Databases (BDD) — Chapter 2

The Relational Model

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Outline

Relational Model Foundations

The Relational Model

The **relational model** represents data using a single concept: the **relation**.

In this model:

- Data is stored in tables

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- The model is based on set theory

The relational model is simple, rigorous, and widely used in modern DBMS.

Relation

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- **Attributes** (columns)

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RELATION: STUDENT

StudentID	LastName	FirstName	Age
125	Amara	Leila	19
203	Ghichi	Faiza	20
65	Bey	Sami	20

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- **Attributes** (columns)
- **Tuples** (rows)

RELATION: STUDENT

StudentID	LastName	FirstName	Age
125	Amara	Leila	19
203	Ghichi	Faiza	20
65	Bey	Sami	20

Each relation has an **identifier** (key) that uniquely identifies its tuples.

Example: STUDENT Relation

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125	Amara	Leila	19
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Example tuple: $\langle 203,\ \textit{Ghichi},\ \textit{Faiza},\ 20 \rangle$

Attribute

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In the STUDENT relation, attributes are:

- StudentID

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Each attribute has exactly one value per tuple.

Domain

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Attribute	Domain
StudentID	integer in [0..300]
Name	string (max length 20)
Age	integer in [16..60]

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Domains enforce **domain integrity**.

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In the STUDENT relation:

- StudentID is the identifier
- No two students share the same StudentID

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TAKES

StudentID	CourseName
365	Algo
65	OS
125	Algo
125	OS

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For a many-to-many association, a separate relation is created.

TAKES

StudentID	CourseName
365	Algo
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Identifier: (StudentID, CourseName)

Relation Schema

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Example schema:

$$\textsc{Student}(\mathit{StudentID}{:}\mathit{int},\ \mathit{LastName}{:}\mathit{string},\ \mathit{FirstName}{:}\mathit{string},\ \mathit{Age}{:}\mathit{int})$$

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Example schema:

$$\textsc{Student}(\mathit{StudentID}{:}\mathit{int},\ \mathit{LastName}{:}\mathit{string},\ \mathit{FirstName}{:}\mathit{string},\ \mathit{Age}{:}\mathit{int})$$

The schema is also called the **intension**.

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Schema vs Population

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Schema (structure only)

StudentID	LastName	FirstName

Population (current tuples)

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125	Amara	Leila
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Database Schema

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Attribute Modeling Rule

In the relational model, attributes must be:

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Complex or multivalued attributes must be transformed.

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Option 2: decompose into atomic attributes (recommended)

- STUDENT(StudentID, LastName, Number, Street, City, PostalCode, Country)

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- STUDENT(StudentID, LastName, Number, Street, City, PostalCode, Country)
- Each component becomes an atomic attribute
- Enables queries on City, Country, etc.

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$$\textsc{Student\_FirstName}(\textit{StudentID},\ \textit{FirstName})$$

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INF100	Mohammed
INF100	Redha
INF120	Yasmine

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Optional Attribute (NULL)

Some attributes may have no value for certain tuples.

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StudentID	LastName	FirstName	Age
INF200	Mohammed	Ahmed	NULL

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NULL represents an unknown or missing value.

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- Relations are defined by attributes, domains, and keys
- Schemas describe structure; populations describe data
- Attributes must be atomic and single-valued

Normalization and Integrity

Why Normalization?

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Normalization is the process that produces **non-redundant** and **consistent** relational schemas.

Motivating Example

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Problems observed:

- EmpName and Dept are repeated

Motivating Example

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Example:

$$\text{EMP\_PROJECT}(\textit{EmpID}, \textit{EmpName}, \textit{Project}, \textit{Dept})$$

Problems observed:

- EmpName and Dept are repeated
- Updating a department requires multiple updates

Running Example: EMPLOYEE_PROJECT

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EMPLOYEE_PROJECT

EmpID	EmpName	Project	StartDate	EndDate	Department
E01	Mohamed	Alpha	01/02/24	30/06/24	IT
E01	Mohamed	Beta	15/03/24	31/12/24	IT
E02	Salima	Alpha	01/02/24	30/06/24	Finance
E03	Yacine	Gamma	10/01/24	30/09/24	IT

Running Example: EMPLOYEE_PROJECT

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E01	Mohamed	Beta	15/03/24	31/12/24	IT
E02	Salima	Alpha	01/02/24	30/06/24	Finance
E03	Yacine	Gamma	10/01/24	30/09/24	IT

This relation stores employee data, project data, and assignments together.

