

Solution

Exercise 1.

Consider the following grammar G:

$$P: A \rightarrow Ba \mid bBc \mid dc \mid bda$$

$$B \rightarrow d$$

a) Construct the set of items LR(0).

We add the rule $A' \rightarrow A$

$$I_0 = \text{Closure}(\{[A' \rightarrow A]\}) = \{[A' \rightarrow A], [A \rightarrow Ba], [A \rightarrow bBc], [A \rightarrow dc], [A \rightarrow bda], [B \rightarrow d]\}$$

$$I_1 = \text{Goto}(I_0, A) = \text{Closure}(\{[A' \rightarrow A]\}) = \{[A' \rightarrow A]\}$$

$$I_2 = \text{Goto}(I_0, B) = \text{Closure}(\{[A \rightarrow Ba]\}) = \{[A \rightarrow B.a]\}$$

$$I_3 = \text{Goto}(I_0, b) = \text{Closure}(\{[A \rightarrow b.Bc], [A \rightarrow b.da]\}) = \{[A \rightarrow b.Bc], [A \rightarrow b.da], [B \rightarrow d]\}$$

$$I_4 = \text{Goto}(I_0, d) = \text{Closure}(\{[A \rightarrow d.c], [B \rightarrow d]\}) = \{[A \rightarrow d.c], [B \rightarrow d]\}$$

$$I_5 = \text{Goto}(I_2, a) = \text{Closure}(\{[A \rightarrow Ba]\}) = \{[A \rightarrow Ba]\}$$

$$I_6 = \text{Goto}(I_3, B) = \text{Closure}(\{[A \rightarrow bB.c]\}) = \{[A \rightarrow bB.c]\}$$

$$I_7 = \text{Goto}(I_3, d) = \text{Closure}(\{[A \rightarrow bd.a], [B \rightarrow d]\}) = \{[A \rightarrow bd.a], [B \rightarrow d]\}$$

$$I_8 = \text{Goto}(I_4, c) = \text{Closure}(\{[A \rightarrow dc]\}) = \{[A \rightarrow dc]\}$$

$$I_9 = \text{Goto}(I_6, c) = \text{Closure}(\{[A \rightarrow bBc]\}) = \{[A \rightarrow bBc]\}$$

$$I_{10} = \text{Goto}(I_7, a) = \text{Closure}(\{[A \rightarrow bda]\}) = \{[A \rightarrow bda]\}$$

SLR(1) parsing table.

0) $A' \rightarrow A$

1) $A \rightarrow Ba$

2) $A \rightarrow bBc$

3) $A \rightarrow dc$

4) $A \rightarrow bda$

5) $B \rightarrow d$

	First	Follow
A	b d	#
B	d	a c

	a	b	c	d	#	A	B
0		Sh3		Sh4		1	2
1					"accept "		
2	Sh5						
3				Sh7			6
4	R(5)		R(5)/ Sh8				
5					R(1)		
6			Sh9				
7	R(5)/ Sh10		R(5)				
8					R(3)		
9					R(2)		
10					R(4)		

c) Analyze the string: bdca #

SLR(1) parsing table is multi-defined, therefore the grammar G is **not SLR(1)** and we cannot perform a deterministic SLR(1) parse.

Exercise 2.

Consider the following grammar G:

$$P: A \rightarrow Ba \mid bBc \mid dc \mid bda$$

$$B \rightarrow d$$

a) Construct the set of LR(1) items.

