Centre Universitaire Abdelhafid Bousouf - Mila

2024-2025 Semestre 1

# Distribution et collecte des eaux :

# A PARTIE II : Assainissement

## – Cours 8 –

## Chapitre 13 : Mathematical models usable in urban sanitation.






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| Sciences et Technologie | GC et hydraulique | 2eme année master | Hydraulique urbaine |

## Objectifs du cours 1

The objective of this chapter is to present the main mathematical models used to analyze, size, simulate and manage urban sanitation networks.

Modeling makes it possible to represent the hydrological and hydraulic processes involved in urban water collection, transport and disposal systems.

At the end of this chapter, the student should be able to:

- Understanding the fundamental principles of mathematical modeling applied to sanitation;
- Distinguish between hydrological, hydraulic and water quality models;
- Identify the main data needed to build a model;
- Understanding the fundamental equations describing flows;
- Interpreting the results of a hydraulic simulation;
- Understanding the principles of calibration and validation;
- Identify the main applications of mathematical models in urban networks.

### XIII.1. Introduction

Urban sanitation networks are complex hydraulic systems whose operation depends on many parameters, including pipe geometry, slopes, wastewater flow rates, rainfall, land use, boundary conditions and the operation of ancillary structures.

Traditional analytical methods allow for numerous sizing calculations. However, when networks become large or exhibit complex operating conditions, these approaches may be insufficient. Mathematical modeling then allows for a simplified reproduction of the hydrological and hydraulic behavior of the system and the study of different operating situations **(Akan & Houghtalen, 2003; Rossman, 2015).**

A mathematical model is a simplified representation of a real system using equations, physical relationships, and parameters. In the field of sanitation, it allows, in particular, the representation of the transformation of precipitation into runoff, the propagation of flows in pipes, water storage, and the operation of hydraulic structures.

Modeling can thus be used to study network operation in dry or wet weather, overload phenomena, overflows, storage capacities, the operation of control structures, and the consequences of different development strategies. **(Chow, 1959; Butler & Davies, 2011).** (See Figure XIII.01).

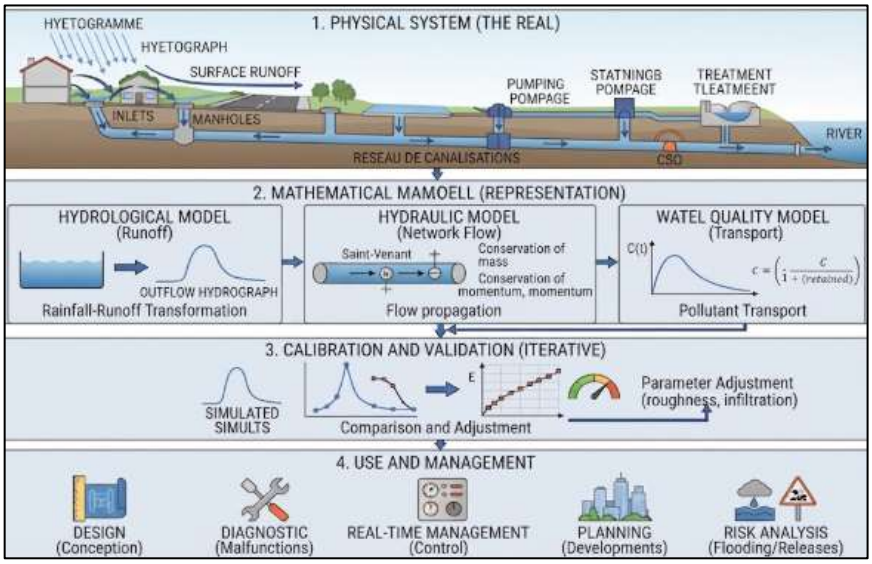



**Figure XIII.01. General principle of mathematical modeling of an urban sanitation network (Rossman, 2015).**

## XIII.2. General Principles of Modeling

Mathematical modeling relies on representing the real system as a set of interconnected elements and equations describing their behavior. A sanitation model typically includes input data, a physical representation of the network, mathematical equations, parameters, initial and boundary conditions, and simulation results. (See Table XIII.01).

The input data may consist of the geometric characteristics of the network, rainfall, wastewater flow rates, catchment characteristics, and operating conditions of the structures. The quality of the results depends heavily on the quality and representativeness of this data (See Figure XIII.02). **(Akan & Houghtalen, 2003; WMO, 2009).**

**Table XIII.01 – Main components of a mathematical sanitation model**

| Component | Examples |
|---|---|
| **Geometric data** | Diameters, lengths, slopes, altitudes |
| **Hydrological data** | Rainfall, hyetograms, IDF curves |

| | |
|---|---|
| **Hydraulic data** | Flow rates, water levels, boundary conditions |
| **Settings** | Roughness, infiltration, runoff |
| **Works** | Basins, weirs, pumps, valves |
| **Results** | Flow rates, levels, velocities, overflow volumes |