Data Redundancy

In the EMPLOYEE_PROJECT relation:

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Example:

- EmpName = Mohamed appears several times

Data Redundancy

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- Employee name and department are repeated for each project
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Example:

- EmpName = Mohamed appears several times
- Department = IT is duplicated

Update Anomaly

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- Forgetting one row leads to inconsistency

Insertion Anomaly

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Example:

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Insertion Anomaly

An **insertion anomaly** occurs when we cannot insert data without other data.

Example:

- A new employee with no project
- Cannot be inserted into EMPLOYEE_PROJECT

Deletion Anomaly

A **deletion anomaly** occurs when deleting data causes loss of other data.

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Example:

- If project Gamma is deleted

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Example:

- If project Gamma is deleted
- Employee Yacine information is lost

Functional Dependency

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Example:

$$EmpID \rightarrow EmpName$$

An employee identifier determines the employee name.

Functional Dependencies: Example

Relation:

$$\text{EMPLOYEE}(\textit{EmpID}, \textit{EmpName}, \textit{Dept}, \textit{DeptPhone})$$

Functional Dependencies: Example

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Valid functional dependencies:

- $\mathit{EmpID} \rightarrow \mathit{EmpName}, \mathit{Dept}$

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Valid functional dependencies:

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- $\mathit{Dept} \rightarrow \mathit{DeptPhone}$

Invalid dependency:

$$\mathit{EmpName} \nrightarrow \mathit{Dept}$$

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Example:

$$Num\_Employe \rightarrow Nom,\ Prenom$$

Armstrong's Axioms

Let X, Y, Z, W be sets of attributes.

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Transitivity:

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$$EmpID \rightarrow Dept \land Dept \rightarrow DeptPhone \Rightarrow EmpID \rightarrow DeptPhone$$

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- **Decomposition:** $X \to YZ \Rightarrow X \to Y$ and $X \to Z$

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- **Composition:** $X \to Y,\ Z \to W \Rightarrow XZ \to YW$

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- **Decomposition:** $X \to YZ \Rightarrow X \to Y$ and $X \to Z$
- **Composition:** $X \to Y,\ Z \to W \Rightarrow XZ \to YW$
- **Pseudo-transitivity:**

$$X \to Y,\ WY \to Z \Rightarrow XW \to Z$$

Inference Rules

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Then:

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Decomposition:

$$EmpID \rightarrow EmpName, Dept \Rightarrow \begin{cases} EmpID \rightarrow EmpName \\ EmpID \rightarrow Dept \end{cases}$$

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An FD is canonical if its right-hand side has a single attribute: $X \rightarrow A$.

Example: $CodeProd \rightarrow (Label, Price)$ is not canonical.

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Example: $(CodeProd, CodeDep) \to CodeProd$

2) Canonical FD

An FD is canonical if its right-hand side has a single attribute: $X \to A$.

Example: $CodeProd \to (Label, Price)$ is not canonical.

By decomposition:

$$CodeProd \to Label \qquad \text{and} \qquad CodeProd \to Price$$

Types of Functional Dependencies

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Example (not elementary):

$$(CodeProd, CodeDep) \to Label$$

is not elementary if

$$CodeProd \to Label$$

holds (because $CodeProd \subset (CodeProd, CodeDep)$).