We add the rule $A' \rightarrow A$

$$I_0 = \text{Closure}(\{[A' \rightarrow A, \#]\}) = \{[A' \rightarrow A, \#], [A \rightarrow Ba, \#], [A \rightarrow bBc, \#], [A \rightarrow dc, \#], [A \rightarrow bda, \#], [B \rightarrow d, a]\}$$

$$I_1 = \text{Goto}(I_0, A) = \text{Closure}(\{[A' \rightarrow A, \#]\}) = \{[A' \rightarrow A, \#]\}$$

$$I_2 = \text{Goto}(I_0, B) = \{[A \rightarrow B.a, \#]\}$$

$$I_3 = \text{Goto}(I_0, b) = \text{Closure}(\{[A \rightarrow b.Bc, \#], [A \rightarrow b.da, \#]\}) = \{[A \rightarrow b.Bc, \#], [A \rightarrow b.da, \#], [B \rightarrow d, c]\}$$

$$I_4 = \text{Goto}(I_0, d) = \text{Closure}(\{[A \rightarrow dc, \#], [B \rightarrow d., \#]\}) = \{[A \rightarrow d.c, \#], [B \rightarrow d., a]\}$$

$$I_5 = \text{Goto}(I_2, a) = \text{Closure}(\{[A \rightarrow Ba., \#]\}) = \{[A \rightarrow Ba., \#]\}$$

$$I_6 = \text{Goto}(I_3, B) = \text{Closure}(\{[A \rightarrow bB.c, \#]\}) = \{[A \rightarrow bB.c, \#]\}$$

$$I_7 = \text{Goto}(I_3, d) = \text{Closure}(\{[A \rightarrow bd.a, \#], [B \rightarrow d., c]\}) = \{[A \rightarrow bd.a, \#], [B \rightarrow d., c]\}$$

$$I_8 = \text{Goto}(I_4, c) = \text{Closure}(\{[A \rightarrow dc., \#]\}) = \{[A \rightarrow dc., \#]\}$$

$$I_9 = \text{Goto}(I_6, c) = \text{Closure}(\{[A \rightarrow bBc., \#]\}) = \{[A \rightarrow bBc., \#]\}$$

$$I_{10} = \text{Goto}(I_7, a) = \text{Closure}(\{[A \rightarrow bda., \#]\}) = \{[A \rightarrow bda., \#]\}$$

LR(1) parsing table .

0) A' → A

1) A → Ba

2) A → bBc

3) A → dc

4) A → bda

5) B → d

	a	b	C	d	#	A	B
0		Sh 3		Sh 4		1	2
1					"accept "		
2	Sh 5						
3				Sh 7			6
4	R(5)		Sh 8				
5					R(1)		
6			Sh 9				
7	Sh 10		R(5)				
8					R(3)		
9					R(2)		
10					R(4)		

c) Analyze the string: bdca #

LR(1) parse table is mono-defined, so the grammar G is **LR(1)** and we can do a deterministic LR(1) parse for the string **bdca #**.

Stack	String	Action
0	bdca #	Sh 3
0b3	dca #	Sh 7

0b3d7	ca #	R(5)
0b3B6	ca #	Sh 9
0b3B6c9	a #	" error "

Exercise 3.

Consider the C pointer grammar, whose numbered productions are given below:

- (0) $S' \rightarrow S$
- (1) $S \rightarrow L = R$
- (2) $S \rightarrow R$
- (3) $L \rightarrow * R$
- (4) $L \rightarrow a$
- (5) $R \rightarrow L$

a)

1) Proof that the grammar G is not SLR(1):

Calculation of follow set:

	follow
S	#
L	# =
R	# =

Calculation of the canonical collection of LR (0) items:

- $I_0 = \{[S' \rightarrow .S], [S \rightarrow .L=R], [S \rightarrow .R], [L \rightarrow .*R], [L \rightarrow .a], [R \rightarrow .L]\}$
- $I_1 = \text{GOTO}(I_0, S) = \{[S' \rightarrow S.]\}$
- $I_2 = \text{GOTO}(I_0, L) = \{[S \rightarrow L.=R], [R \rightarrow L.]\}$
- $I_3 = \text{GOTO}(I_0, R) = \{[S \rightarrow R.]\}$
- $I_4 = \text{GOTO}(I_0, *) = \{[L \rightarrow *.R], [L \rightarrow .*R], [L \rightarrow .a], [R \rightarrow .L]\}$
- $I_5 = \text{GOTO}(I_0, a) = \{[L \rightarrow a.]\}$
- $I_6 = \text{GOTO}(I_2, =) = \{[S \rightarrow L.=R], [L \rightarrow .*R], [L \rightarrow .a], [R \rightarrow .L]\}$
- $I_7 = \text{GOTO}(I_4, R) = \{[L \rightarrow *R.]\}$
- $I_8 = \text{GOTO}(I_4, L) = \{[R \rightarrow L.]\}$
- $\text{GOTO}(I_4, *) = I_4$
- $\text{GOTO}(I_4, a) = I_5$
- $I_9 = \text{GOTO}(I_6, R) = \{[S \rightarrow L=R.]\}$
- $\text{GOTO}(I_6, L) = I_8$
- $\text{GOTO}(I_6, *) = I_4$
- $\text{GOTO}(I_6, a) = I_5$

