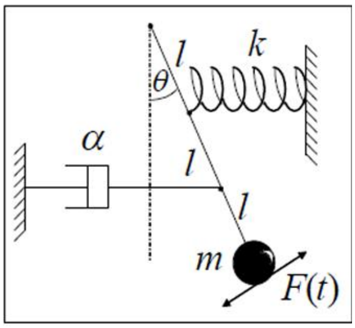
University Center of Mila

Institute of Science and Technology

Department Process Engineering

Exercise series 3: Forced vibration with one degree of freedom

Exercise 1:



In the system shown, the sphere is a point mass, and the rod has a total length of 3l and negligible mass. With $F(t) = F_0 \cos(\omega_e t)$.

1- Find the kinetic energy T, the potential energy U, and the dissipation function D ($\theta << 1$).

2- Find the Lagrangian and then the equation of motion.

3- Using the complex representation, find the steady-state solution of the equation of motion.

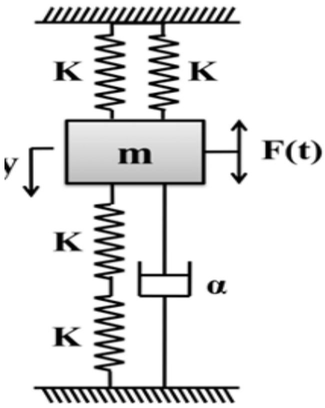
(Specify its amplitude A and its phase ϕ).

4- Deduce the resonance frequency ω_r.

5- Give the cutoff frequencies ω_{e1}, ω_{e2}, and the bandwidth B for low damping: $\delta << \omega_0$.

6- Calculate ω_r, B, and the quality factor if m=1 Kg, k=15 N/m, l=0.5 m, α=0.5 N.s/m, g=10 m.s^{-2}.

Exercise 2



A mechanical system consists of mass m=0.5 kg and a shock absorber with a friction coefficient α = 2 kg/s connected to springs with the same stiffness constant K=20N/m. The system is subjected to an external excitation of motion $F(t) = F_0 \cos(\omega e t)$.

1-Calculate the equivalent stiffness constant Keq.

2- Find the kinetic energy T, the potential energy U, and the dissipation function D.

3- Find the Lagrangian then the equation of motion.

4- Find its solution in steady state (Specify its amplitude A and its phase φ).

5- Give the resonance condition and the resonance pulsation ω_r.

6- Give the bandwidth B for low damping: $\delta \ll \omega 0$

Exercise 3: A mechanical system consists of a rod of length 2L and negligible mass that can rotate around an axis passing through O in the plane of the figure. A point mass m is welded to the end of the rod. At both ends of this rod. Also attached are a spring of stiffness K and a damper with a viscous friction coefficient α. The system is subjected to an external force $F_{ext} = F_0 \sin(\omega e t)$. When the rod is moved away from its equilibrium position, the system performs oscillations of low amplitudes

. 1- Determine the kinetic energy T, the potential energy U and the dissipation function D.

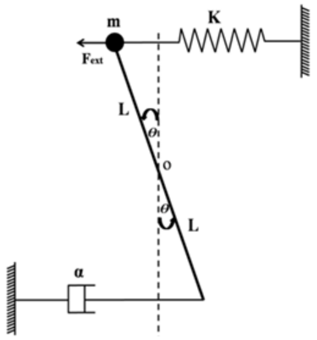
2- Calculate the moment of force $\mathcal{M}F$.

3- Establish the differential equation of motion at θ, determine the constants δ, ω_0 and A_0.

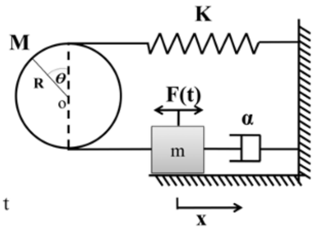
4- Find the permanent solution of the equation of motion (Specify its amplitude A and its phase φ).

5- Write the amplitude resonance condition and give the resonance pulsation ωR.

6- For $\delta = 0$, what happens?



***Exercise 4*:** In the system opposite, a homogeneous disk of mass M and radius R can rotate freely with an angle θ around its fixed axis. The mass m on the horizontal plane is connected to a damper of coefficient α and to the disk by an inextensible and non-slip wire. At equilibrium, the spring was not deformed. A sinusoidal excitation F(t) = F0cos(ωet) is applied to the mass m. The moment of inertia of the disk around its axis is: $J/0 = ½\ MR^2$.



1- Find the kinetic energy T, the potential energy U, and the dissipation function D as a function of the variable x.

2- Find the Lagrangian and deduce the equation of motion.

3- Give its solution in steady state by specifying the amplitude A and the phase φ.

4- Deduce the resonance pulsation ωR and give the quality factor Q of the weakly damped system.

5- Graphically represent the variation of the amplitude A as a function of ωe.