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So the direct dependencies here are:

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Functional Dependency Graph

A **functional dependency graph** is a directed graph where:

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- Nodes represent attributes

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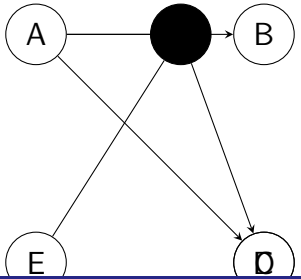

FD Graph and Minimal FD Graph

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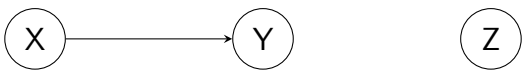

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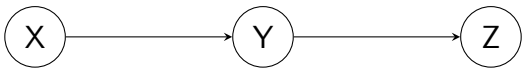

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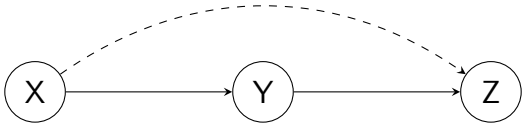

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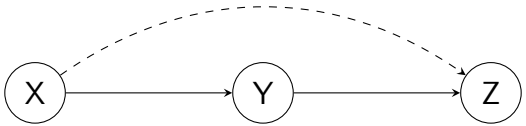

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Transitive Closure of Functional Dependencies

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  - For each $Y \rightarrow Z$ in F
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- Until no change

Example: Attribute Closure

Let:

$$F = \{AB \to C,\ BC \to AD,\ D \to E,\ CG \to B\}$$

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$$[AB]^+ = \{A, B, C, D, E\}$$

Minimal Cover

A set of functional dependencies F is a **minimal cover** if:

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Note: minimal covers are not unique.

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A minimal cover removes all redundancies from F.

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Example:

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Minimal cover:

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Minimal Cover Algorithm

Step 1: Decompose right-hand sides

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Step 1: Decompose right-hand sides
Step 2: Remove redundant dependencies
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The result is a minimal cover equivalent to F.

Minimal Cover — Example 1

Let the relation schema be:

$$R(A, B, C)$$

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$$[\textit{EmpID}]^+ = \{\textit{EmpID}, \textit{EmpName}, \textit{Dept}\} \Rightarrow \textit{EmpID} \text{ is a key}$$

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Primary key chosen:

$$\textit{Registration}$$

From Functional Dependencies to Normal Forms

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Normal forms are based on functional dependencies.

Normal Forms

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- Third Normal Form (3NF)

Goal: remove redundancy and anomalies.

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Heath's Theorem
If:

$$X \to Y$$

then the decomposition:

$$R(X, Y, Z) \Rightarrow R_1(X, Y),\ R_2(X, Z)$$

is lossless.

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A decomposition preserves dependencies if:
All functional dependencies in F can be enforced without performing joins.
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Goal:

- Maintain integrity constraints
- Avoid expensive join operations

First Normal Form (1NF)

A relation is in **First Normal Form** if:

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- All attributes are atomic

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Example (not in 1NF):

$$Phones = \{0551, 0662\}$$

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1NF decomposition:

$$\text{EMP\_PHONE}(EmpID,\ Phone)$$

Second Normal Form (2NF)

A relation is in **Second Normal Form** if:

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A relation is in **Second Normal Form** if:

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Example:

$$(EmpID, Project) \rightarrow StartDate$$

but:

$$EmpID \rightarrow EmpName$$

Third Normal Form (3NF)

A relation is in **Third Normal Form** if:

Third Normal Form (3NF)

A relation is in **Third Normal Form** if:

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Example:

$$EmpID \rightarrow Dept \rightarrow DeptPhone$$

Normalization Process (Bernstein Algorithm)

The Bernstein algorithm produces a normalized schema in 3NF.

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- Lossless-join decomposition in 3NF

Bernstein Algorithm — Steps

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4. Ensure that at least one relation contains a key of R
5. Remove redundant relations if any

Bernstein Algorithm — Example

Given:

$$R(A, B, C, D) \quad \text{and} \quad F = \{A \to B,\ B \to C,\ C \to D\}$$

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Generated relations:

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This decomposition is lossless and dependency-preserving.

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1NF solution:

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Integrity Constraints

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Age $\in [18, 60]$
Entity integrity:
EmpID cannot be NULL
Referential integrity:
EmpID in ASSIGNMENT must exist in EMPLOYEE

From the Relational Model to SQL

Wrap-up

Questions

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