The parsing table:

	=	*	a	#	S	L	R
0		Sh 4	Sh 5		1	2	3
1				" Accept "			
2	Sh 6/ R(5)			R(5)			
3				R(2)			
4		Sh 4	Sh D5			8	7
5	R(4)			R(4)			
6		Sh 4	Sh 5			8	9
7	R(3)			R(3)			
8	R(5)			R(5)			
9				R(1)			

The parsing table is multi-defined so the grammar is not SLR(1).

2) Proof that the grammar G is LR(1):

Calculation of the canonical collection of LR(1) items:

- $I_0 = \{[S' \rightarrow .S, \#], [S \rightarrow .L=R, \#], [S \rightarrow .R, \#], [L \rightarrow . *R, =], [L \rightarrow .a, =], [R \rightarrow .L, \#], [L \rightarrow . *R, \#], [L \rightarrow .a, \#]\}$
- $I_1 = \text{GOTO}(I_0, S) = \{[S' \rightarrow S., \#]\}$
- $I_2 = \text{GOTO}(I_0, L) = \{[S \rightarrow L.=R, \#], [R \rightarrow L., \#]\}$
- $I_3 = \text{GOTO}(I_0, R) = \{[S \rightarrow R., \#]\}$
- $I_4 = \text{GOTO}(I_0, *) = \{[L \rightarrow *.R., =], [R \rightarrow .L., =], [L \rightarrow . *R., =], [L \rightarrow .a., =], [L \rightarrow *.R., \#], [R \rightarrow .L., \#], [L \rightarrow . *R., \#], [L \rightarrow .a., \#]\}$
- $I_5 = \text{GOTO}(I_0, a) = \{[L \rightarrow a., =], [L \rightarrow a., \#]\}$
- $I_6 = \text{GOTO}(I_2, =) = \{[S \rightarrow L.=R., \#], [R \rightarrow .L., \#], [L \rightarrow . *R., \#], [L \rightarrow .a., \#]\}$
- $I_7 = \text{GOTO}(I_4, R) = \{[L \rightarrow *R., =], [L \rightarrow *R., \#]\}$
- $I_8 = \text{GOTO}(I_4, L) = \{[R \rightarrow L., =], [R \rightarrow L., \#]\}$
- $\text{GOTO}(I_4, *) = I_4$
- $\text{GOTO}(I_4, a) = I_5$
- $I_9 = \text{GOTO}(I_6, R) = \{[S \rightarrow L.=R., \#]\}$
- $I_{10} = \text{GOTO}(I_6, L) = [R \rightarrow L., \#]$
- $I_{11} = \text{GOTO}(I_6, *) = \{[L \rightarrow *.R., \#], [R \rightarrow .L., \#], [L \rightarrow . *R., \#], [L \rightarrow .a., \#]\}$
- $I_{12} = \text{GOTO}(I_6, a) = \{[L \rightarrow a., \#]\}$
- $I_{13} = \text{GOTO}(I_{11}, R) = \{[L \rightarrow *R., \#]\}$
- $\text{GOTO}(I_{11}, L) = I_{10}$
- $\text{GOTO}(I_{11}, *) = I_{11}$

• $GOTO(I_{11}, a) = I_{12}$

The analysis table:

	=	*	A	#	S	L	R
0		Sh 4	Sh 5		1	2	3
1				" Accept "			
2	Sh 6			R(5)			
3				R(2)			
4		Sh 4	Sh 5			8	7
5	R(4)			R(4)			
6		Sh 11	Sh 12			10	9
7	R(3)			R(3)			
8	R(5)			R(5)			
9				R(1)			
10				R(5)			
11		Sh 11	Sh 12			10	13
12				R(4)			
13				R(3)			

The parsing table is mono-defined, therefore the grammar is LR(1).

