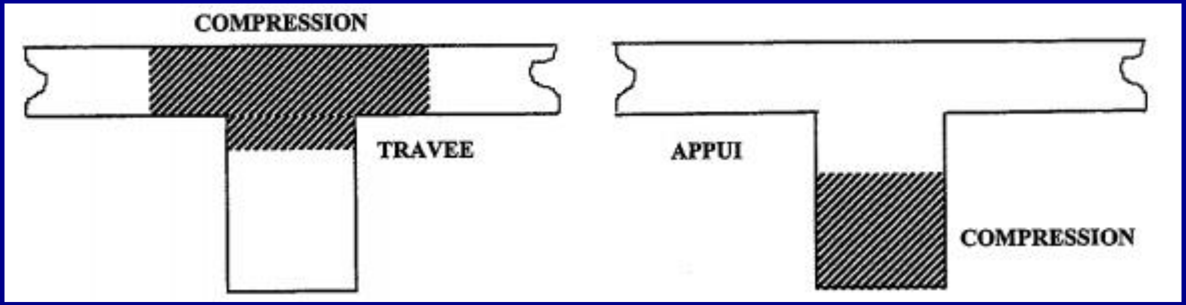
- **1/10** of the span of the span(Ly/10)





In the analysis of a T-section, it is essential to determine whether the compressed part affects only the flange (low stress) or if it also involves the web (high stress). In other words, one must ascertain the position of the neutral axis. To achieve this, the balanced moment Mt, resisted by the flange, will be calculated

"A beam is most commonly associated with a floor; within a span, a portion of this floor bears compressive forces. The beam-floor system is thus capable of supporting loads greater than those borne by the beam alone."



On supports, only the bearing of the beam operates under compression; therefore, the analysis of a T-beam is applicable only within a span."

- Terminology:

III.2. Calculation at Ultimate Limit State (E.L.U):

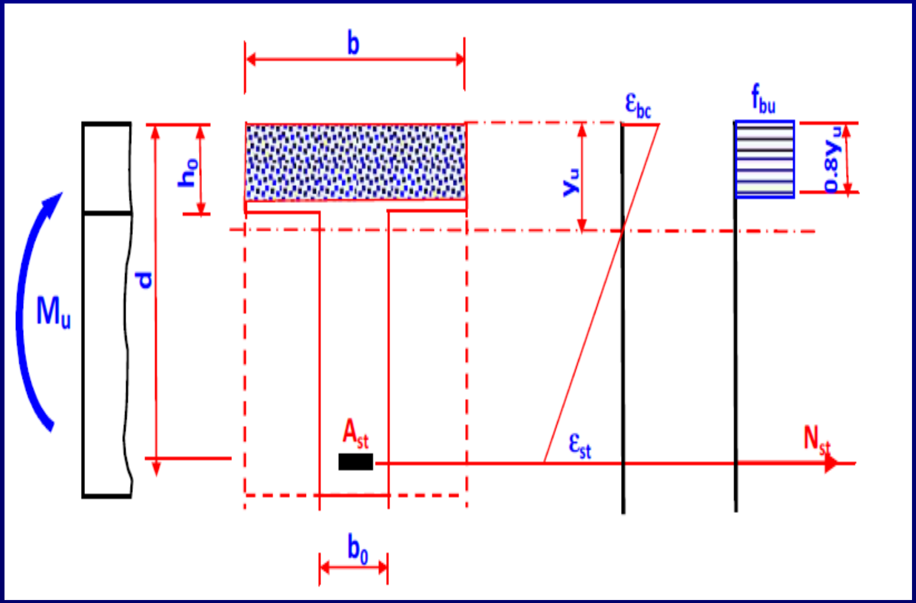
□ **The moment balanced by the table alone, Mt:**

