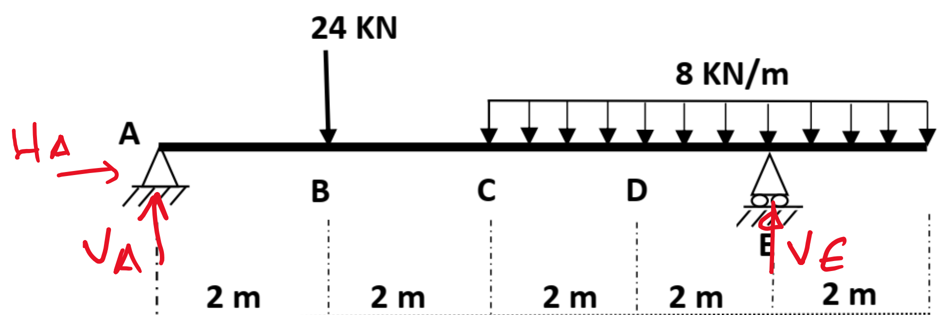
Exercice 02 :

Utilisez ***la méthode moments des aires*** *pour déterminer* ***la flèche*** V_C *au point* ***C,*** *et* ***la flèche*** V_F *au point* ***F*** *de la poutre illustrée à la Figure 2.*

EI *constant, avec* $\mathbf{E = 2 \times 10^5}$ ***MPa,*** *et* $\mathbf{I = 50 \times 10^6}$ ***mm⁴***

