Tutorial Worksheet No.1

Exercise 1.

a) Let A and B be two sets in a universal set U such that
$A = \{1, 2, 3\} \qquad B = \{1, 2, 4\}$

- 1-a) Find the following :

$$A \cup B, \quad A \cap B, \quad A - B, \quad A \times B, \quad P(A),$$
$$A \cap \emptyset, \quad card(P(A)), \quad card(A \times B), \quad (A \cup B) - (A \cap B)$$

- 1-b) Represent graphically $A \times B$

b) Let E and F be two sets defined by

$$E = \{\{c\}, \{c, e\}\}, \quad F = \{\{a\}, \{a, b\}\}$$

- Prove that $(E = F) \Rightarrow (c = a) \wedge (e = b)$.

Exercise 2.

Let A and B be a two subset in a set U

1. Determine the following sets $X = (A \cap B) \cup (C_U^A \cap B)$, $Y = (C_U^A \cup C_U^B) \cap (C_U^A \cup B)$
2. Show the following equivalence : $A \subset B \Leftrightarrow A \cap B = A \Leftrightarrow A \cup B = B$
3. Prove that $A - B = A - (A \cap B) = (A \cup B) - B$.

Exercise 3.

Let S be a relation on $\mathbb{R}$defined by

$$\forall a, b \in \mathbb{R} \mid ab = 2a - 1$$

1. Is the pair $(1, 1)$ an element of S ?
2. Is S reflexive for $a = 1.$?
3. What is the condition in the element (a, b) to satisfy the symmetric property of S ?
4. What is the condition on the elements a, b and c to satisfy the transitive property of S ?

Exercise 4.

Let $a \geq 0$ and let $\Re$ be a relation from E to F on $I = [0, +\infty)$ defined by

$$\forall (x,y) \in E \times F \mid \frac{ae^{x-2}}{y} - \frac{e^y}{x} = 0$$

1. Determine a such that the relation $\Re$ is reflexive.
2. By replacing the found value of a in $\Re$. Is $\Re$ an equivalence relation ?

Exercise 5.

Let $\mathcal{R}$be a relation on $\mathbb{R}$ defined by : $\forall (x,y) \in \mathbb{R}^2,\ xRy \Rightarrow cos^2(x) + sin^2(y) = 1$

1. Show that the relation $\mathcal{R}$ is an equivalence relation.
2. Find the class of equivalence $[x]$.

Home work exercise.

Let $\mathcal{S}$ be a relation defined on $\mathbb{N}$ by : $\forall k_1,\ k_2 \in \mathbb{N},\ k_1 \mathcal{S} k_2 \Rightarrow k_1\, divise\, k_2$

- Prove that $\mathcal{S}$ is a partial order relation on $\mathbb{N}$.

Exercise 6.

Let E and F be two subsets on $\mathbb{R}$ where $E = \{-1, 1, 2, 3\}$, and let f be a function defined by

$$f:\ E \longrightarrow F$$
$$x \longmapsto f(x) = x^2 + 2$$

1. Determine F and deduce $f(1),\ f(-1)$ and $f^{-1}(6)$. Is f injective ?
2. Determine the image of 3 and the pre-image of 5.
3. Is there an image of -2 under f.

Exercise 7

Let f be a function defined by

$$f:\ \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto f(x) = x^2 + 2$$

1. Is f injective ? Surjective ?
2. Specify the case in which the function f is bijective.
3. Find $f^{-1}([2, 6])$.

Exercise 8

Let f and g be two functions defined by

$$f: \mathbb{R}^{-\{\frac{1}{2}\}} \longrightarrow \mathbb{R}^{-\{2\}} \qquad g: \mathbb{R}^{-\{2\}} \longrightarrow \mathbb{R}^{-\{1\}}$$
$$x \longmapsto f(x) = 2x+1 \qquad x \longmapsto g(x) = \frac{x}{x-2}$$

1. Determine $g \circ f$
2. Show that the function $g \circ f$ is bijective.
3. Determine $(g \circ f)^{-1}$

Exercise 9

Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined by $g(x) = x^2 - 4x + 4$.

1. Check that $\forall a \in \mathbb{R}, g(2+a) = g(2-a)$. Deduce that $g(x)$ is not injective.
2. Show that $\forall x \in \mathbb{R}, g(x) \geq 0$. Is $g(x)$ surjective ?