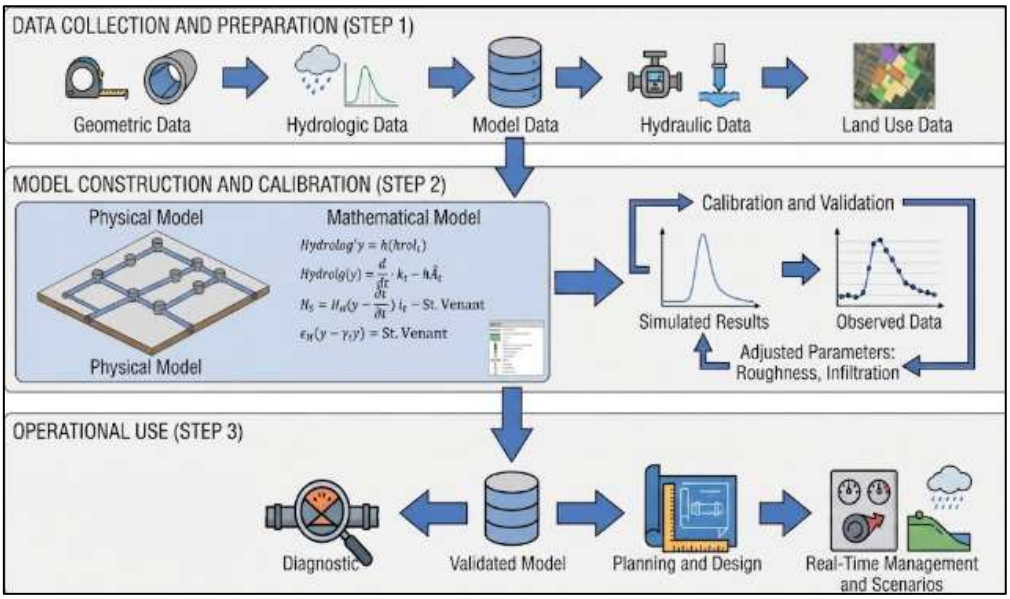



**Figure XIII.02 – General chain of construction and use of a sanitation model.**

## XIII.3. Classification of models used in sanitation

The mathematical models used in sanitation can be grouped into several categories according to the phenomena they represent. The main categories are hydrological models, hydraulic models, and water quality models. (See Table XIII.02).

Integrated models can also combine several of these approaches to represent the overall functioning of an urban system **(Butler & Davies, 2011; Schütze et al., 2004).**

**Table XIII.02. Classification of the main models used in sanitation**

| Model type | Phenomenon represented | Main applications |
|---|---|---|
| **Hydrological** | Rainfall-runoff transformation | Urban runoff |
| **Hydraulic** | Flow propagation | Behaviors, perspectives, works |

| | | |
|---|---|---|
| **Water quality** | Transport and transformation of pollutants | Pollution from discharges |
| **1D** | Flow along an axis | Pipe networks |
| **2D** | Flow over a surface | Urban flooding |
| **1D–2D Coupled** | Network-surface interaction | Floods and overflows |

### XIII.3.1. Hydrological Models

Hydrological models primarily describe the transformation of precipitation into runoff. They allow, in particular, the determination of runoff volume, peak flow, and hydrograph generated by an urban catchment **(Akan & Houghtalen, 2003; WMO, 2009).**

Commonly used approaches include the rational method, conceptual rainfall-runoff models, reservoir models, and distributed or semi-distributed hydrological models (Akan & Houghtalen, 2003). (See Figure XIII.03).

The rational method is one of the most widely used simplified approaches for estimating peak flow in small urban watersheds. It can be expressed by the following relationship:

$$Q_p = C \cdot i \cdot A \qquad \textbf{XIII.01}$$

Where: ( $Q_p$) represents the peak flow, (C) the runoff coefficient, (i) the rainfall intensity and (A) the catchment area **(Akan & Houghtalen, 2003).**

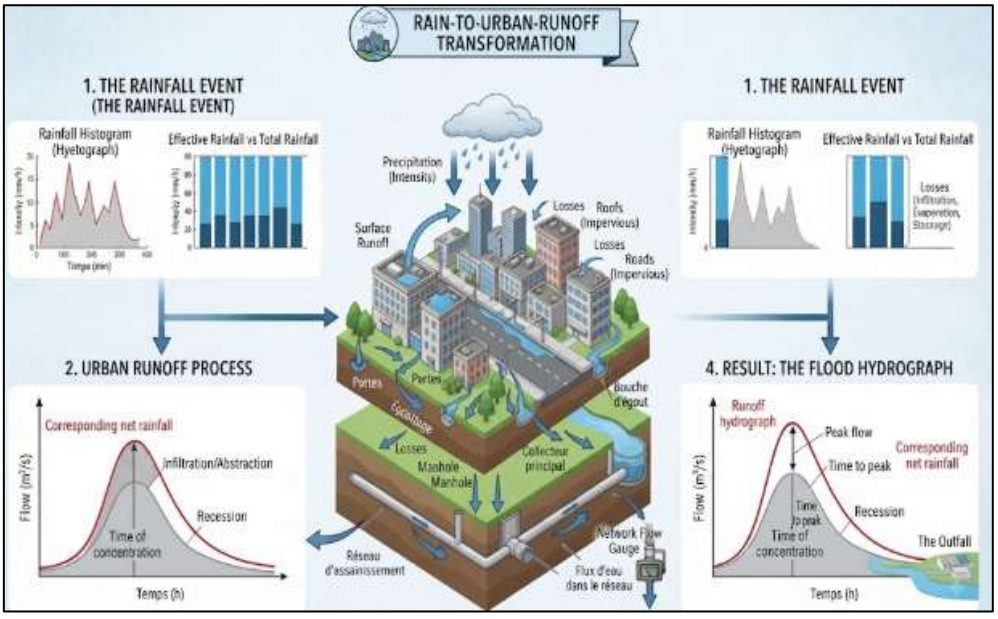


**Figure XIII.03. Transformation of rain into urban runoff (WMO, 2009).**

## III.4. Hydraulic Models

Hydraulic models allow us to represent the propagation of flows in pipes, canals, and network structures. They use the fundamental principles of conservation of mass and momentum to determine the evolution of flow rates, velocities, and water levels.

