Série N° 3

Exercice 3:

l'operateur A définie sur $C^{\infty}(\mathbb{R})$, soit $f(x) \in C^{\infty}(\mathbb{R})$

donc $Af(x) = f''(x) - f'(x)$.

Les valeurs propres λ et les fonctions propres f de A telque:

$f''(x) - f'(x) = \lambda f(x)$.

$$Af(x) = \lambda f(x) \Leftrightarrow f''(x) - f'(x) - \lambda f(x) = 0 \quad \dots (*)$$

$$\Leftrightarrow f''(x) - f'(x)$$

l'equation $(*)$, c'est une eq diff d'ordre 2 à coeff constantes homogène, alors la solution est sous la forme $f(x) = e^{rx}$.

$\Rightarrow r^2 - r - \lambda = 0$, (......., continuer ~~des calcules~~ la résolution de l'equation caractéristique (depend de λ)....

Exercice 4:

1) $f(x,y)$ est harmonique $\Leftrightarrow \dfrac{\partial^2 f(x,y)}{\partial x^2} + \dfrac{\partial^2 f(x,y)}{\partial y^2} = 0$.

Calculer $\dfrac{\partial f}{\partial x} = \dots$, $\dfrac{\partial^2 f}{\partial x^2} = \dots$

$\dfrac{\partial f}{\partial y} = \dots$, $\dfrac{\partial^2 f}{\partial y^2} = \dots$

Exercice 5:

on a $\Delta u(x,y) = \dfrac{\partial^2 u(x,y)}{\partial x^2} + \dfrac{\partial^2 u(x,y)}{\partial y^2}$. , Les coordonnées polaires sont r et φ.

on pose: $\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \Rightarrow \begin{cases} r^2 = x^2 + y^2, \quad r = \sqrt{x^2+y^2}. \\ \varphi = \arccos\dfrac{x}{r}, \text{ ou } \varphi = \arcsin\dfrac{y}{r} \end{cases}$

donc:
$$\frac{\partial^2 u(x,y)}{\partial x^2} = \frac{\partial^2 u(r,\varphi)}{\partial r^2}\left(\frac{\partial r}{\partial x}\right)^2 + 2\frac{\partial^2 u(r,\varphi)}{\partial r \partial \varphi}\left(\frac{\partial r}{\partial x}\frac{\partial \varphi}{\partial x}\right) + \frac{\partial^2 u(r,\varphi)}{\partial \varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)^2 + \frac{\partial u}{\partial r}\frac{\partial^2 r}{\partial x^2} + \frac{\partial u}{\partial \varphi}\frac{\partial^2 \varphi}{\partial x^2}.$$

$$\frac{\partial^2 u(x,y)}{\partial y^2} = \frac{\partial^2 u(r,\varphi)}{\partial r^2}\left(\frac{\partial r}{\partial y}\right)^2 + 2\frac{\partial^2 u}{\partial r \partial \varphi}\left(\frac{\partial r}{\partial y}\frac{\partial \varphi}{\partial y}\right) + \frac{\partial^2 u}{\partial \varphi^2}\left(\frac{\partial \varphi}{\partial y}\right)^2 + \frac{\partial u}{\partial r}\frac{\partial^2 r}{\partial y^2}$$

$$+ \frac{\partial u}{\partial \varphi} \cdot \frac{\partial^2 \varphi}{\partial y^2}.$$

$\frac{\partial r}{\partial y} = \ldots$, $\frac{\partial^2 r}{\partial y} = \ldots$, $\frac{\partial r}{\partial x} = \ldots$, $\frac{\partial^2 r}{\partial x^2} = \ldots$

$\frac{\partial \varphi}{\partial x} = \frac{\partial}{\partial x}\left(\arcsin \frac{y}{\sqrt{x^2+y^2}}\right) = \ldots$, $\frac{\partial^2 \varphi}{\partial x^2} = \ldots$, $\frac{\partial \varphi}{\partial y} = \ldots$

$\frac{\partial^2 \varphi}{\partial y^2} = \ldots$

On remplace Laplacien par les résultats.