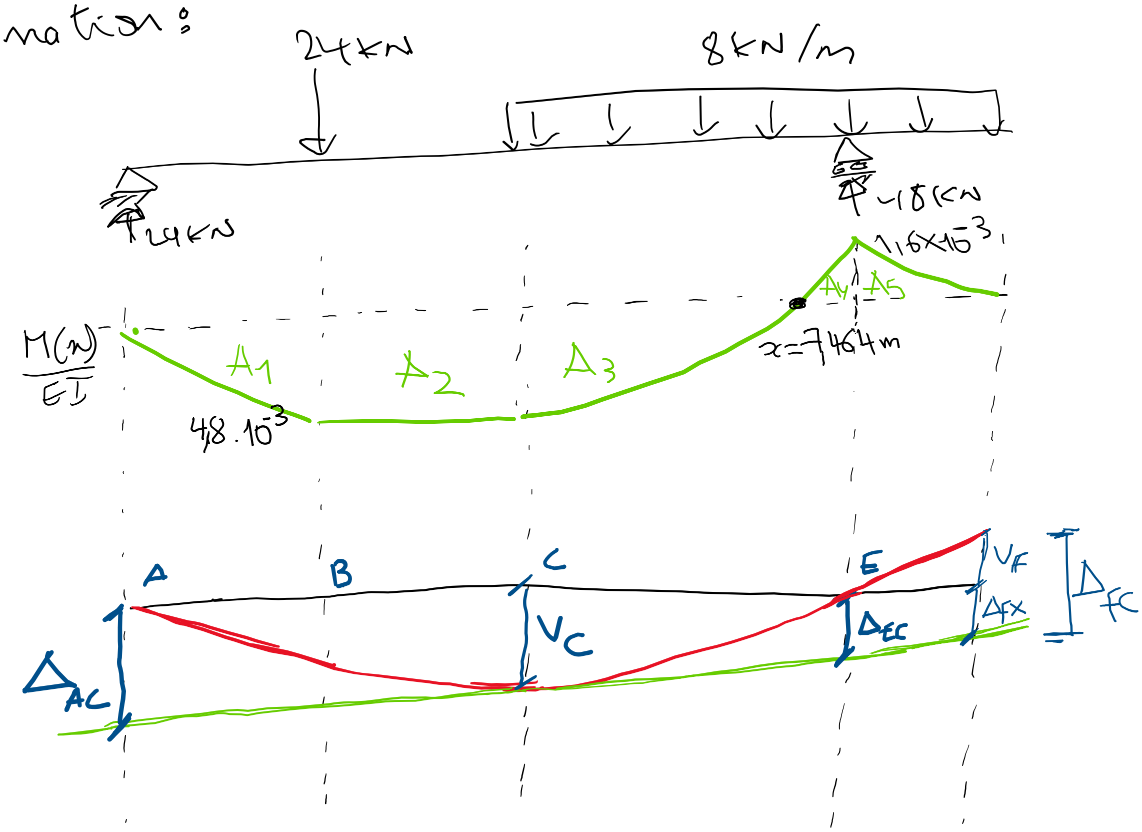


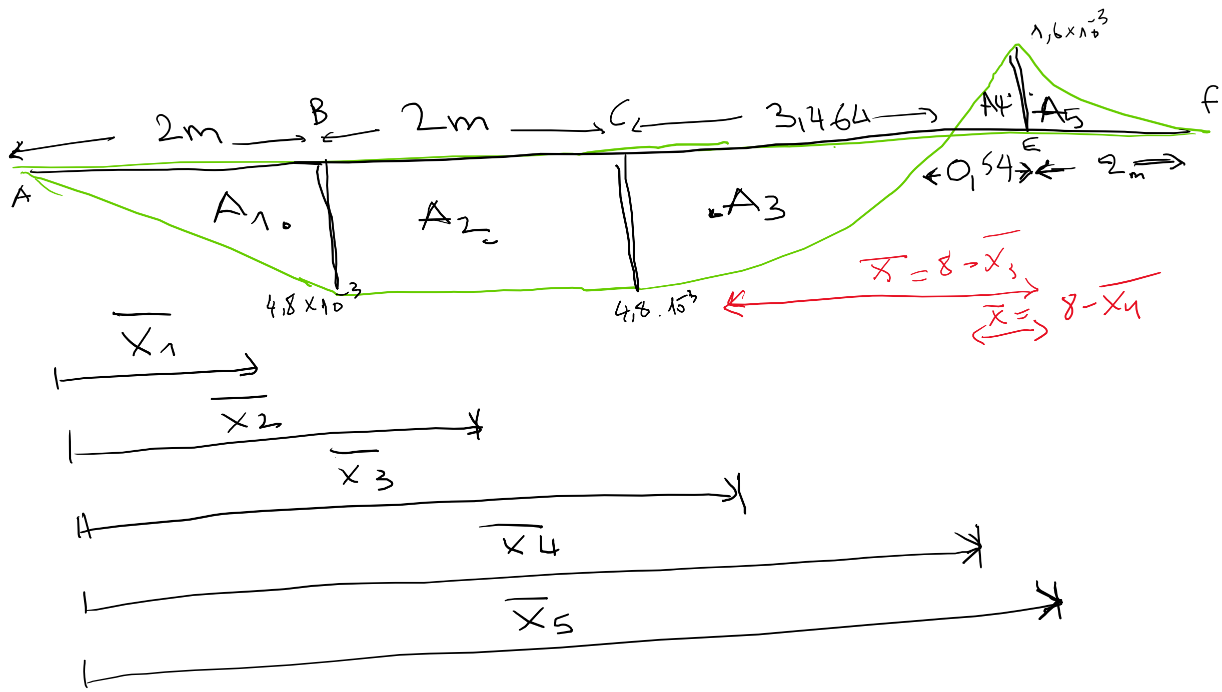
1/ Calculer les réactions d'appuis

$\sum F_x = 0 \Rightarrow H_A = 0$

$\sum F_y = 0 \Rightarrow V_A = 24KN \qquad V_E = 48KN$

2/ Tracer le diagramme $\frac{M}{EI}$, et l'Allure de la déformation :





$A_1 = 4{,}8 \times 10^{-3}$, $\overline{X}_1 = 1{,}333\ m$

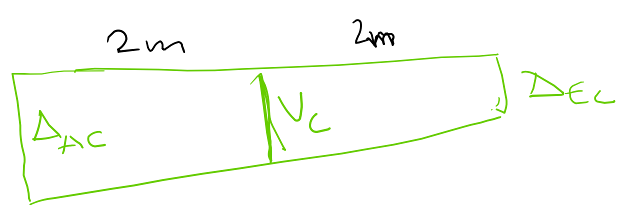
$A_2 = 9{,}6 \times 10^{-3}$, $\overline{X}_2 = 2+1 = 3\ m$

$A_3 = \frac{2}{3}\left(4{,}8.10^{-3} \times 3{,}464\right) = 11{,}08 \times 10^{-3}$, $\overline{X}_3 = 2+2+\left(\frac{3}{8} \times 3{,}464\right) = 5{,}3m$

$A_4 = \frac{1}{3}\left(1{,}6 \times 10^{-3} \times 0{,}54\right) = 0{,}288 . 10^{-3}$, $\overline{X}_4 = 7{,}464 + \left(\frac{3}{4}(0{,}54)\right) = 7{,}87\ m$