Hydraulic models can be simplified when flow conditions are relatively stable or use a dynamic approach when temporal and spatial variations need to be taken into account with greater accuracy **(Butler & Davies, 2011; Rossman, 2015).**

### XIII.4.1. Simplified hydraulic models

Simplified models use hydraulic relationships established from geometric characteristics and flow conditions.

For open surface flows, the Manning-Strickler formula constitutes a fundamental relationship used to link flow rate to cross-sectional characteristics and hydraulic slope **(Chow, 1959).**

$$Qp = K_s \cdot A \cdot R_h^{2/3} \cdot I^{1/2} \qquad \text{XIII.02}$$

Where: (Ks) is the Strickler coefficient, (A) the wetted area, (Rh) the hydraulic radius and (I) the hydraulic slope **(Chow, 1959).**

### XIII.4.2. Dynamic Hydraulic Models

When temporal variations in flow rate and water level need to be represented, dynamic hydraulic models use the equations of conservation of mass and momentum. In particular, the Saint-Venant equations provide a fundamental basis for representing unsteady free-channel flows **(Saint-Venant, 1871; Chow, 1959).**

The continuity equation can be expressed in the form:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \qquad \text{XIII.03}$$

Where (q) represents a possible lateral contribution (Chow, 1959). (See figure XIII.04).

These models are particularly useful for the study of complex networks, overload phenomena, pressurized pipes, storage basins, control structures and urban overflows (Butler & Davies, 2011; Rossman, 2015).

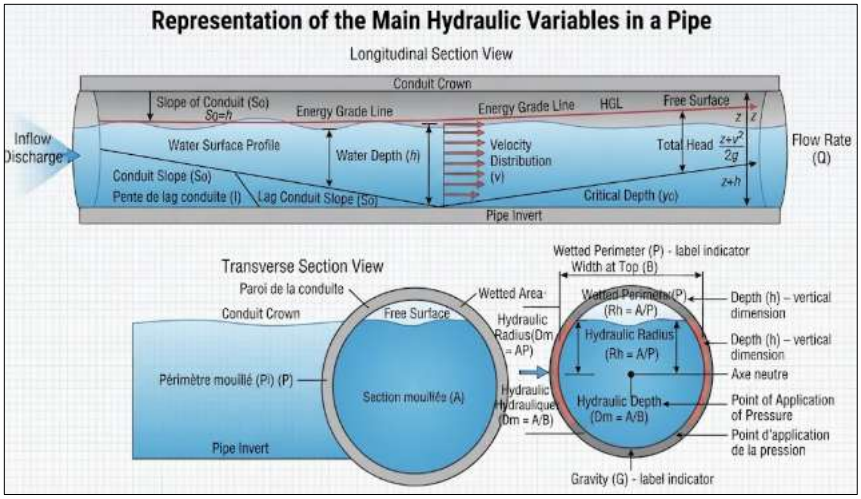



**Figure XIII.04. Representation of the main hydraulic variables in a pipe.**

## XIII.5. Urban runoff models

Urban runoff is an important component of stormwater and combined sewer network modeling. It depends in particular on the impervious surface area, slope, surface type, soil characteristics, infiltration capacity, and precipitation characteristics. (See Table XIII.03; Figure XIII.05).

Urbanization generally leads to an increase in impermeable surfaces and a reduction in infiltration. It can therefore cause an increase in runoff volume as well as a faster hydrological response of the catchment (Akan & Houghtalen, 2003).

**Table XIII.03: Main urban areas considered in a runoff model (Rossman, 2015).**

| Surface | Dominant hydrological behavior |
|---|---|
| **Roofs** | Significant runoff |
| **Roads** | Significant runoff |
| **Parking** | Variable runoff |

| | |
|---|---|
| **Green spaces** | Infiltration and runoff |
| **Permeable soils** | Significant infiltration |

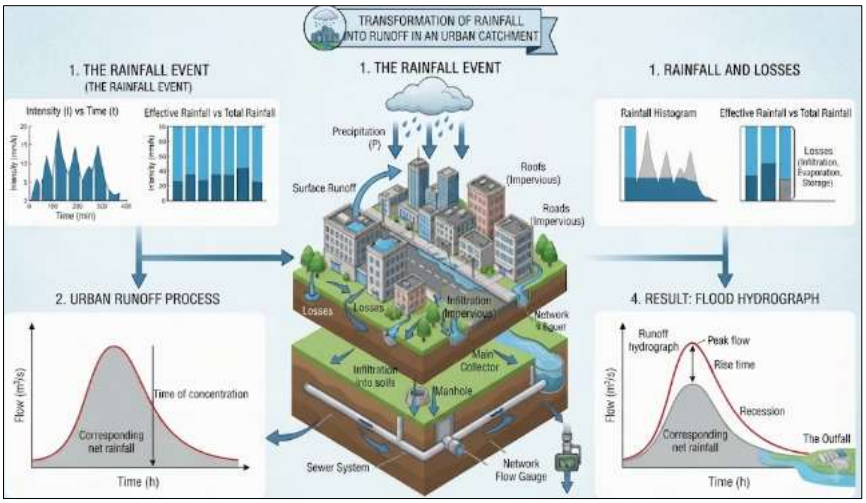



**Figure XIII.05. Transformation of rainfall into runoff in an urban catchment.** (Akan & Houghtalen, 2003).

## XIII.6. Wastewater Network Modeling

In wastewater networks, flow rates are primarily related to water consumption and domestic, industrial, and similar discharges. The total flow rate may also include extraneous water from infiltration and intrusions into the network (Butler & Davies, 2011).

