University of Mila	**Academic Year: 2025/2026**
Institute of Nature and life Sciences	**1^{st} Year NLS, Semester 1**
Department of NLS	**Mathematics Statistics**

Duration: 02 hours | # Model Answer | **18/01/2026**

Exercise 1: (Course Questions) 07 Pts

1. The domain of definition of $f(x) = \sqrt{x^2 - 3x + 2}$ is $D_f =]-\infty, 1[\cup]2, +\infty[$.
   False. 0.5
   Justification: We need $x^2 - 3x + 2 \geq 0$. The roots of $x^2 - 3x + 2 = 0$ are $x_1 = 1$ and $x_2 = 2$. The expression is positive outside the roots **including** them. Thus $D_f =]-\infty, 1] \cup [2, +\infty[$. 0.5

2. If $f'_+(x_0)$ and $f'_-(x_0)$ exist and are finite $\Longrightarrow f$ is differentiable at x_0.
   False. 0.5
   Justification: For f to be differentiable, the two limits must exist, be finite, **and be equal** $(f'_+(x_0) = f'_-(x_0))$. 0.5

3. $f(x) = x^2 - 1$ admits a local maximum at $x_0 = 0$.
   False. . 0.5
   Justification: $f'(x) = 2x \Rightarrow f'(0) = 0$. Since $f''(x) = 2 > 0$, the point $x_0 = 0$ is a **local minimum**. 0.5

4. The series
   $$\sum_{n=1}^{+\infty} \ln\left(1 + \frac{1}{n}\right) \text{ is convergent}$$
   False. . 0.5
   Justification: The term u_n can be rewritten in the form
   $$u_n = \ln\left(1 + \frac{1}{n}\right) = \ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln(n).$$
   0.5
   Let
   $$S_n = \sum_{k=1}^{n} u_k.$$
   Then
   $$\begin{aligned} S_n &= (\ln 2 - \ln 1) + (\ln 3 - \ln 2) + \cdots + (\ln(n+1) - \ln n) \\ &= \ln(n+1) - \ln(1). \qquad \boxed{0.25} \end{aligned}$$
   Since $\ln(1) = 0$, we obtain
   $$S_n = \ln(n+1).$$

Thus,
$$\lim_{n\to+\infty} S_n = +\infty.\boxed{0.25}$$
Therefore, the series
$$\sum_{n=1}^{+\infty} \ln\left(1+\frac{1}{n}\right)$$
diverges.

5. If $\lim_{n\to+\infty} u_n = 0$, then the series $\sum u_n$ is convergent.
False. . $\boxed{0.5}$
Justification: This is a **necessary** condition but not sufficient if
$$\lim_{n\to+\infty} u_n \neq 0,$$
then the series $(\sum u_n)_n$ is **divergent**. $\boxed{0.5}$

6. The Riemann series $\sum \frac{1}{n^\alpha}$ is convergent iff $\alpha \geq 1$.
False. $\boxed{0.5}$
Justification: The series converges if and only if $\alpha > 1$. If $\alpha = 1$, it diverges. $\boxed{0.5}$

7. If $\sum u_n$ converges and $\sum v_n$ diverges, then $\sum(u_n + v_n)$ diverges.
True. $\boxed{0.5}$

Exercise 2 (04 pts)

I. Calculate the limit:
$$\lim_{x\to 0} \frac{e^x - 1}{x(e^x+1)}$$

First method
Using the fundamental limit
$$\lim_{x\to 0} \frac{e^x-1}{x} = 1 \qquad \boxed{0.5}$$
$$\lim_{x\to 0} \left(\frac{e^x-1}{x}\right) \cdot \frac{1}{e^x+1} = 1 \cdot \frac{1}{e^0+1} = \frac{1}{2} \qquad \boxed{0.5}$$
Or by the second method
$$\lim_{x\to 0} \frac{e^x - 1}{x(e^x+1)}$$
As $x \to 0$, we have:
$$e^x - 1 \to 0 \quad \text{and} \quad x(e^x+1) \to 0,$$
so the limit is of the indeterminate form $\frac{0}{0}$. We apply L'Hôpital's rule.
$$\lim_{x\to 0} \frac{e^x - 1}{x(e^x+1)} = \lim_{x\to 0} \frac{(e^x-1)'}{(x(e^x+1))'} \qquad \boxed{0.5}$$
$$= \lim_{x\to 0} \frac{e^x}{(e^x+1) + xe^x} = \frac{1}{2} \qquad \boxed{0.5}$$
II. Function $f(x,y) = \ln(1+x+y^2)$

- **Domain:** $D_f = \{(x,y) \in \mathbb{R}^2 \mid 1+x+y^2 > 0\}$. $\boxed{0.5}$

- **Gradient $\nabla f(0,1)$:**

$$\frac{\partial f}{\partial x} = \frac{1}{1+x+y^2} \quad \boxed{0.25} \quad \Rightarrow f'_x(0,1) = \frac{1}{2} \quad \boxed{0.25}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{1+x+y^2} \quad \boxed{0.25} \Rightarrow f'_y(0,1) = 1 \quad \boxed{0.25}$$

. $\nabla f(0,1) = (\frac{1}{2}, 1)$.