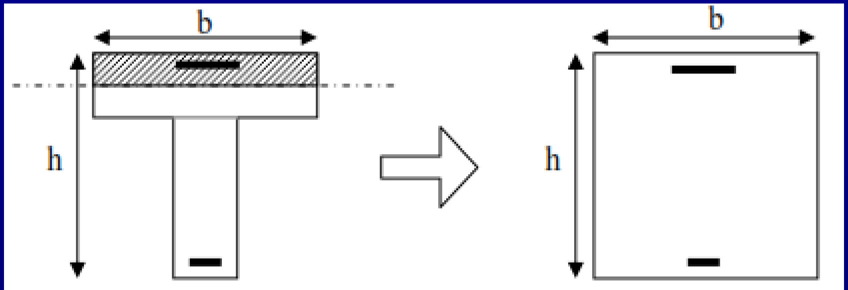
$$M_t = b h_0 f_{bu}(d - \frac{h_0}{2})$$

Two cases may arise:

The first case (neutral axis within the table): Mu ≤ Mt: if $0.8y_u \le h_0$

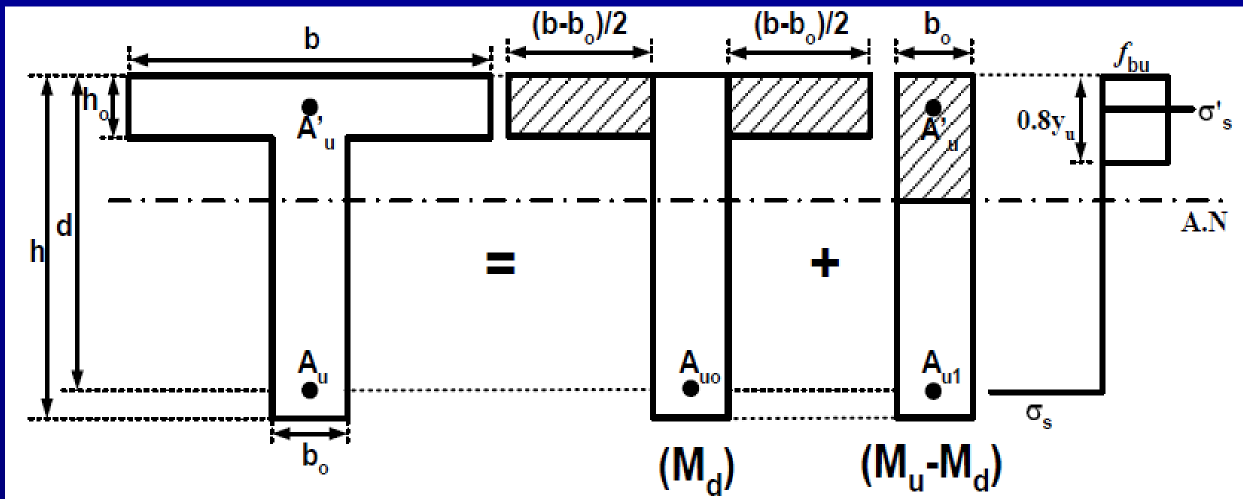
As the tensile concrete does not contribute to the strength calculations, the calculation is conducted as if the section were rectangular with a width 'b' and an effective height 'd'

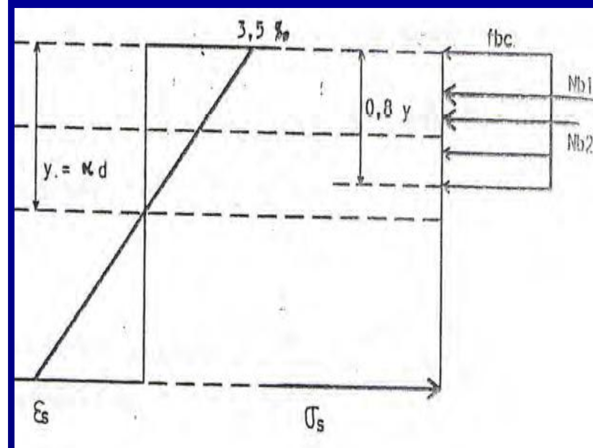




Calculation principle at the Ultimate Limit State (ULS) of a T-section, with : **Mt ≥ Mu**

□ **The second case : $M_u > M_t$** : Compression involves both the flange and a portion of the web, which is the case that truly corresponds to a T-beam. The ultimate moment is then taken up by both the overhangs of the flange and the web of the beam.





