....

Exercise 1 – Very Simple Refraction Velocities

In a seismic refraction survey, the following **first arrival times** are measured:

Distance x (m)	Time t (ms)
0	0
100	60
200	120

Assume that these arrivals are **direct waves in the first layer** (no refraction yet).

Tasks

1. Compute the **seismic velocity of the first layer**.
2. If another arrival appears later with an apparent velocity of **3000 m/s**, which one is faster: direct wave or refracted wave?

Solution

1. Use the formula $v = \frac{\Delta x}{\Delta t}$.

Take 0 → 200 m:

- Δx = 200 m
- Δt = 120 ms = 0.120 s

$$v_1 = \frac{200}{0.120} = \frac{200}{0.12}$$

Compute carefully:

- 200 / 0.12
  - → 2000 / 1.2
  - → $\approx 1666.7\,\mathrm{m/s}$

So, $v_1 \approx$ **1670 m/s**

2. The refracted wave has apparent velocity **3000 m/s**, which is greater than 1670 m/s.

✔ The **refracted wave is faster** than the direct wave.

Exercise 2 – Critical Angle

A weathered layer has P-wave velocity $v_1 = 1500\,\mathrm{m/s}$.
The underlying bedrock has velocity $v_2 = 3000\,\mathrm{m/s}$.

Tasks

1. Is refraction possible (i.e., is $v_2 > v_1$)?
2. Compute the **critical angle** i_c (in degrees).

Solution 2

1. Yes, refraction is possible if $v_2 > v_1$.

   Here, 3000 > 1500 → ✔ Yes, possible.
2. Critical angle formula:

$$\sin i_c = \frac{v_1}{v_2} = \frac{1500}{3000} = 0.5$$

$$i_c = \arcsin(0.5) = 30°$$

✔ **Critical angle = 30°**

Exercise 3 — Seismic Refraction Travel-Time Diagram

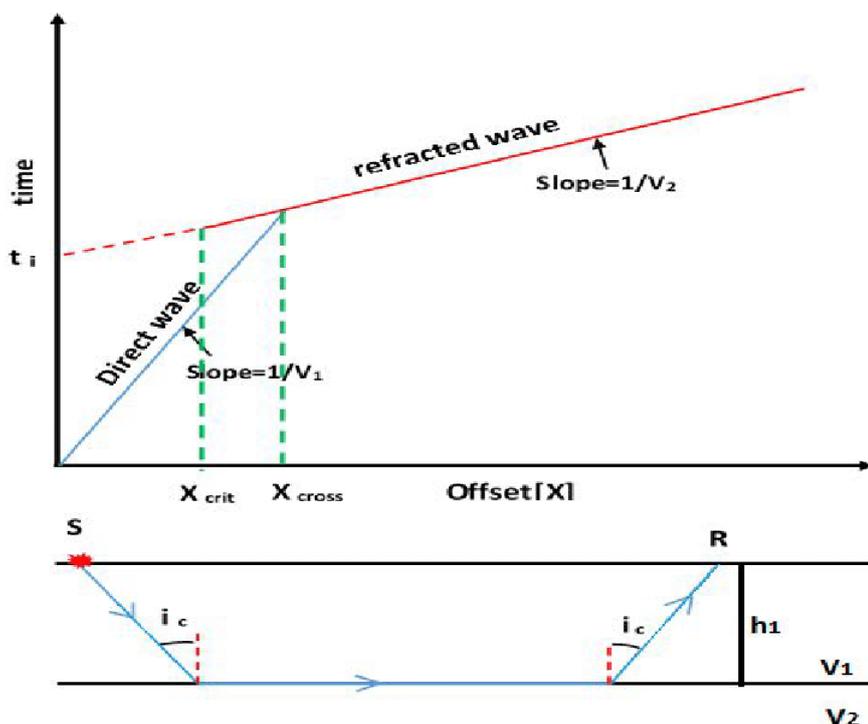A refraction survey shows the travel-time curve above.