The total flow rate can be represented by:

$$\boldsymbol{Q_{tot} = Q_{wastewater} + Q_{ind} + Q_{parasite}} \qquad \textbf{XIII.04}$$

$Q_{tot}$: Total flow rate passing through the wastewater collector (in $m^3$/sor L/s).

$Q_{EU}$: Domestic wastewater flow rate (related to user consumption and discharge).

$Q_{ind}$: Flow rate of industrial and similar discharges connected to the system.

$Q_{parasite}$: Flow rate of extraneous water (originating from groundwater infiltration and clear water intrusion into the network). (Butler & Davies, 2011).

The models also make it possible to represent daily variations in flow rates and to identify peak periods likely to cause hydraulic constraints in certain parts of the network. (See Figure XIII.06).

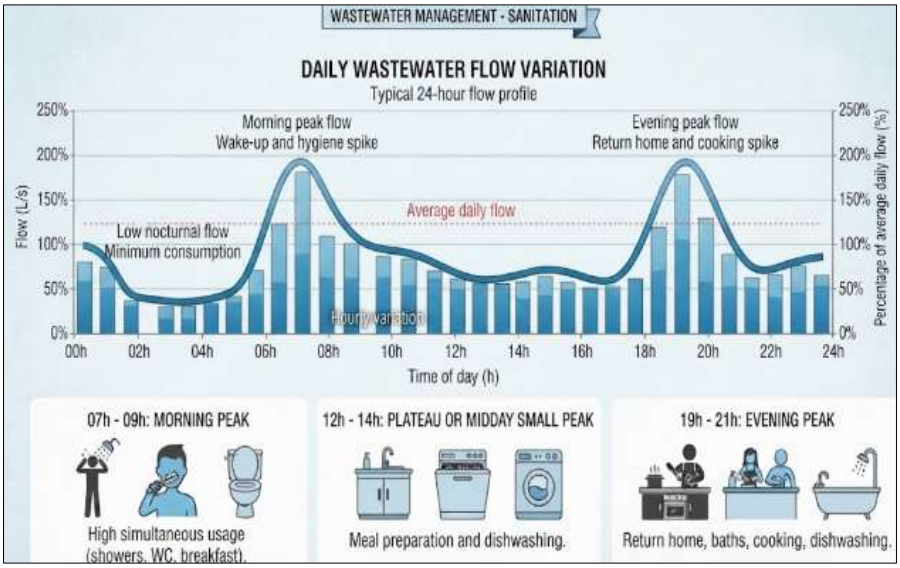



**Figure XIII.06. Daily variation in wastewater flow rate.**

## XIII.7. Modeling of hydraulic structures

Mathematical models make it possible to represent the main ancillary structures of wastewater networks, including weirs, storage basins, pumping stations, valves, weirs and control devices **(Butler & Davies, 2011; Rossman, 2015).** (See Table XIII.04; Figure XIII.07).

For a storage basin, the fundamental principle is based on the conservation of volume:

$$\frac{dV}{dt} = Q_{in}(t) - Q_{out}(t) \qquad \textbf{XIII.05}$$

Where: V: represents the stored volume, $Q_{in:}$ the incoming flow rate and $Q_{out:}$ the outgoing flow rate.

**Table XIII.04. Main structures that can be represented in a hydraulic model.**

| Work | Main function |
|---|---|
| **Spillway** | Limitation or diversion of flow rates |
| **Retention basin** | Temporary storage |
| **Pump** | Water level rise |
| **Valve** | Flow regulation |

| | |
|---|---|
| **Orifice** | Flow control |
| **Threshold** | Level regulation |

