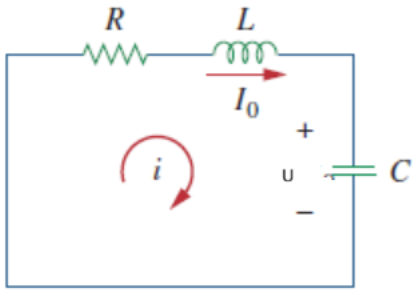
LW3: LRC SERIES FREE OSCILLATIONS

1. OBJECTIVES:

Observe and study the electrical oscillations in an RLC circuit.

2. THEORY:

A series RLC circuit is a linear system consisting of a resistor (R), an inductor (L), and a capacitor (C) connected in series, as shown in the accompanying figure. If the capacitor is initially charged by an electrical generator, the charge will oscillate over time between the inductor and the capacitor in the absence of any external voltage; the resulting oscillation is known as free oscillation. If, instead, additional voltage is supplied to the circuit by a generator during the oscillation, the process is referred to as forced oscillation. In this laboratory experiment, we will focus exclusively on free oscillations.



According to second Kirchhoff's law, in a closed circuit, the sum of the voltage drops across all current-consuming components equals the sum of the voltages supplied by the voltage sources. In our case, the voltage source is zero.

$$\boxed{u_L + u_R + u_C = 0} \qquad (1)$$

Where,

$$\begin{cases} u_R = R \cdot i \\ \quad and \\ u_L = L \cdot \dfrac{di}{dt}, \end{cases} \rightarrow \boxed{R \cdot i + L \cdot \frac{di}{dt} + u_C = 0}$$

We have $i = \frac{dq}{dt}$, $q = C \cdot u_c$

$$\boxed{\ddot{u_c} + \frac{R}{L}\dot{u_c} + \frac{1}{L \cdot C}u_c = 0} \qquad (2)$$

3. Operation regimes

3. 1. Harmonic regime (Natural beat)

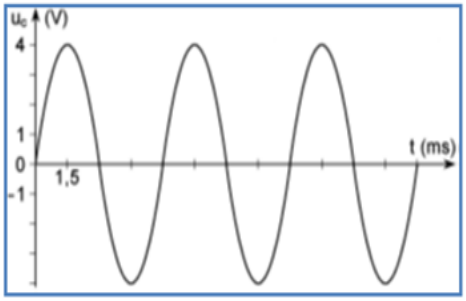
For an RLC circuit with R=0, the regime is free and undamped; it is also referred to as aperiodic or harmonic regime. Equation (2) will write by,

$$\ddot{u_c} + \frac{1}{L \cdot C}u_c = 0$$

$$\omega_n^2 = \frac{1}{L \cdot C} \rightarrow w_n = \sqrt{\frac{1}{L \cdot C}} : \textit{is the natural beat (the resonant beat)}$$

$$\ddot{u_c} + \omega_0^2 \cdot u_c = 0$$

$$U_C(t) = U_{0C}\cos(w_n t + \varphi),$$
$$(if\ at\ t = 0, \varphi = 0)$$
$$\boxed{U_C(t) = U_{0C}\cos(w_n t)} \quad (3)$$
$$w_n = \frac{2\pi}{T_n} \rightarrow T_n = 2\pi \cdot \sqrt{L \cdot C}$$



3. 2. Non harmonic regimes

When R is non-zero, the harmonic regime disappears and gives way to other oscillation regimes.

The solution of equation 333 can be rewritten as

$$\ddot{u_c} + 2\alpha\dot{u_c} + \omega_n^2 u_c = 0$$

$\alpha = \frac{R}{2L}$ is called damping factor

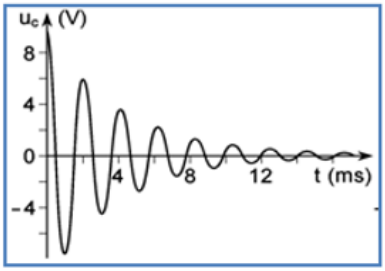
The characteristic equation is: $\lambda^2 + 2\alpha\lambda + \omega_0^2 = 0$

The solutions of characteristic equation are: $\lambda_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

The solution to differential equation (2) depends on the nature of $\lambda_{1,2}$ and leads to different regimes.