Measured values:

- Slope of first segment → $V_1 = 400 \ m/s$
- Slope of second segment → $V_2 = 1800 \ m/s$
- Intercept time → $t_i = 0.12 \ s$

Question:

Compute the depth to the refractor using:

$$t_i = \frac{2h \cos i_c}{V_1} \qquad \text{and} \qquad \sin i_c = \frac{V_1}{V_2}$$

Solution 4

✔️ **Corrected Answer:**

Step 1 — Critical angle:

$$\sin i_c = \frac{400}{1800} = 0.2222$$

$$i_c = 12.84°$$

$$\cos i_c = 0.974$$

Step 2 — Solve for depth:

$$h = \frac{t_i V_1}{2 \cos i_c}$$

$$h = \frac{0.12 \times 400}{2 \times 0.974}$$

$$h = \frac{48}{1.948} \approx 24.6 \; m$$

$$\boxed{h \approx 25 \; m}$$

Exercise 5   Head Wave Travel Time

Given:

- $v_1 - 1500 \; \text{m/s}$
- $v_2 - 3000 \; \text{m/s}$
- $h - 100 \; \text{m}$
- Offset $x = 2000 \; \text{m}$

Step 1: Critical angle

$$\sin i_c - \frac{1500}{3000} - 0.5 \rightarrow i_c - 30°$$

Step 2: Travel-time formula

$$t - \frac{2h \cos i_c}{v_1} + \frac{x}{v_2}$$

$$\cos 30° - 0.866$$

Vertical legs:

$$t_1 - \frac{2 \times 100 \times 0.866}{1500} - \frac{173.2}{1500} \approx 0.115 \; s$$

Along refractor:

$$t_2 - \frac{2000}{3000} - 0.667 \; s$$

Total:

$$t \approx 0.115 + 0.667 - 0.782 \; s$$

Answer

Head-wave arrival time ≈ 0.78 s

Exercise 6 — Reflection Travel-Time Curve

A seismic reflection event has a **zero-offset time** $T_0=1.2$ s and **velocity** $V=2200$ m/s

Question:

1. Write the hyperbolic travel-time equation.
2. Compute travel time at offset x=800 m.

✔️ **Corrected Answer:**

1. Equation:

$$T(x) = \sqrt{T_0^2 + \frac{x^2}{V^2}}$$

2. Plug in values:
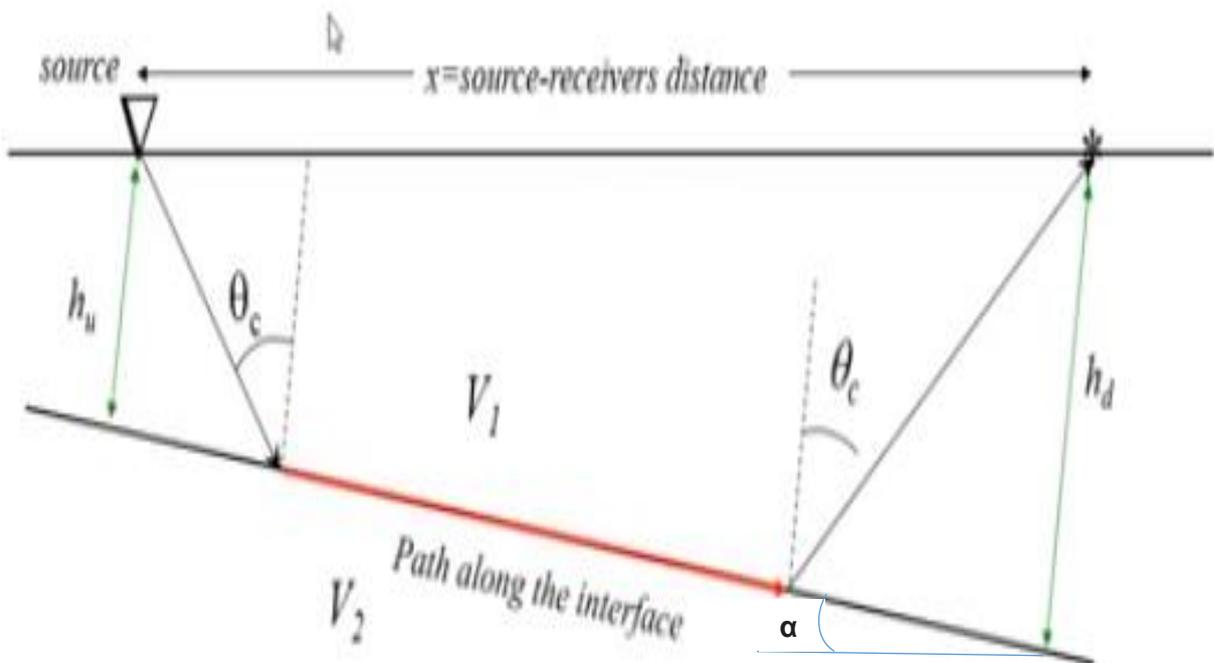
$$T(800) = \sqrt{1.2^2 + \frac{800^2}{2200^2}}$$

$$\frac{800^2}{2200^2} = 0.1323$$

$$T = \sqrt{1.44 + 0.1323} = \sqrt{1.5723}$$

$$T = 1.25\ s$$

$$\boxed{T(800) = 1.25\ s}$$

Exercise 7: Find dip, velocities & depths from travel-time data

Exercise 7: Find dip, velocities & depths from travel-time data

A seismic refraction survey is performed over a **dipping two-layer system.**
From the **first-arrival travel-time curves**, you obtain:

- Direct wave velocity (surface layer):

$$V_1 = 1800 \text{ m/s}$$

- Apparent refractor velocity **up-dip:**

$$V_u = 5000 \text{ m/s}$$

- Apparent refractor velocity **down-dip:**

$$V_d = 3500 \text{ m/s}$$

- Intercept times for the refracted branches:

$$t_{0u} = 0.06 \text{ s}, \quad t_{0d} = 0.10 \text{ s}$$

Assume a simple 2-layer model with a planar dipping interface and $V_2 > V_1$.

Required:

1. Critical angle i_c
2. Dip angle α
3. True refractor velocity V_2
4. Depth to the refractor beneath the **up-dip** and **down-dip** shot points (h_u, h_d)
5. Say which shot is located over the **shallow** end a.⤓ which over the **deeper** end of the refractor.

✅ Step 1 – Use apparent velocities to find i_c and α

For an inclined refractor:

$$V_u = \frac{V_1}{\sin(i_c - \alpha)}, \quad V_d = \frac{V_1}{\sin(i_c + \alpha)}$$

Rearrange:

$$i_c - \alpha = \sin^{-1}\left(\frac{V_1}{V_u}\right), \quad i_c + \alpha = \sin^{-1}\left(\frac{V_1}{V_d}\right)$$

Compute the two angles.

1.1 Compute $i_c - \alpha$

$$\frac{V_1}{V_u} = \frac{1800}{5000} = 0.36$$

$$i_c - \alpha = \sin^{-1}(0.36) \approx 21.1°$$

1.2 Compute $i_c + \alpha$

$$\frac{V_1}{V_d} = \frac{1800}{3500} \approx 0.5143$$

$$i_c + \alpha = \sin^{-1}(0.5143) \approx 31.0°$$

SEISMIC PROSPECTING METHODS EXERCISES --------- UNIV MILA 2025-2026

....

Now solve the system:

$$\begin{cases} i_c - \alpha \approx 21.1° \\ i_c + \alpha \approx 31.0° \end{cases}$$

Add them:

$$2i_c \approx 52.1° \quad \Rightarrow \quad i_c \approx 26.05°$$

Subtract them:

$$2\alpha \approx 9.9° \quad \Rightarrow \quad \alpha \approx 4.95°$$

Answers (1 & 2):

- Critical angle:

$$i_c \approx 26°$$

- Dip angle:

$$\alpha \approx 5°$$

✅ Step 2 – Compute the true refractor velocity V_2

From Snell's law at critical angle:

$$\sin i_c = \frac{V_1}{V_2} \quad \Rightarrow \quad V_2 = \frac{V_1}{\sin i_c}$$

Compute:

- $\sin 26° \approx 0.438$

So:

$$V_2 \approx \frac{1800}{0.438} \approx 4100 \text{ m/s}$$

Answer (3):

$$V_2 \approx 4.1 \text{ km/s}$$

✅ Step 3 – Depths from intercept times

For a 2-layer model (even with a dip), the intercept-time method gives:

$$h_u = \frac{V_1 t_{0u}}{2\cos i_c}, \quad h_d = \frac{V_1 t_{0d}}{2\cos i_c}$$

First compute $\cos i_c$:

- $\cos 26° \approx 0.899$

3.1 Depth beneath up-dip shot

$$h_u = \frac{1800 \times 0.06}{2 \times 0.899} = \frac{108}{1.798} \approx 60.1 \text{ m}$$

3.2 Depth beneath down-dip shot

$$h_d = \frac{1800 \times 0.10}{2 \times 0.899} = \frac{180}{1.798} \approx 100.1 \text{ m}$$

Answer (4):

- $h_u \approx 60$ m
- $h_d \approx 100$ m