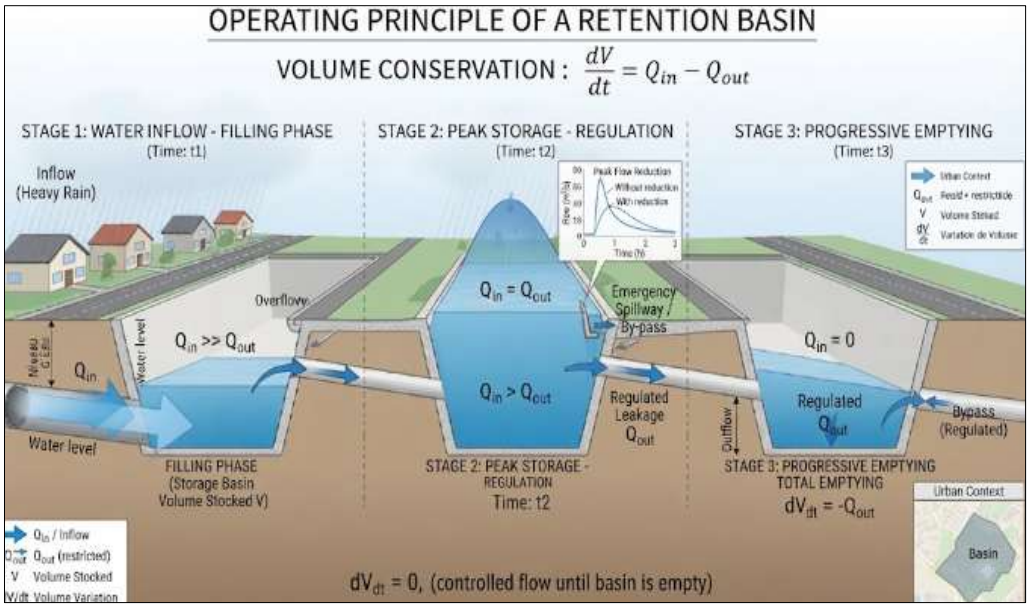



**Figure XIII. 07. Operating principle of a retention basin.**

## XIII.8. Water Quality Models

Wastewater network modeling can also incorporate water quality and pollutant transport. The components studied may include suspended solids, biochemical oxygen demand, chemical oxygen demand, nitrogen, phosphorus, and certain microorganisms.

The transport of a constituent can be represented using a general advection-diffusion-reaction equation that takes into account the transport, dispersion, and transformations of the pollutant within the network **(Schütze et al., 2004).** (See the figure XIII.08).

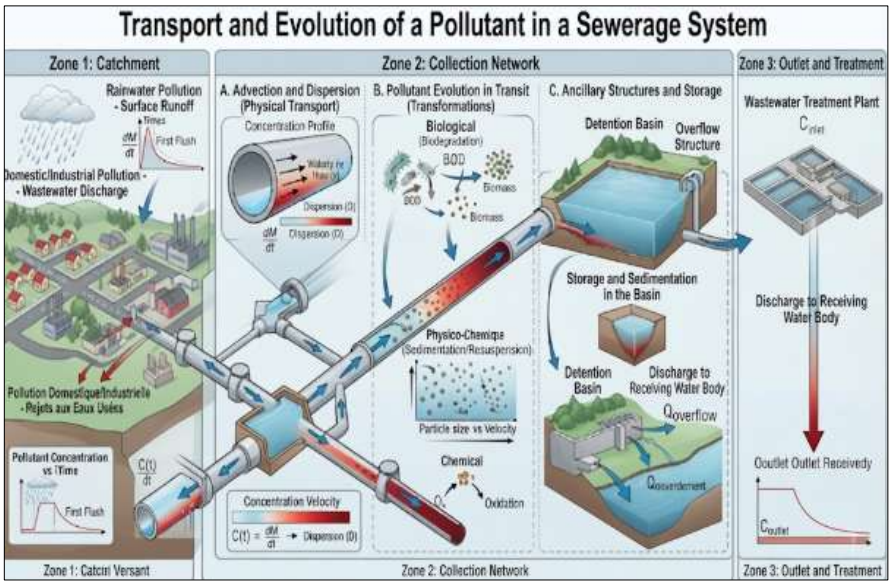



**Figure XIII.08. Transport and evolution of a pollutant in a sanitation network.**

## XIII.9. 1D, 2D and coupled modeling

The choice of model dimension depends on the phenomenon being studied and the level of detail required. One-dimensional models are generally used to represent flows in pipes, while two-dimensional models allow for the representation of water propagation on urban surfaces **(Butler & Davies, 2011; Rossman, 2015).**

### XIII.9.1. One-dimensional (1D) model

In a 1D model, the flow is primarily represented along the axis of the pipes or channels. This approach is particularly well-suited to the simulation of collection networks and the analysis of flow rates, water levels, and overload phenomena **(Rossman, 2015).**

### XIII.9.2. Two-dimensional (2D) model

2D models allow us to represent the propagation of water over a surface. They are particularly useful for studying sewer overflows, street flows, and urban areas susceptible to flooding **(Butler & Davies, 2011).**

### XIII.9.3. Coupled 1D–2D models

1D–2D coupled models allow for the simultaneous representation of flows within the network and their propagation across the surface. This approach is particularly useful for

analyzing urban flooding and the interactions between pipes and the urban surface (Butler & Davies, 2011). (See Table XIII.05; Figure XIII.09).

**Table XIII.05. Comparison of 1D and 2D modeling approaches**

| Approach | Domain represented | Main application |
|---|---|---|
| **1D** | Pipes and canals | Sewerage networks |
| **2D** | Urban surface | Flooding and runoff |
| **1D–2D** | Network + surface | Integrated urban flooding |