3) Proof that the grammar G is LALR(1):

The states that share the same core are:

- I_4 and $I_{11} \rightarrow$ after merging, we obtain : $I_{411} = \{[L \rightarrow *.R,=#], [R \rightarrow .L,=#], [L \rightarrow *.R,=#], [L \rightarrow .a,=#]\}$
- I_5 and $I_{12} \rightarrow$ after merging, we obtain: $I_{512} = \{[L \rightarrow a.,=#]\}$
- I_7 and $I_{13} \rightarrow$ after merging, we obtain: $I_{713} = \{[L \rightarrow *.R.,=#]\}$
- I_8 and $I_{10} \rightarrow$ after merging, we obtain: $I_{810} = \{[R \rightarrow L.,=#]\}$

Calculation of the canonical collection LR(1) after merging items that have the same core:

- $I_0 = \{[S' \rightarrow .S, \#], [S \rightarrow .L=R, \#], [S \rightarrow .R, \#], [L \rightarrow *.R, =], [L \rightarrow .a, =], [R \rightarrow .L, \#]\}$

- $I_1 = \text{GOTO}(I_0, S) = \{[S' \rightarrow S.,\#]\}$
- $I_2 = \text{GOTO}(I_0, L) = \{[S \rightarrow L.=R,\#], [R \rightarrow L.,\#]\}$
- $I_3 = \text{GOTO}(I_0, R) = \{[S \rightarrow R.,\#]\}$
- $I_{411} = \text{GOTO}(I_0, *) = \{[L \rightarrow *.R,=#], [R \rightarrow .L,=#], [L \rightarrow *.R,=#], [L \rightarrow .a,=#]\}$
- $I_{512} = \text{GOTO}(I_0, a) = \{[L \rightarrow a.,=#]\}$
- $I_6 = \text{GOTO}(I_2, =) = \{[S \rightarrow L.=R,\#], [R \rightarrow .L,\#], [L \rightarrow *.R,\#], [L \rightarrow .a,\#]\}$
- $I_{713} = \text{GOTO}(I_{411}, R) = \{[L \rightarrow *.R.,=#]\}$
- $I_{810} = \text{GOTO}(I_{411}, L) = \{[R \rightarrow L.,=#]\}$
- $\text{GOTO}(I_{411}, *) = I_{411}$
- $\text{GOTO}(I_{411}, a) = I_{512}$
- $I_9 = \text{GOTO}(I_6, R) = \{[S \rightarrow L=R.,\#]\}$
- $\text{GOTO}(I_6, L) = I_{810}$
- $\text{GOTO}(I_6, *) = I_{411}$
- $\text{GOTO}(I_6, a) = I_{512}$
- $I_{713} = \text{GOTO}(I_{411}, R) = \{[L \rightarrow *.R.,\#]\}$
- $\text{GOTO}(I_{411}, L) = I_{810}$
- $\text{GOTO}(I_{411}, *) = I_{411}$
- $\text{GOTO}(I_{411}, a) = I_{512}$

The parsing table:

	=	*	a	#	S	L	R
0		Sh 411	D512		1	2	3
1				" Accept "			
2	Sh 6			R(5)			
3				R(2)			
411		Sh 411	Sh 512			810	713
512	R(4)			R(4)			
6		Sh 411	Sh 512			810	9
713	R(3)			R(3)			
810	R(5)			R(5)			
9				R(1)			

The parsing table is mono-defined, therefore the grammar is LALR(1).

b) analysis of the string $a=*a$ using the two methods LR(1) and LALR(1).

