Abd El Hafid Boussouf University – Mila
Institute of Science and Technology
Department of Process Engineering

January 2026

Heat Transfer Final Exam - Correction

Exercise 1: (13 marks)

1.1. heat flux transferred from the steam to the plate bottom surface:

Using Newton's law of cooling: $q'' = h_{steam}.(T_{\infty,2} - T_1) = 30.(100 - 80) = \mathbf{600\ W/m^2}$ 1

1.2. Deduce the rate of heat transfer entering the plate:

We have: $q'' = \frac{q}{A} \rightarrow q = q''.A \rightarrow q = 600 \times 0{,}2 = \mathbf{120\ W}$ 1

2.1 Write the governing equation for steady one-dimensional heat conduction in the plate:

The heat conduction equation, for constant k, is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \dot{q} = \frac{\rho c_p}{k} . \frac{\partial T}{\partial t}$$

Then, for steady-state, one-dimensional conduction without heat generation:

$$\frac{\partial^2 T}{\partial x^2} = 0$$

1

2.2 Determine the temperature gradient ($\left.\frac{dT}{dy}\right|_{inside}$) inside the plate :

Inside the solid plate, we have: conduction heat transfer,

Then, from Fourier's law: $q'' = -k_{plate}.\frac{dT}{dy} \rightarrow \frac{dT}{dy} = -\frac{q''}{k_{plate}} \rightarrow \frac{dT}{dy} = -\frac{600}{237} \rightarrow \frac{dT}{dy} = \mathbf{-2{,}53\ K/cm}$ 1

Means that: **temperature decreases upward through the plate**.

2.3 Calculate the top surface temperature of the plate :

We have: $\left|\frac{dT}{dy}\right| = \frac{\Delta T}{thickness} \rightarrow \Delta T = T_2 - T_1 = \left|\frac{dT}{dy}\right|.thickness \rightarrow T_2 = \left|\frac{dT}{dy}\right|.thickness - T_1$

$\rightarrow T_2 = (-2{,}53) \times 0{,}01 + 80 \rightarrow \mathbf{T_2 = 79{,}97\ °C}$ 1

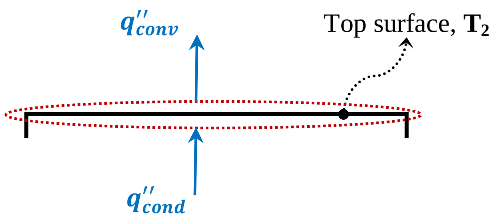
Due to the __very high thermal conductivity__ of aluminum, the __temperature drop across the plate is negligible__.

3.1 The energy balance at the top surface of the plate :

The energy balance is: $E_{in} + E_g = E_{st} + E_{out}$

At steady state, no generation of heat: $E_{st} = 0$ and $E_g = 0$

Then, $E_{in} = E_{out} \rightarrow q''_{cond,\ plate} = q''_{conv,\ air}$ 1



3.2 Determine the air-side convection coefficient (h_{air}) :

We have: $q''_{cond,\ plate} = q''_{conv,\ air} \rightarrow q''_{cond,\ plate} = h_{air}.(T_2 - T_{\infty,1})$

$h_{air} = \frac{q''_{cond,\ plate}}{(T_2 - T_{\infty,1})} \rightarrow h_{air} = \frac{600}{(79{,}97 - 25)} \rightarrow \mathbf{h_{air} = 10{,}92\ \frac{W}{m^2.K}}$ 1

3.3 Calculate the temperature gradient ($\left.\frac{dT}{dy}\right|_{top\ surface}$) in the air at the top surface of the plate :

Using Fourier's law in the air adjacent to the top surface (because, flow air considered with no motion):

$$q'' = -k_{air}.\left.\frac{dT}{dy}\right|_{top\ surface} \rightarrow \left.\frac{dT}{dy}\right|_{top\ surface} = -\frac{q''}{k_{air}} \rightarrow \left.\frac{dT}{dy}\right|_{top\ surface} = -\frac{600}{0{,}0243}$$

$$\rightarrow \left.\frac{dT}{dy}\right|_{top\ surface} = -2{,}47 \times 10^3\ K/m$$