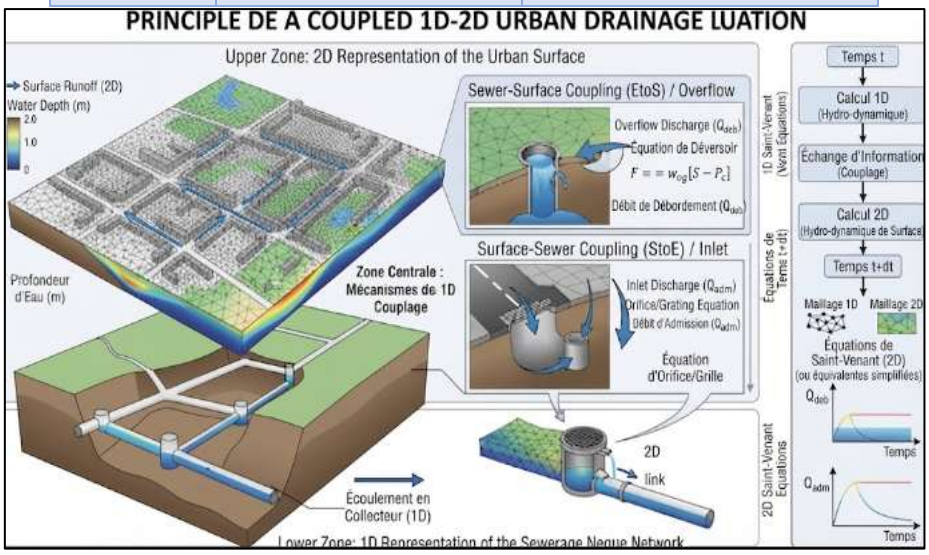



**Figure XIII.09. Principle of a coupled 1D–2D model of an urban sanitation network.**

## XIII.10. Data required for modeling

Building a model requires the collection and preparation of geometric, hydrological, hydraulic, and land-use data. The accuracy and reliability of this data directly affect the quality of the resulting simulations (Akan & Houghtalen, 2003; Butler & Davies, 2011). (See Table XIII.06; Figure XIII.10).

**Table XIII.06. Data required for the construction of a sanitation model.**

| Category | Main data |
|---|---|
| **Geometry** | Diameters, lengths, slopes, altitudes |
| **Hydrology** | Rainfall, hyetograms, IDF |
| **Hydraulic** | Flow rates, levels, boundary conditions |
| **Watersheds** | Surface areas, waterproofing, slopes |
| **Works** | Characteristics of hydraulic structures |

| | |
|---|---|
| **Observations** | Measured flow rates and levels |

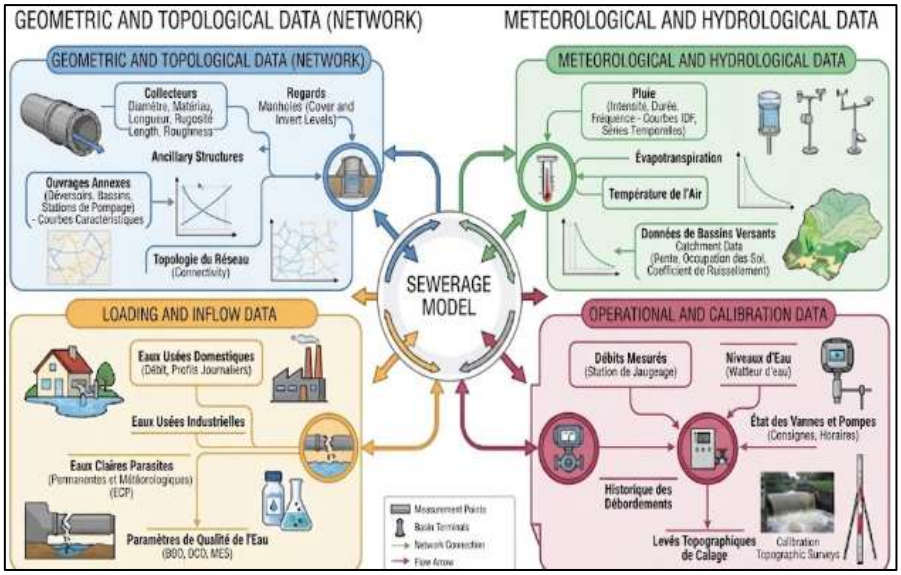



**Figure XIII.10. Main categories of data required for a sanitation model.**

## XIII.11. Calibration and validation of models

Calibration is a fundamental step in modeling. It consists of adjusting certain parameters of the model so that the simulated results satisfactorily reproduce the available observations. (See Table XIII.07; Figure XIII.11).

The parameters that can be adjusted include, in particular, roughness coefficients, runoff parameters, infiltration parameters and certain characteristics of hydraulic structures.

Validation then involves testing the model using a dataset independent of the one used for calibration. This allows us to assess the model's ability to accurately reproduce the system's behavior for other events. **(Butler & Davies, 2011).**

**Table XIII.07. Distinction between calibration and validation (Butler & Davies, 2011).**

| Stage | Objective | Data used |
|---|---|---|
| **Shimming** | Adjust the settings | reference observation data |
| **Validation** | Check performance | Independent data |