3. 2. 1. Underdamped Regime (Pseudo-periodic) ($\alpha < \omega_0$)

This regime occurs when $\alpha < \omega_0$, where the characteristic equation yields complex solutions of the form $s_{1,2} = -\alpha \pm i\omega_d$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ is called damping beat, The solution to differential equation (5) is given by



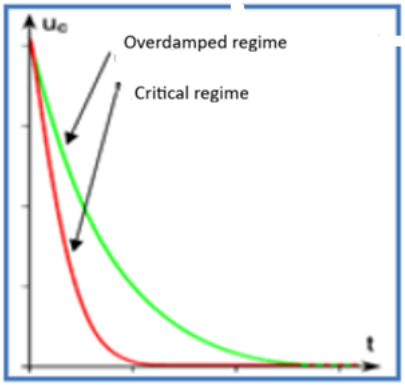
$$u_c(t) = e^{-\alpha t}[A_1\cos(\omega_d t) + A_2\sin(\omega_d t)],\ (if\ at\ t = 0, \varphi = 0)$$
$$u_c(t) = u_{0c}e^{-\alpha t}(\cos(\omega_d t)) \quad (4)$$

3. 2. 2. Critically Damped Regime ($\alpha = \omega_0$)

When $R = 2\sqrt{\frac{L}{C}}$, the solutions of characteristic equation have the form

$s_{1,2} = -\alpha$

The solution to differential equation (5) is given by



$$x(t) = (A_1 + A_2 t)e^{-\alpha t},$$
$$(if\ at\ t = 0, u_c = u_{0c}, and\ \dot{u}_c = 0) \rightarrow$$
$$u_c(t) = u_{0c}(1 + \alpha t)e^{-\alpha t} \quad (5)$$

3. 2. 3. Overdamped Regime (Aperiodic)

This regime occurs when $\alpha > \omega_0$, where the characteristic equation yields real solutions of the formss_1 *and* s_2, The solution to differential equation (5) is given by

$$\boxed{u_C(t) = \frac{u_{0c}}{s_2 - s_1}(s_2 e^{s_1 t} - s_1 e^{s_2 t})} \quad (6)$$

Where: $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

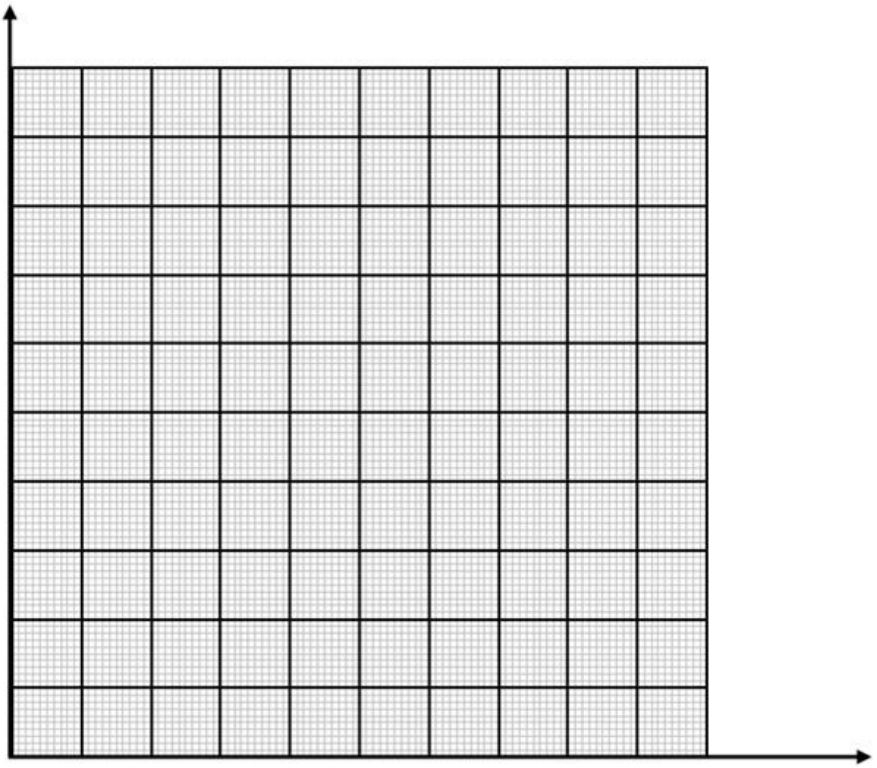
Full name	*Group*	*…… / 20*

4. 1. The influence of *C* on voltage u_C (t):

a. Complete the following table:

R=0,					
C× 10^{-9}(F)	**100**	**500**	**1000**	**1500**	**2000**
$T(s)$					
$T^2(s^2)$					

b. Plot the graph $T^2 = f(C)$



d. Determine graphically the Value of Inductance (**L(mH)**)

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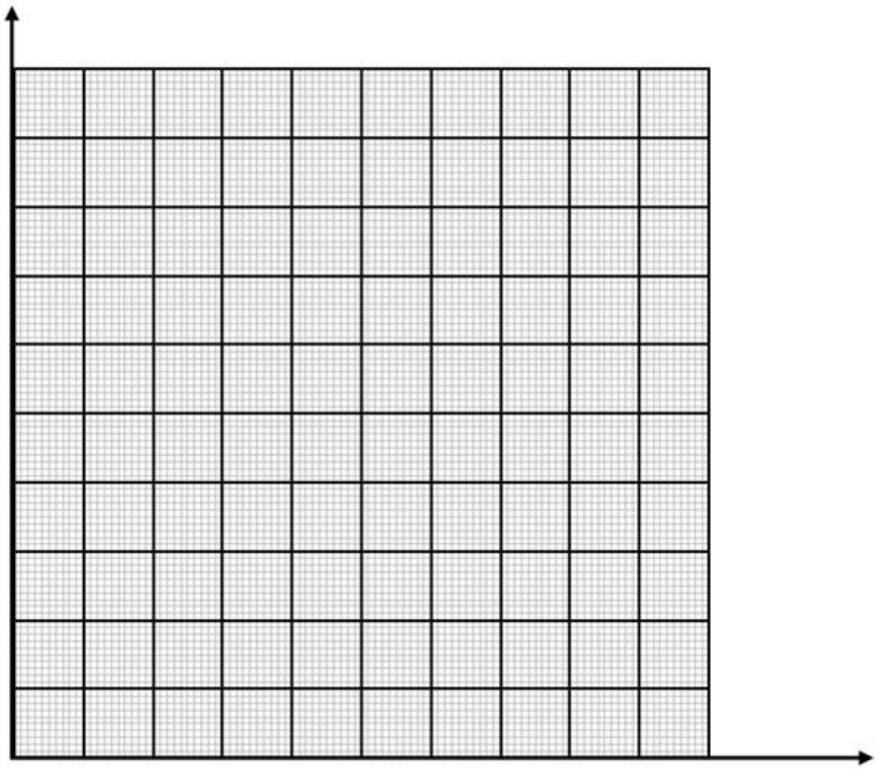
4. 2. The influence of R on voltage u_C(t) :

1. Complete the following tables:

$\mathbf{C = 100 \times 10^{-9}(F)}$, $\mathbf{L = 300 \times 10^{-3}(H)}$, $\mathbf{R = 100(\Omega)}$ **find the frequency** ***f*****=**……					
Amplitude (V)					
Time(s)					
$\mathbf{C = 100 \times 10^{-9}(F)}$, $\mathbf{L = 300 \times 10^{-3}(H)}$, $\mathbf{R = 300(\Omega)}$ **find the frequency** ***f*****=**……					
Amplitude (V)					
Time(s)					

Plot on same figure $A = f(t)$ for the two values of R.



4. 3. Critical regime:

4. 3. 1. For the previous case ($\mathbf{C = 100 \times 10^{-9}(F)}$, $\mathbf{L = 300 \times 10^{-3}(H)}$), Find analytically $\boldsymbol{R_c}$ which gives the critical regime.

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4. 3. 2. For the critical value $\boldsymbol{R_c}$, determine the time at which $\boldsymbol{u_c}$ becomes zero.

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4. 3. 3. Choose a value of R greater than $\boldsymbol{R_c}$ and determine the time at which $\boldsymbol{u_c}$ becomes zero.

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Conclusion:

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