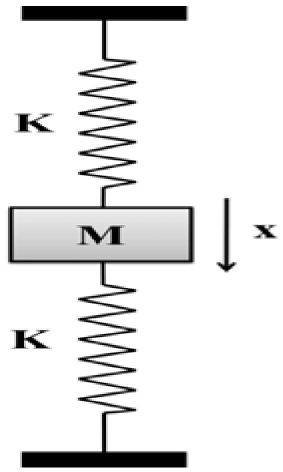
University Center of Mila

Institute of Science and Technology

Department Process Engineering

Exercise series 2: Free undamped vibration with one degree of freedom

Exercise 1:



Consider a system modeled by a mass M and two springs of stiffness K. (See figure opposite).

1- What is the type of system?

2- Find the kinetic energy T and the potential energy U.

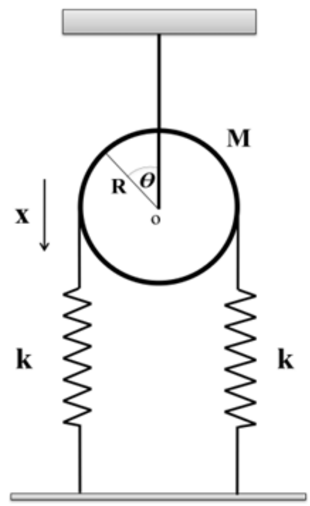
3- Establish the differential equation of motion as a function of x.

4- Give the final solution if: $x(0) = 1\ cm$; $\dot{x}(0) = 0$.

5- Calculate the total energy.

Numerical values: M=0.5 kg, K=100 N/m.

Exercise 2:



Let a pulley (hollow cylinder) of mass M and radius R rotate freely around its fixed axis. The pulley is suspended at its center by an inextensible rope from a fixed frame. On either side of the pulley, two springs of the same stiffness k are fixed to its periphery. The other end of the two springs is connected to the ground. The moment of inertia Jcylinder/O = MR^2 is given.

1- What is the type of the system?

2- Find the kinetic energy T and the potential energy U.

3- Determine the differential equation of motion as a function of θ.

4- Find the final solution by taking as initial conditions: $\theta(0) = 0$;$\dot{\theta}(0) = 5\omega_0$.

5- Calculate the total energy

Numerical values: M = 1 kg, K = 50 N/m, R = 0.2 m.

Exercise 3: Consider the mechanical system shown in the figure opposite. The rod of negligible mass and length 2L. The rod performs small amplitude oscillations around a fixed axis passing through point O. The middle of the rod (point b) is connected to a frame by a spring of stiffness K, and its free end carries a mass M.

1- What is the number of degrees of freedom of the system under study?

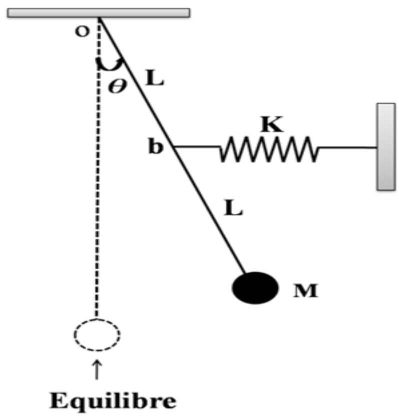
2- Find the kinetic energy T and the potential energy U of the system (for the variable θ).

3- Deduce the differential equation of motion of this system, using the variable θ, and specify its natural frequency f_0.

4- The solution to the differential equation is of the form: $\theta(t) = A\sin(\omega_0 t + \varphi)$.

Determine the amplitude A and the initial phase φ if at time $t = 0$

$$\theta(0) = \frac{\pi}{20}\ ;\ \dot{\theta}(0) = 0$$



Exercise 4: A mechanical system consists of a bar of mass M and length L oscillating around joint O in the plane of the figure. At the end of this bar, a spring of stiffness constant K is placed. At equilibrium, the bar is vertical and the spring of stiffness K is at rest.

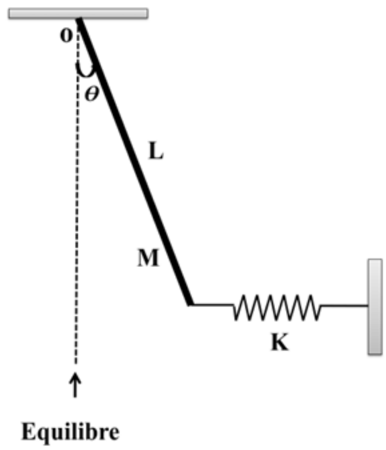
1- Give the moment of inertia of the system (using Huygens' Theorem)

2- Give the expression for the kinetic energy T and the potential energy U of the system in the case of weak oscillations (for variable θ).

3- Determine the differential equation of motion as a function of θ.

4- Find the final solution using the following initial conditions:

$$\theta(0) = \frac{\pi}{22}\ ;\ \dot{\theta}(0) = 0.$$

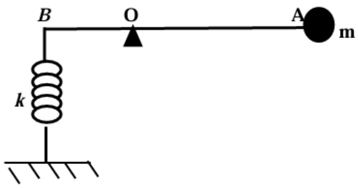


Exercise 5: A mass m is attached to the end of a bar of negligible mass resting at O'(AO $= \frac{2}{3}$ l)

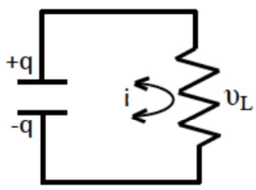
whose other end is attached to a spring of constant stiffness k

(OB= $\frac{1}{3}$ l)

Calculate the period of the small oscillations of m around its equilibrium position.



Exercise 6: Analyze the L-C circuit (without resistance) by deriving the differential equation for the movement of electric charges. Determine the oscillation frequency and the equivalent spring constant k, drawing parallels between the mechanical and electrical circuits.



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