Parsing the string $*a=a$ using LR(1) method		
Pile	Chaîne	Action
0	$a=*a\#$	Sh 5
0a5	$=*a\#$	R(4)
0L2	$=*a\#$	Sh 6
0L2=6	$*a\#$	Sh 11
0L2=6*11	$a\#$	Sh 12
0L2=6*11a12	$\#$	R(4)
0L2=6*11L10	$\#$	R(5)
0L2=6*11R13	$\#$	R(3)
0L2=6L10	$\#$	R(5)
0L2=6R9	$\#$	R(1)
0S1	$\#$	"Accepter"

Parsing the string $*a=a$ using LALR(1) method		
Pile	Chaîne	Action
0	$a=*a\#$	Sh 512
0a512	$=*a\#$	R(4)
0L2	$=*a\#$	Sh 6
0L2=6	$*a\#$	Sh 411
0L2=6*411	$a\#$	Sh 512
0L2=6*411a512	$\#$	R(4)
0L2=6*411L810	$\#$	R(5)
0L2=6*411R713	$\#$	R(3)
0L2=6L810	$\#$	R(5)
0L2=6R9	$\#$	R(1)
0S1	$\#$	"Accepter"

$S' \rightarrow S$

(1) $S \rightarrow L = R$

(2) $S \rightarrow R$

(3) $L \rightarrow * R$

(4) $L \rightarrow a$

(5) $R \rightarrow L$

b) analysis of the string $a=aa$ using the two methods LR(1) and LALR(1).

Parsing the string $*a=a$ using LR(1) method		
Pile	Chaîne	Action
0	$a=aa\#$	Sh 5
0a5	$=aa\#$	R(4)
0L2	$=aa\#$	Sh 6
0L2=6	$aa\#$	Sh 12
0L2=6a12	$a\#$	"Erreur"

Parsing the string $*a=a$ using LALR(1) method		
Pile	Chaîne	Action
0	$a=aa\#$	Sh 512
0a512	$=aa\#$	R(4)
0L2	$=aa\#$	Sh 6
0L2=6	$aa\#$	Sh 412
0L2=6a512	$a\#$	"Erreur"

Exercise 4.

Proof that the grammar G is not SLR(1), nor LR(1), nor LALR(1):

We begin with the LR(1) parsing:

LR(1) parsing table :

Augmented grammar G'.

$$A' \rightarrow A \#$$

$$1) A \rightarrow a A a$$

$$2) A \rightarrow b A b$$

$$3) A \rightarrow \varepsilon$$

Set of LR(1) items:

$$I_0 = \{[A' \rightarrow \cdot A, \#], [A \rightarrow \cdot a A a, \#], [A \rightarrow \cdot b A b, \#], [A \rightarrow \cdot, \#]\}$$

$$I_1 = \text{GOTO}(I_0, A) = \{[A' \rightarrow A \cdot, \#]\}$$

$$I_2 = \text{GOTO}(I_0, a) = \{[A \rightarrow a \cdot A a, \#], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$$I_3 = \text{GOTO}(I_0, b) = \{[A \rightarrow b \cdot A b, \#], [A \rightarrow \cdot a A a, b], [A \rightarrow \cdot b A b, b], [A \rightarrow \cdot, b]\}$$

$$I_4 = \text{GOTO}(I_2, A) = \{[A \rightarrow a A \cdot a, \#]\}$$

$$I_5 = \text{GOTO}(I_2, a) = \{[A \rightarrow a \cdot A a, a], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$$I_6 = \text{GOTO}(I_2, b) = \{[A \rightarrow b \cdot A b, a], [A \rightarrow \cdot a A a, b], [A \rightarrow \cdot b A b, b], [A \rightarrow \cdot, b]\}$$

