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Second Year LMD Mathematics and Applied Mathematics 2025/2026

Subject : *Algebra 4*
Instructor : *Y. Halim*

Series of Exercises No. 1

Exercise 1 :

Let

$$E = \mathbb{R}_2[X] = \left\{a_0 + a_1X + a_2X^2 : a_0, a_1, a_2 \in \mathbb{R}\right\}.$$

Consider :

$$F = \{P \in E : P'(0) = 0\}.$$

1. Justify that F is a hyperplane of E, and deduce its dimension.
2. Give all complements of F in E.

Exercise 2 :

Determine the linear form f defined on $\mathbb{R}^3$ such that

$$f(1,1,1) = 0, \quad f(2,0,1) = 1, \quad f(1,2,3) = 4.$$

Give a basis of $ker(f)$, the kernel of f.

Exercise 3 :

In $\mathbb{R}^4$, consider the vectors :

$$v_1 = (1,1,1,1), \quad v_2 = (0,1,1,1), \quad v_3 = (0,0,1,1), \quad v_4 = (0,0,0,1).$$

1. Show that $B = \{v_1, v_2, v_3, v_4\}$ is a basis of $\mathbb{R}^4$.
2. Determine the dual basis of B.

Exercise 4 : (2023 Quiz)

Let $\varphi_1, \varphi_2, \varphi_3$ be linear forms on $\mathbb{R}_2[X]$ defined by

$$\varphi_1(P) = P(0), \quad \varphi_2(P) = P'(1), \quad \varphi_3(P) = \int_0^1 P(t)dt.$$

1. Show that the family $\{\varphi_1, \varphi_2, \varphi_3\}$ is a basis of the dual of $\mathbb{R}_2[X]$.
2. Determine its antidual basis.

Exercise 5 :

Let $E = \mathbb{R}_2[X]$. Consider the family $\mathcal{F} = \{f_0, f_1, f_2\}$ of elements of E^* defined by, for $j = 0, 1, 2$

$$\forall P \in E, \quad f_j(P) = P(j).$$

1. Show that the family $\mathcal{F}$ is a basis of E^*.
2. Determine the antidual basis B of $\mathcal{F}$.