$A_5 = \frac{1}{3}\left(1{,}6 \times 10^{-3} \times 2\right) = 1{,}07 \times 10^{-3}$, $\overline{X}_5 = 8 + \frac{1}{4}(2) = 8{,}5\ m.$

*) $V_C = \frac{\Delta_{AC} + \Delta_{EC}}{2}$



$\Delta_{AC} = \overline{X}_{AG} . \left[\frac{M}{EI}\right]_{x_A}^{x_C}$

$\rightarrow \Delta_{AC} = (A_1 . \overline{X}_1) + (A_2 . \overline{X}_2)$

$\Delta_{AC} = (4{,}8 \times 10^{-3} \times 1{,}333) + (9{,}6 \times 10^{-3} \times 3) = 0{,}0352\ m$

$\rightarrow \Delta_{EC} = \overline{X}_{EG} \times \text{Aire}\left[\frac{-M}{EI}\right]_{x_E}^{x_C}$

$= A_3(8 - \overline{X}_3) + A_4(8 - \overline{X}_4)$

$= 11{,}08 \times 10^{-3}(8 - 5{,}3) + 2{,}88 \times 10^{-4}(8 - 7{,}87)$

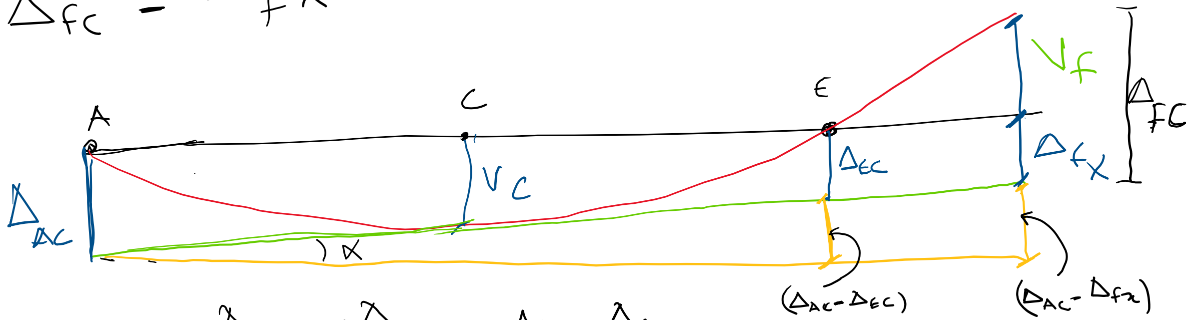
$\Delta_{EC} = 0{,}0299$

$$\Rightarrow V_C = \frac{0{,}0352 + 0{,}0299}{2} = 0{,}0325\,m$$

$$\Rightarrow \boxed{V_C = 32{,}5\ mm} \downarrow$$

*) Calculer V_E ?

$$V_E = \Delta_{fc} - \Delta_{fx}$$



$$tg\alpha = \frac{\Delta_{AC} - \Delta_{EC}}{L_{AE}} = \frac{\Delta_{AC} - \Delta_{fx}}{L_{Af}}$$

$$\Rightarrow \Delta_{fx} = \Delta_{AC} - \left(\frac{\Delta_{AC} - \Delta_{EC}}{L_{AE}}\right) \cdot L_{AF}$$

$$= 0{,}0352 - \left(\frac{0{,}0352 - 0{,}0299}{8}\right) \times 10$$

$$\Delta_{fx} = 0{,}0352 - 0{,}00625 = 0{,}0285\ m$$

*) $\Delta_{fc} = \overline{X}_{fG} \cdot \text{Aire}\left[\frac{-M}{EI}\right]_f^C$

$$= A_3 \cdot (10 - \overline{X}_3) + A_4(10 - \overline{X}_4) + A_5(10 - \overline{X}_5)$$

$$= 11{,}08 \times 10^{-3}(10 - 5{,}3) - 2{,}88 \times 10^{-4}(10 - 7{,}87) - 1{,}07 \times 10^{-3}(10 - 8{,}50)$$

$$= 11{,}08(4{,}7) \cdot 10^{-3} + 2{,}88(2{,}13) \cdot 10^{-4} + 1{,}07(1{,}5) \cdot 10^{-3}$$

$$\Delta_{fc} = 49{,}857 \times 10^{-3}\ m$$

$$\Rightarrow V_E = 49{,}857 \times 10^{-3} - 28{,}5 \times 10^{-3} = 21{,}357 \times 10^{-3}\ m$$

$$\Rightarrow \boxed{V_E = 21{,}357\ mm} \uparrow$$