

Exercise 1 (Course Questions) 07pts

State whether the following statements are **True** or **False**, justify your answers.

- ❶ The domain of definition of the function $f(x) = \sqrt{x^2 - 3x + 2}$ is $D_f =]-\infty, 1[\cup]2, +\infty[$.
- ❷ If we have : $\begin{cases} f'_+(x_0) \text{ exists and finite.} \\ f'_-(x_0) \text{ exists and finite.} \end{cases} \implies f \text{ is differentiable at the point } x_0$
- ❸ The function $f(x) = x^2 - 1$ admits a local maximum at the point $x_0 = 0$.
- ❹ The series
$$\sum_{n=1}^{+\infty} \ln \left(1 + \frac{1}{n} \right) \text{ is convergent}$$
- ❺ If $\lim_{n \rightarrow +\infty} u_n = 0$, then the series $\sum u_n$ is convergent.
- ❻ The Riemann series $\sum \frac{1}{n^\alpha}$ is convergent if and only if $\alpha \geq 1$.
- ❼ If $\sum u_n$ converges and $\sum v_n$ diverges, then the series $\sum (u_n + v_n)$ diverges.

Exercise 2

I. Calculate the following limit:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x(e^x + 1)}$$

II. Let the function f defined by

$$f(x, y) = \ln(1 + x + y^2)$$

- Find the domain of definition of the function f .
- Calculate the gradient of f at the point $(0, 1)$.

$$\nabla f(0, 1)$$

- Compute the second partial derivatives of f .

Exercise 3

- ❶ Recall the integration by parts technique for a definite integral.
- ❷ Calculate the definite integral

$$I = \int_0^{\frac{\pi}{2}} x \sin x \, dx.$$

- ❸ Make the change of variable to calculate the following integral:

$$\int \frac{1 - \sqrt{x}}{\sqrt{x}} dx$$

Exercise 4

A survey was conducted in eighty-two (82) hospitals in a country regarding cases of a rare and unknown disease. The results are collected in the following table:

x_i	0	1	2	3	4	5
n_i	4	10	28	22	12	6

- ❶ Determine the character studied and give its nature
- ❷ Calculate the cumulative absolute frequency $n_i^{c\uparrow}$, relative frequency f_i and cumulative relative frequency $f_i^{c\uparrow}$ (present the calculations in a table).
- ❸ Calculate mode (M_0), mean ($\bar{x}$) and median (Me).
- ❹ Determine quartiles (Q_1), (Q_3) and interquartile range (I_Q).
- ❺ Calculate the range (e), variance ($V(X)$), standard deviation (σ_X).