- **Second Partial Derivatives:**

$$f''_{xx} = \frac{-1}{(1+x+y^2)^2} \quad \boxed{0.5}$$

$$f''_{yy} = \frac{2(1+x+y^2) - 4y^2}{(1+x+y^2)^2} \quad \boxed{0.5}$$

$$f''_{xy} = \frac{-2y}{(1+x+y^2)^2} \quad \boxed{0.25}$$

$$f''_{yx} = \frac{-2y}{(1+x+y^2)^2} \quad \boxed{0.25}$$

Exercise 3

1) Recall the integration by parts technique for a definite integral.

$$\int_a^b f(x)\, g'(x)\, dx = \left[f(x)g(x)\right]_a^b - \int_a^b f'(x)\, g(x)\, dx. \quad \boxed{0.5}$$

2) Calculate

$$I = \int_0^{\frac{\pi}{2}} x \sin x dx$$

Let $u = x \Rightarrow du = dx$ $\boxed{0.25}$ and $dv = \sin x dx \Rightarrow v = -\cos x$ $\boxed{0.25}$.

$$I = [-x\cos x]_0^{\pi/2} + \int_0^{\pi/2} \cos x dx \quad \boxed{0.25} = [0-0] + [\sin x]_0^{\pi/2} = 1 \quad \boxed{0.25}$$

3) Change of variable for

$$J = \int \frac{1-\sqrt{x}}{\sqrt{x}} dx$$

Let $t = \sqrt{x}$ $\boxed{0.5}$ $\Rightarrow dt = \frac{1}{2\sqrt{x}}dx \Rightarrow 2dt = \frac{dx}{\sqrt{x}}$ $\boxed{0.25}$.

$$J = \int (1-t)2dt \quad \boxed{0.25} = 2t - t^2 + C \quad \boxed{0.25} = 2\sqrt{x} - x + C \quad \boxed{0.25}$$

$\boxed{\text{Exercise 4}}$

1) Character: Number of disease cases. $\boxed{0.25}$ **Nature:** Quantitative discrete. $\boxed{0.25}$

2) Statistical Table ($N = 82$)**:**

$$f_i = \frac{n_i}{N},$$

$$f_i^{c\uparrow} = \frac{n_i^{c\uparrow}}{N}.$$

x_i	n_i	$n_i^{c\uparrow}$	f_i	$f_i^{c\uparrow}$	$x_i n_i$	$x_i^2 n_i$
0	4	4	0.049	0.049	0	0
1	10	14	0.122	0.171	10	10
2	28	42	0.341	0.512	56	112
3	22	64	0.268	0.780	66	198
4	12	76	0.146	0.926	48	192
5	6	82	0.073	1.000	30	150
Sum	**82**	-	**1.00**	-	**210**	**662**

$\boxed{0.5}$ $\boxed{0.5}$ $\boxed{0.5}$

3)

- **Mode** (M_0)**:** $M_0 = 2$ $\boxed{0.25}$ (highest frequency $n_i = 28$).
- **Mean** ($\bar{x}$)**:**

$$\begin{aligned}
\bar{x} &= \frac{1}{N}\sum_{i=1}^{k} x_i n_i \qquad \boxed{0.25} \\
&= \frac{4\times 0 + 1\times 10 + 28\times 2 + 22\times 3 + 12\times 4 + 6\times 5}{82} \\
&= \frac{210}{82} \approx 2.56 \qquad \boxed{0.25}
\end{aligned}$$

- **Median** (Me)**:** $\frac{N}{2} = 41$ Since N is even, then

$$Me = \frac{x_{\frac{N}{2}} + x_{\frac{N}{2}+1}}{2} \qquad \boxed{0.25}$$

$$\frac{N}{2} \quad \text{and} \quad \frac{N}{2}+1$$

$$\frac{N}{2} = \frac{82}{2} = 41 \qquad \frac{N}{2}+1 = 42$$

so $Me = \frac{2+2}{2}$ $\boxed{0.25}$

4) Quartiles and Interquartile Range
The first quartile corresponds to the value of rank:

$$\frac{N}{4} = \frac{82}{4} = 20.5 \qquad \boxed{0.25}$$

Since $14 < 20.5 \leq 42$, the corresponding value is:

$$\boxed{Q_1 = 2} \qquad \boxed{0.25}$$

Third quartile Q_3
The third quartile corresponds to the value of rank:

$$\frac{3N}{4} \qquad \boxed{0.25} = \frac{3 \times 82}{4} = 61.5$$

Since $42 < 61.5 \leq 64$, the corresponding value is:

$$\boxed{Q_3 = 3} \qquad \boxed{0.25}$$

Interquartile range
The interquartile range is defined by:

$$I_Q = Q_3 - Q_1 \qquad \boxed{0.25}$$

$$\boxed{I_Q = 3 - 2 = 1} \qquad \boxed{0.25}$$

5)

- **range:**

$$\mathrm{e} = x_{\max} - x_{\min} = 5 \qquad \boxed{0.25}$$

- **Variance $V(X)$:**

$$V(X) = \frac{1}{N} \sum x_i^2 n_i - \bar{x}^2 \qquad \boxed{0.25}$$

  then by the statistical table

$$V(X) = \frac{1}{82} 662 - (2.56)^2 \approx 8.07 - 6.55 = 1.52 \qquad \boxed{0.25}$$

  .

- **Standard Deviation σ_X:**

$$\sigma_X = \sqrt{V(X)} \qquad \boxed{0.25}$$

  so

$$\sqrt{1.52} \approx 1.23 \qquad \boxed{0.25}$$

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