Tutorial Worksheet No.2

Exercise 1.

1. Write in algebraic form the following :

$$z_1 = 2(1+5i)(1+2i)^2 \qquad z_2 = 3i\overline{(5i-3)}(11+\frac{2}{i}) \qquad z_3 = \frac{(4+2i)^3}{1+i}$$

$$z_4 = \left(\overline{\frac{2+3i}{1+2i}}\right) \qquad\qquad z_5 = \overline{\left(\frac{2+3i}{1+2i}\right)^2}$$

2. Simplify the following :

   (a) $\dfrac{1+i}{1-i} - (1+2i)(2+2i) + \dfrac{3-i}{1+i}$

   (b) $2i(i-1) + \left(\overline{\sqrt{3}+i}\right)^3 + (1+i)(\overline{1+2i})$

3. Find $z \in \mathbb{C}$ such that $z\overline{z} = i(z-1)+1$

4. Find $z \in \mathbb{C}$ such that $z^2 - i \in \mathbb{R}$

Exercise 2.

Let z and w be a two complex numbers. Prove the following :

1. $|1+iz| = |1-iz| \Rightarrow z \in \mathbb{R}$

2. $w\overline{z} + \overline{w}z = 2Re(w\overline{z})$

3. $\overline{z} = -z \Rightarrow Re(z) = 0$

4. $z\overline{z} = z + 3\overline{z} \Rightarrow z$ is real.

Exercise 3.

1. Write in trigonometric (polar) form the following :

$$z_1 = -1 + \sqrt{3}i \qquad z_2 = 6i \qquad z_3 = -\frac{1}{2} + \frac{i}{2}$$

$$z_4 = \frac{3-i}{2+i} \qquad z_5 = \frac{2+i}{3-2i} \qquad z_6 = \frac{-1}{2+2i}$$

2. Write in algebraic form $z = (3+3i)^8$

3. Calculate $Im((i+1)^8 z^2)$ for $z = x+iy$.

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Exercise 4.

Write $1 + i$ in trigonometric form and then :

1. Calculate $(1 + i)^{24}$

2. Show that $(1 + i)^2$ is purely imaginary

3. Let $z = (1 + i)^{4n}$. Prove that if n is even, then $z = 4^n$ and if n is odd, $z = -4^n$.

Exercise 5.	Exercise 6.
Give the square roots of the following complex numbers : $z_1 = i \qquad z_2 = 2 \qquad z_3 = 2 - 3i \qquad z_4 = 3 + 2i \qquad z_4 = -\frac{1}{2} - \frac{3}{5}i$	Prove the following : $e^{i\theta} + e^{2i\theta} = 2\cos(\frac{\theta}{2})e^{i\frac{3}{2}\theta}$ $\dfrac{\tan(\varphi) - i}{\tan(\varphi) + i} = e^{i(\pi + 2\varphi)}$

Exercise 7.

Let $z \in \mathbb{C}$. Solve the following equations :
$$z^2 = 1 + i$$
$$z^4 = 1 + i$$
$$z^2 + 2z = -2$$
$$z^2 - \sqrt{3}z - i = 0$$
$$z^2 - (1 + 2i)z + i - 1 = 0$$
$$z^4 + 10z^2 + 169 = 0$$
$$iz^2 + (1 - 5i)z + 6i - 2 = 0$$

Exercise 8.

1. Solve for $z \in \mathbb{C}$ the following :
$$z^5 = 1$$
$$z^3 = 2 + i$$
$$z^5 = 1 - i$$
$$z^5 = \bar{z}$$
$$z^{2n} - z^n + 1 - i = 0$$

2. Calculate the nth roots of i and $1 + i$

Home work

1. Calculate $(1 - i\sqrt{3})^{21}$

2. Determine the modulus and the argument of $z = \dfrac{1}{1 + i\tan(\varphi)}$

3. Solve the following :
   (a) $z^6 - (1 - i)z^3 - i - \frac{1}{2} = 0$
   (b) $(z + 1)^n = (z - 1)^n$
   (c) $z^n = \bar{z}$, where $n \geq 1$

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