$$I_7 = \text{GOTO}(I_3, A) = \{[A \rightarrow b A \cdot b, \#]\}$$

$$I_8 = \text{GOTO}(I_3, a) = \{[A \rightarrow a \cdot A a, b], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$$I_9 = \text{GOTO}(I_3, b) = \{[A \rightarrow b \cdot A b, b], [A \rightarrow \cdot a A a, b], [A \rightarrow \cdot b A b, b], [A \rightarrow \cdot, b]\}$$

$$I_{10} = \text{GOTO}(I_4, a) = \{[A \rightarrow a A a \cdot, \#]\}$$

$$I_{11} = \text{GOTO}(I_5, A) = \{[A \rightarrow a A \cdot a, a]\}$$

$$I_5 = \text{GOTO}(I_5, a) = \{[A \rightarrow a \cdot A a, a], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$$I_6 = \text{GOTO}(I_5, b) = \{[A \rightarrow b \cdot A b, a], [A \rightarrow \cdot a A a, b], [A \rightarrow \cdot b A b, b], [A \rightarrow \cdot, b]\}$$

$$I_{12} = \text{GOTO}(I_6, A) = \{[A \rightarrow b A \cdot b, a]\}$$

$$I_8 = \text{GOTO}(I_6, a) = \{[A \rightarrow a \cdot A a, b], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$$I_9 = \text{GOTO}(I_6, b) = \{[A \rightarrow b \cdot A b, b], [A \rightarrow \cdot a A a, b], [A \rightarrow \cdot b A b, b], [A \rightarrow \cdot, b]\}$$

$$I_{13} = \text{GOTO}(I_7, b) = \{[A \rightarrow b A b \cdot, \#]\}$$

$$I_{14} = \text{GOTO}(I_8, A) = \{[A \rightarrow a A \cdot a, b]\}$$

$$I_5 = \text{GOTO}(I_8, a) = \{[A \rightarrow a \cdot A a, a], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$$I_6 = \text{GOTO}(I_8, b) = \{[A \rightarrow b \cdot A b, a], [A \rightarrow \cdot a A a, b], [A \rightarrow \cdot b A b, b], [A \rightarrow \cdot, b]\}$$

$$I_{15} = \text{GOTO}(I_9, A) = \{[A \rightarrow b A \cdot b, b]\}$$

$$I_8 = \text{GOTO}(I_9, a) = \{[A \rightarrow a \cdot A a, b], [A \rightarrow \cdot a A a, a], [A \rightarrow \cdot b A b, a], [A \rightarrow \cdot, a]\}$$

$I_9 = \text{GOTO}(I_9, b) = \{ [A \rightarrow b.Ab, b], [A \rightarrow . aAa, b], [A \rightarrow . bAb, b], [A \rightarrow ., b] \}$

$I_{16} = \text{GOTO}(I_{11}, a) = \{ [A \rightarrow aAa ., a] \}$

$I_{17} = \text{GOTO}(I_{12}, b) = \{ [A \rightarrow bAb ., a] \}$

$I_{18} = \text{GOTO}(I_{14}, a) = \{ [A \rightarrow aAa ., b] \}$

$I_{19} = \text{GOTO}(I_{15}, b) = \{ [A \rightarrow bAb ., b] \}$

LR(1) parsing table :

	A	B	#	A
0	Sh2	Sh 3	R(3)	1
1			"Accept"	
2	Sh 5/ R(3)	Sh 6		4
3	Sh 8	Sh 9/ R(3)		7
4	Sh 10			
5	Sh 5/ R(3)	Sh 6		11
6	Sh 8	Sh 9/ R(3)		12
7		Sh 13		
8	Sh 5/ R(3)	Sh 6		14
9	Sh 8	Sh 9/ R(3)		15
10			R(1)	
11	Sh 16			
12		Sh 17		
13			R(2)	
14	Sh 18			
15		Sh 19		
16	R(1)			
17	R(2)			
18		R(1)		
19		R(2)		

The LR(1) parsing table is **multi-defined** , therefore the grammar G is not LR(1). Consequently, the grammar G is neither SLR(1) nor LALR(1).