The balanced forces are equal to:

(Wings): N_{b1} =(b-b0) h0 fbc ; **(web)** : N_{b1} =0.8αdb0 fbc. ; **As**: N_s =As.σs

- "The moments balanced by the wings are:

$$M_d = (b - b_0)\, h_0\, f_{bu} \left(d - \frac{h_0}{2}\right) = \frac{b - b_0}{b} \,.\, M_t$$

$$A_{uo} = \frac{M_d}{\left(d - \frac{h_0}{2}\right) \sigma_s}$$

- he moments balanced by the web,**bo h of the beam are:** $M_{âme} = M_u - M_d$

we calculate : $\mu_u = \dfrac{M_u - M_d}{b_o d^2 f_{bu}}$

- $\mu_{bu} \leq \mu_l$ → Asc = 0 (section without compressed steel Asc=0)

$A_{u1} = \dfrac{M_u - M_d}{z_u \dfrac{f_e}{\gamma_s}}$ and $z_u = d(1 - 0.4\alpha_u)$ and $\alpha_u = 1.25(1 - \sqrt{1 - 2\mu_u}$

The total steel section is then: $A_u = A_{uo} + A_{u1} = \dfrac{M_d}{(d - \dfrac{h_o}{2})\dfrac{f_e}{\gamma_s}} + \dfrac{M_u - M_d}{d(1 - 0.4\alpha_u)\dfrac{f_e}{\gamma_s}}$

- $\mu_{bu} > \mu_l$ (section with compressed steel Asc ≠ 0)

Si 0.8 yℓ > ho alors: $M_{b\ell} = \mu_\ell b_o d^2 f_{bu}$ The moment taken by the compressed concrete in the web, wi $z_l = d\,(1 - 0{,}4\,.\,\alpha_l)$

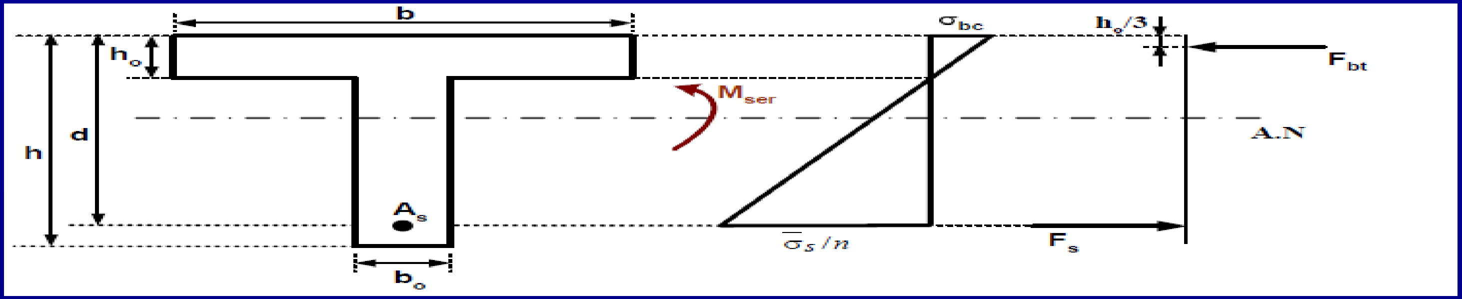
$$A_u = \frac{M_d}{(d - \frac{h_o}{2})\frac{f_e}{\gamma_s}} + \frac{M_{b\ell}}{d(1 - 0.4\alpha_\ell)\frac{f_e}{\gamma_s}} + \frac{M_u - M_d - M_{b\ell}}{(d - d')\frac{f_e}{\gamma_s}}$$

and : $A_{sc} = \dfrac{M_u - M_d - M_{bl}}{(d - d')\sigma_{sc}}$ with : **σsc = f(ξsc)**

- if 0.8 yℓ ≤ ho We reduce it to a rectangular section (b,h) avec Asc ≠ 0

III.3. Verification at Serviceability Limit State (S.L.S): The principle is the same as for the case of a simply rectangular section. The two fundamental unknowns that must be determined for the verification are 'y' and 'I'."

