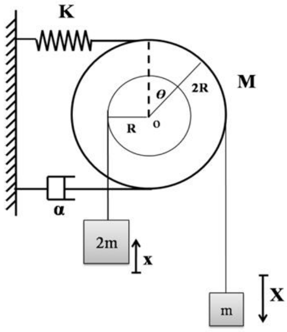
University Center of Mila

Institute of Science and Technology

Department Process Engineering

**Exercise series 3: Free damped vibration with one degree of freedom**

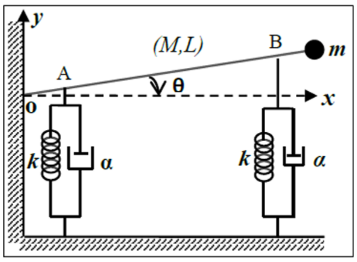
## Exercise 1:




A homogeneous disk of mass M and radius 2R is connected at its rim to a spring with stiffness K and a damper with damping coefficient α. A mass 2m is suspended from a string wound around the rim of the disk, and another mass m is suspended from a string wound around a groove of radius R cut into the surface of the disk. The strings are assumed to be inextensible and non-slipping.
The disk can rotate freely about its fixed axis.The moment of inertia of the disk about its axis is: $J/0 = ½ MR^2$.
Given: α = 8 N·s/m, K = 2 N/m, M = 2m = 1 kg, m = 0.5 kg

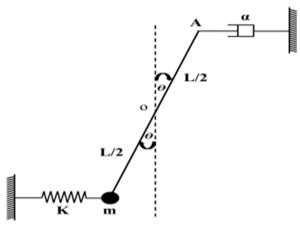
1- Find the kinetic energy T, the potential energy U, and the dissipation function ED for $\theta \ll 1$. (At equilibrium, the spring was not deformed).
2- Establish the differential equation of motion.
3- Deduce the natural angular frequency ω0 and the damping coefficient θ.
4- Determine the nature of the motion.
5- What is the maximum value of θ that must not be exceeded to cause oscillation?

## Exercise 2




A system consists of a rod of length L and negligible mass, which can rotate in a vertical plane around its fixed axis O.
Point A is connected to a fixed frame by a damper with a viscous friction coefficient α.
At the other end of the rod, a point mass m is attached and connected to a second fixed frame by a spring of stiffness K.
We consider the case of free oscillations of small amplitude.
1.What is the number of degrees of freedom of the studied system? Justify your answer.
2.Calculate the kinetic energy (T), the potential energy (U), and the dissipation function (ED) as functions of the variable θ.
3. Establish the differential equation of motion in the regime of small (weak) damping, and find its solution.
4. After 15 periods, the amplitude of the motion decreases by 25% of its initial value.
Calculate the logarithmic decrement D.
5. Deduce the number of periods for which the total energy decreases by the same percentage.

**Exercise 3**: Given the vibrating mechanical system shown in the adjacent figure.

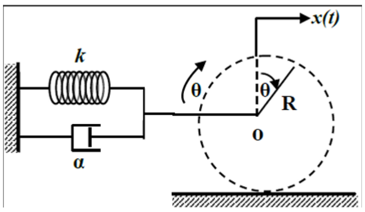



Let G be the center of gravity of a bar of mass M and length L.
1)Determine the differential equation of motion, and deduce the expressions for $\omega_0$ (the natural angular frequency) and δ (the damping coefficient).
2)Write the equation of motion in the case where $\delta < \omega_0$. Given data: $J_G = \frac{1}{12}ML^2, OA = L_1, OB = L_2$.

***Exercise 4:*** Consider a mechanical system composed of a disk (mass M, radius R) that can roll without slipping on a horizontal plane, connected to a spring of stiffness k and a damper with viscous friction coefficient α.

1)Determine the differential equation of motion in terms of δ (the damping coefficient) and $\omega_0$ (the natural angular frequency), and deduce $\omega_0$.

2)Find the solution of the equation of motion when $\delta = \omega_0$. $J/o = \frac{1}{2} MR^2$




***Exercise 5:***

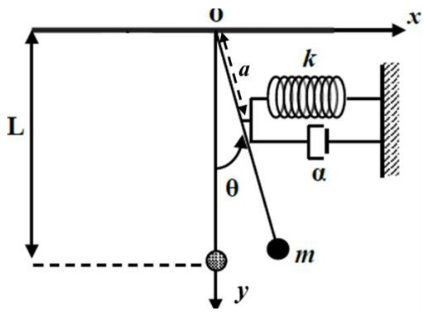
Consider a mass m fixed at the end of a rod of negligible mass and length L. The rod performs small-amplitude oscillations around a fixed axis passing through point O and perpendicular to the plane of motion.

1)Establish the differential equation of motion.

2)Determine the natural angular frequency of the system.

3)Find the equation of motion.

4)Write the solution of the equation in the case $\delta < \omega_0$.




***Exercise 6:***

In the opposite system, the bar of mass m and length 3l can rotate around the axis passing through O. We symbolize all the friction by a viscous friction damper with coefficient α. At equilibrium, the rod is horizontal. We move the rod away from the vertical by a sufficiently small angle to admit that sin θ = θ

1. Find the differential equation of motion.
2. What is the value that the coefficient of friction α must not exceed to have an oscillatory motion?

Calculate this value if m = 1 kg and k = 1 N/m.

3. Assuming that takes the value calculated in the previous question. What is the nature of the motion? Give the time equation of this motion θ(t) knowing that initially, θ (0) = 5° and θ (0) = 0°/s.