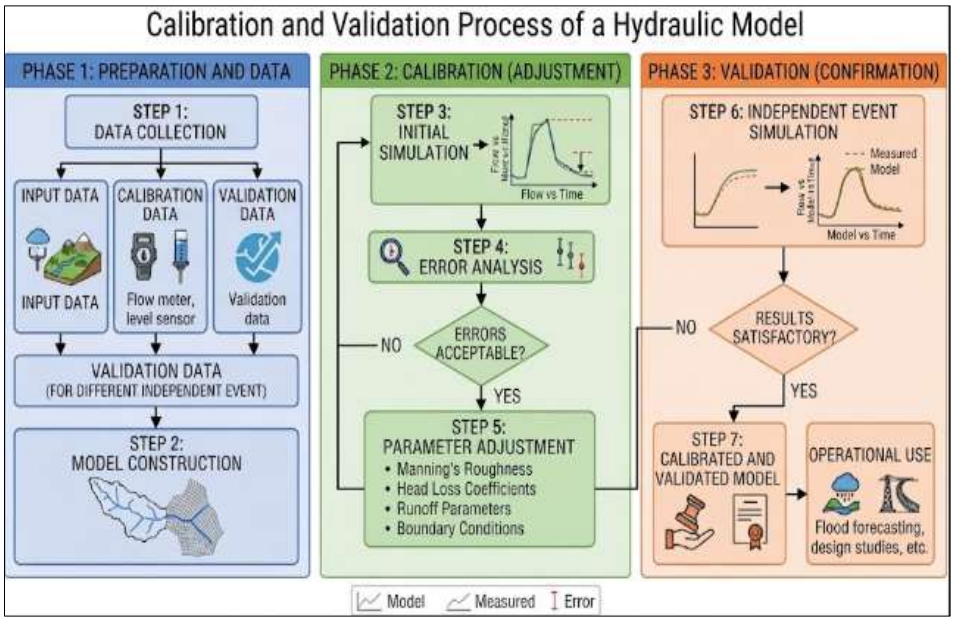



**Figure XIII.11. Calibration and validation process of a hydraulic model (Davies, 2011)**

## XIII.12. Analysis of simulation results

The results of a sanitation model can be presented in different forms: hydrographs, water levels, velocities, flows, stored volumes, overload time, overflow volumes and flood maps.

The analysis should not be limited to the maximum flow rate. It is also necessary to examine the duration of the surges, the levels reached, the volumes overflowed, and the operating conditions of the structures. The interpretation must always take into account the assumptions and uncertainties associated with the model (Butler & Davies, 2011). (See Figure XIII.12).

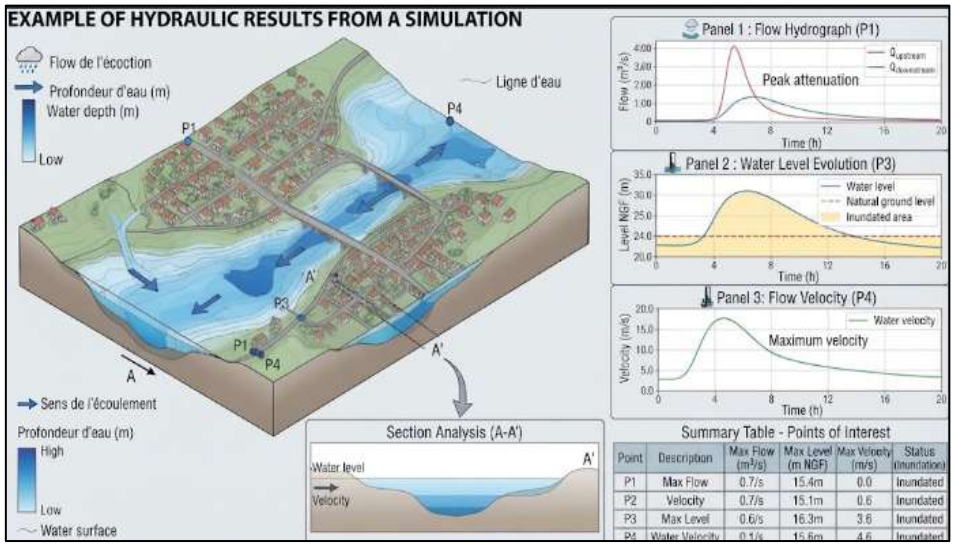



**Figure XIII.12. Example of hydraulic results from a simulation (Rossman, 2015).**

## XIII.13. Main modeling software

Several software programs are used for modeling urban sanitation systems. These tools allow for the representation of catchment areas, pipe networks, hydraulic structures, precipitation, and boundary conditions.

Among the commonly used software is EPA SWMM, developed by the United States Environmental Protection Agency. It allows for the simulation of urban runoff, stormwater and combined sewer systems, storage facilities, and overload phenomena **(Butler & Davies, 2011; Rossman, 2015).** (See Table XIII.08).

**Table XIII.08. Examples of software used in hydraulic and hydrological modeling.**

| Software | Main domain |
|---|---|
| **EPA SWMM** | Urban runoff and sanitation networks |
| **InfoWorks ICM** | Integrated modeling of urban networks |
| **MIKE+** | Modeling of urban hydraulic systems |
| **SewerGEMS** | Analysis and design of sanitation networks |
| **HEC-HMS** | Hydrological modeling of watersheds |

## XIII.14. Advantages and limitations of modeling

Mathematical modeling makes it possible to analyze the overall functioning of a network, to simulate different rainfall scenarios, to identify sensitive areas and to evaluate the effectiveness of different development solutions.

It also serves as a decision-making tool for diagnosing, rehabilitating, extending networks, and preventing urban flooding. In particular, it allows for the comparison of several options before their implementation on the ground.

However, a model remains a simplified representation of the real system. The main uncertainties are related to data quality, model parameters, simplifying assumptions, measurement errors, and initial and boundary conditions **(Butler & Davies, 2011).** (See Table XIII.09).

**Table XIII.09. Main advantages and limitations of modeling.**

| Benefits | Boundaries |
|---|---|
| **Overall network analysis** | Dependence on data quality |

| | |
|---|---|
| **Simulation of several scenarios** | Uncertainty of the parameters |
| **Identification of sensitive areas** | Simplification of phenomena |
| **Comparison of solutions** | Uncertainty of precipitation |
| **Overload forecasting** | Need for calibration and validation |

## XIII.15. Practical applications of modeling

Mathematical modeling can be used in the design, diagnosis, rehabilitation, and management of urban sanitation networks. It allows, in particular, for the verification of hydraulic capacities, the identification of deficiencies, and the comparison of different development solutions.

