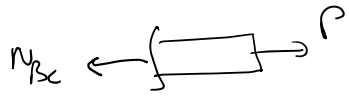
Série TD03

EXERCICE 01

a) $W = \frac{1}{2}\int_A^B \frac{N_{AB}^2}{EA_{AB}}dx + \frac{1}{2}\int_B^C \frac{N_{BC}^2}{EA_{BC}}dx$

$N_{BC} = P$,



$N_{AB} = P$,

$A_{AB} = \frac{\pi D_1^2}{4} = \frac{3{,}14 \cdot 30^2}{4} = 706{,}5\ mm^2 = 706{,}5 \cdot 10^{-6}\ m^2$

$A_{BC} = \frac{\pi D_2^2}{4} = \frac{3{,}14 \cdot 20^2}{4} = 314\ mm^2 = 314 \cdot 10^{-6}\ m^2$

$E = 200\ GPa = 200 \cdot 10^3\ MPa = 200 \cdot 10^9 \cdot Pa$

$W = \frac{1}{2}\int_0^{1{,}50} \frac{P^2}{EA_{AB}}dx + \frac{1}{2}\int_0^{1{,}20} \frac{P^2}{EA_{BC}}dx$

$W = \frac{P^2}{2EA_{AB}}(1{,}5) + \frac{P^2}{2EA_{BC}}(1{,}20) = \frac{P^2}{2E}\left(\frac{1{,}5}{706{,}5 \times 10^{-6}} + \frac{1{,}2}{314 \cdot 10^{-6}}\right)$

$= \frac{(30\,000)^2}{2 \cdot 200 \cdot 10^9 \cdot 10^{-6}}\left(\frac{1{,}5}{706{,}5} + \frac{1{,}2}{314}\right) = \frac{900 \cdot 10^6}{2 \cdot 200 \cdot 10^3}(0{,}00212 + 0{,}003821)$

$= 13{,}37\ N.m\ (J)$.

b) on Applique le théorème de CASTIGLIANO:

Castigliano : 1 forme → $\frac{\partial W}{\partial P_i} = \delta_i$; 2 forme → $\frac{\partial W}{\partial \delta_i} = P_i$

$W = \frac{1}{2}\int_0^{1{,}5} \frac{P^2}{2EA_{AB}}dx + \frac{1}{2}\int_0^{1{,}2} \frac{P^2}{2EA_{BC}}dx = \frac{P^2}{2E}\left(\frac{l_{AB}}{A_{AB}} + \frac{l_{BC}}{A_{BC}}\right)$

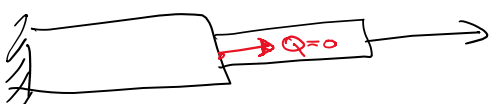
$\delta_C = \frac{\partial W}{\partial P} = \frac{2P}{2E}\left(\frac{l_{AB}}{A_{AB}} + \frac{l_{BC}}{AB}\right)$

$P = 30kN \Rightarrow \delta_C = \frac{2 \cdot 30 \cdot 10^3}{2 \cdot 200 \cdot 10^3}\left(\frac{1500}{706{,}5} + \frac{1200}{314}\right)$

$U_C = \delta_C = 0{,}891\ mm$

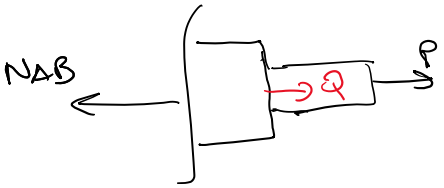
c) Calculer le déplacement $U_B = \delta_B$:

on Applique une charge fictive Q dans "B" :



$N_{BC} = P$

$N_{AB} = (P+Q)$



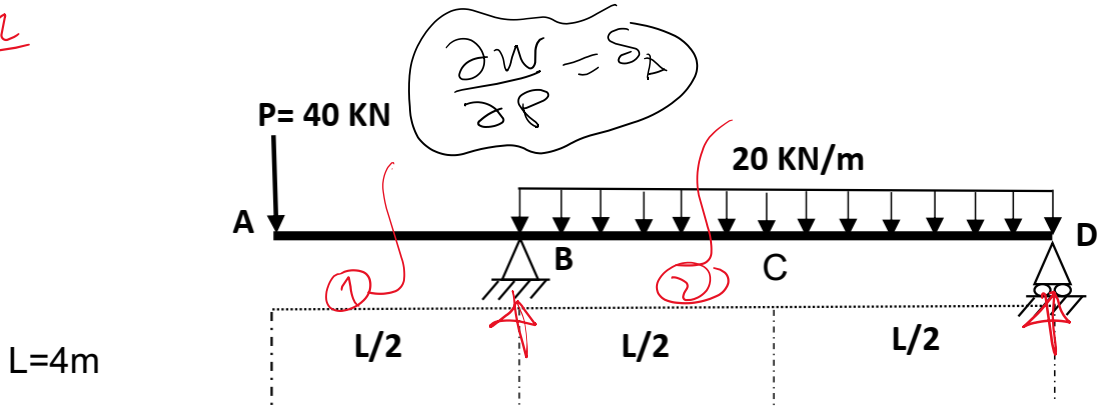
$$w = \frac{1}{2}\int_0^{1,5} \frac{(Q+P)^2}{EA_{AB}}\,dx + \frac{1}{2}\int_0^{1,2} \frac{P}{EA_{BC}}\,dx$$

$$\frac{\partial w}{\partial Q} = u_B = \delta_B = \frac{1}{2}\int_0^{1,5} \frac{2Q+2P}{EA_{AB}}\,dx = \int_0^{1,5} \frac{P}{EA_{AB}}\,dx$$

$$\frac{\partial w}{\partial Q} = u_B = \frac{P}{EA_{AB}}\cdot(1,5) = \frac{30\,000}{200\cdot10^9 \cdot 706,5\cdot10^{-6}}\cdot 1,5 = 3,184\cdot10^{-4}\,m$$

$$\Rightarrow \boxed{u_B = 0,318\ mm}$$

Exercício 2



L=4m

$w = f(M(w))$

$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_A = 0$