III.3.1. Neutral Axis Position and Inertia: : The static moment with respect to an axis passing through the center of gravity of the section (the neutral axis) is zero.



To determine the location of the neutral axis 'y,' it is necessary to perform an initial calculation arbitrary to determine the sign of: $\frac{by^2}{2} + nA_{sc}(y-d') - nA_{st}(d-y)$ By replacing 'y' with 'h0.

(case 1) if The sign is positive, ($y \leq h_0$): When the neutral axis is within the compression flange, the calculations are identical to those of a rectangular section (the equations remain unchanged)

(case 2):if The sign is négative, ($y > h_0$) : When the neutral axis is within the web (T-section), The equation to determine 'y' becomes $\frac{by^2}{2} - \frac{(b-b_0)(y-h_0)^2}{2} + nA_{sc}(y-d') - nA_{st}(d-y) = 0$

So, the equation for the quadratic moment of inertia will be:

$$I = \frac{by^3}{3} - \frac{(b-b_0)(y-h_0)^3}{3} + nA_{st}(d-y)^2 + nA_{sc}(y-d')^2$$

$\sum \mathbf{M_i}$=Ms (service moment)

We have: $M_b + M_{ASC} + M_{AST} = M_{ser}$

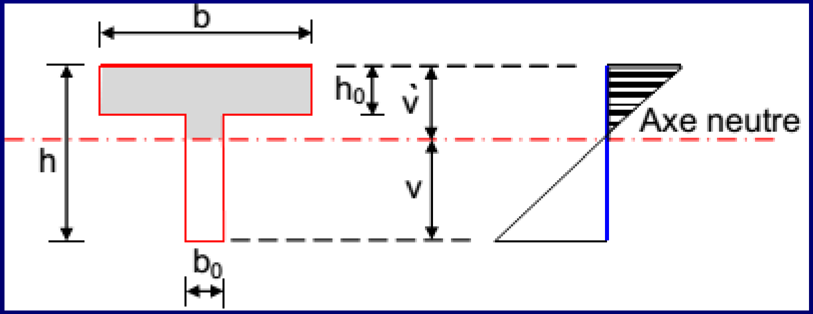
$I = I_b + n \,.\, I_{st} + n \,.\, I_{sc}$

So stress will be : $\sigma_{bc} = \dfrac{M_{ser} \times y}{I_{GZ}}$ and $\sigma_{st} = \dfrac{n.\, M_{ser}.\,(d-y)}{I}$

The checks are: $\left.\begin{array}{l} \sigma_{bc} \prec \overline{\sigma_{bc}} \\ \sigma_{st} \prec \overline{\sigma_{st}} \end{array}\right\} E.L.S$ verified

III.3.2. Non-Brittleness Condition: : The non-brittleness condition leads to placing a minimal section of tension reinforcement for a given formwork dimension."

$$A_{min} = \frac{I}{0{,}81 \,.\, h \,.\, v} \,.\, \frac{f_{t28}}{f_e}$$



$$I_{Gz} = b_0 \frac{h^3}{3} + (b - b_0)\frac{h_0^3}{3} - \left[b_0 h + (b - b_0) h_0\right] v'^2$$

$$v' = \frac{b_0 h^2 + (b - b_0) h_0^2}{2\left[b_0 h + (b - b_0) h_0\right]}$$

$$v = h - v'$$

$$A_{min} = \frac{I_{Gz}}{(d - \frac{h_0}{3}) v} \frac{f_{t28}}{f_e}$$

-**Application :** Consider a T-section subjected to a flexural moment at the Ultimate Limit State (E.U.L)

:M_u = 0,8 MN.m et à l'E.L.S M_{ser} = 0,5 MN.m

if : f_{c28} = 20 MPa ; FeE400 ;

d = 65 cm ; d' = 4 cm.

harmful cracking