(1)

4.1 Calculate the Reynolds number of the air flow :

$$Re = \frac{\rho.U.L}{\mu} \rightarrow Re = \frac{1{,}2\times4\times0{,}5}{1{,}8\times10^{-5}} \rightarrow Re \approx 1{,}33 \times 10^5$$

(1)

4.2 The flow regime:

Since: $Re \approx 1{,}33 \times 10^5 < 5 \times 10^5$

So: the flow regime is: **laminar**.

(0,5)

4.3 The Nusselt number :

For laminar flow over a flat plate: $Nu = \frac{h.L}{k} = 0.664 Re_L^{0.5} Pr^{\frac{1}{3}}$

$$\rightarrow Nu = \frac{h.L}{k} = 0.664 \times (1{,}33 \times 10^5) \times (0{,}71)^{\frac{1}{3}} \rightarrow Nu \approx 213 \rightarrow h_{air} \approx 10{,}35\ \frac{W}{m^2.K}$$

Comparison: The theoretical value is **significantly close to** the value obtained in Question 3.

(1,5)

5.1 The modified energy balance at the top surface including radiation :

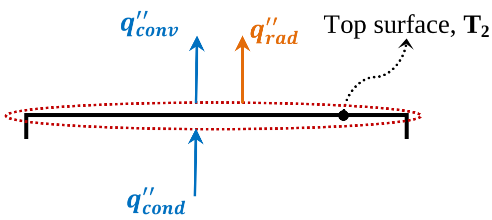
Including radiation: The energy balance is: $E_{in} + E_g = E_{st} + E_{out}$

At steady state, no generation of heat: $E_{st} = 0$ and $E_g = 0$

Then, $E_{in} = E_{out} \rightarrow q''_{cond,\ plate} = q''_{conv,\ air} + q''_{rad}$

So, $q''_{cond} = h_{air}.(T_2 - T_{\infty,1}) + \varepsilon.\sigma.(T_2^4 - T_{\infty,1}^4)$

(1)



5.2 Discuss the influence of radiation on the surface temperature:

→ Radiation increases total heat loss.

(1)

→ Plate surface temperature decreases.

→ Effect becomes significant at higher temperatures.

Exercise 2: (7 marks)

Assumptions:

→ Steady-state conditions.

→ One-dimensional radial heat conduction.

→ Constant thermal conductivity.

→ No internal heat generation.

(2)

For a cylindrical wall under the given assumptions, the governing equation is given by:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

(1)

General solution after two integrations: $T(r) = C_1 \ln(r) + C_2$

(1)

Apply boundary conditions: $T(r_1) = T_1$ and $T(r_2) = T_2$

Substituting into the general solution:

$$\begin{cases} T_1 = C_1 \ln(r_1) + C_2 \\ T_2 = C_1 \ln(r_2) + C_2 \end{cases}$$

This gives that: $\begin{cases} C_1 = \frac{T_2 - T_1}{\ln(\frac{r_2}{r_1})} \\ C_2 = T_1 - C_1 \ln(r_1) \end{cases}$ 1

Then, temperature distribution is: $T(r) = T_1 + \frac{T_2 - T_1}{\ln(\frac{r_2}{r_1})} \ln(\frac{r}{r_1})$ 1

Substitute numerical values:

$r_1 = 0{,}15\ m;\ r_2 = 0{,}20\ m;\ r = 0{,}175\ m;\ T_1 = 60\ °C;\ T_2 = 200\ °C$

$T(0{,}175) = 60 + \frac{200 - 60}{0{,}288} \times 0{,}154 \ \rightarrow\ T(0{,}175) \approx 135\ °C$ 1