In the field of stormwater, it also allows the study of source management solutions, retention basins, infiltration systems and various regulation structures.

Finally, modeling is an important tool for analyzing urban flood risks, particularly when network overflows need to be represented on the urban surface **(Akan & Houghtalen, 2003; Rossman, 2015).** (See Table XIII.10; Figure XIII.13).

**Table XIII.10. Main applications of modelling in urban sanitation**

| Application | Objective |
|---|---|
| **Design** | Check the sizing |
| **Diagnosis** | Identify the shortcomings |
| **Rehabilitation** | Compare the solutions |
| **Stormwater management** | Optimize storage and infiltration |
| **Urban flooding** | Identify vulnerable areas |

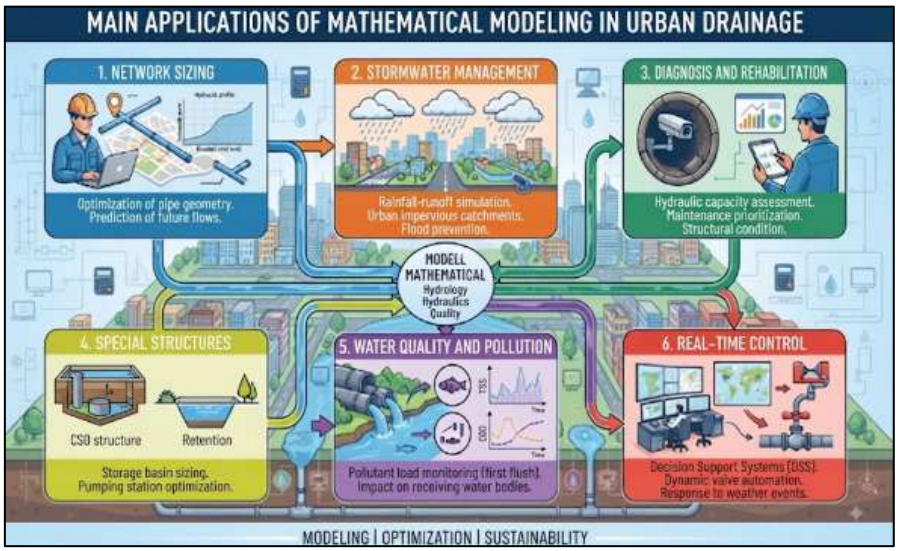



**Figure XIII.13. Main applications of mathematical modelling in urban sanitation.**

## XIII.16. General approach to a modeling study

A hydraulic modelling study must follow a structured approach including data collection, data control, model construction, parameterization, simulation, calibration, validation and analysis of different scenarios **(Butler & Davies, 2011).** (See figure XIII.14).

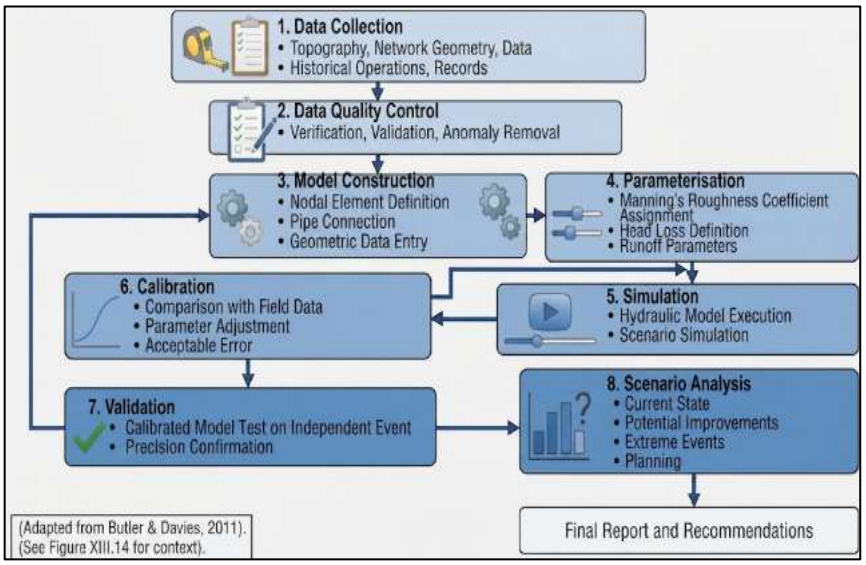



**Figure XIII.14. General flowchart of a modelling study of an urban sanitation network.**

The approach can be summarized as follows:

**Data collection → Model construction → Parameterization → Simulation → Calibration → Validation → Scenarios → Analysis → Development solutions.**

**Conclusion**

Mathematical modeling is now an essential tool for the analysis and management of urban sanitation systems. It allows us to represent hydrological processes, the propagation of flows in networks and, in some cases, the transport and transformation of pollutants.

Hydrological models allow the transformation of precipitation into runoff flow rates, while hydraulic models allow the analysis of flow propagation within pipes and structures. Coupled 1D–2D approaches also enable the study of interactions between the sewer network and surface runoff.

However, the reliability of a model depends heavily on data quality, the choice of assumptions, parameterization, calibration, and validation. Modeling should therefore be considered a decision-making tool whose results must be interpreted taking into account uncertainties and the model's domain of validity.

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