Chapter 4: Metrological Characteristics of Measuring Instruments (6 Weeks)

1 Measurement: Vocabulary and Notation

1.1 Definitions

- The quantity intended to be measured is called the **measurand**.
- Measurement (mesurage) is defined as the set of operations enabling the experimental determination of one or more values that can reasonably be attributed to a quantity.

For example, when measuring the resistance R of a linear passive dipole, the measurand is the resistance R, and the measurement is carried out using an ohmmeter.

- The **true value** (M_{true}) of the measurand is the value that would be obtained if the measurement were perfect. Since no measurement is ever perfect, this value is always unknown.
- The **measurement result** is the set of values attributed to a measurand, together with all relevant available information. A complete measurement result includes the associated measurement uncertainty, indicating the interval within which the probable values of the measurand lie.

In metrology, the symbol m is often used to denote the measured numerical value, while M denotes the complete measurement result (i.e., an interval of values).

Since no measurement is perfect, there is always a measurement error:

$$\varepsilon = m - m_0$$

where m_0 is a reference value. If the reference value is the true value of the measurand, the error is unknown.

Remark: In everyday French language, the word "measurement" has several meanings. For this reason, the term *mesurage* was introduced to denote the act of measuring. Nevertheless, the term "measurement" is widely used in expressions such as measuring instrument, measuring device, unit of measurement, and measurement method.

1.2 Error and Uncertainty

Every measurement is affected by error; it is impossible to perform perfectly exact measurements. To quantify the degree of approximation, errors must be estimated and their effects on final results evaluated. This is the purpose of error analysis or uncertainty analysis.

The **absolute error** of a measured quantity is defined as:

$$\varepsilon = |m - m_0|$$

The **relative error**, which characterizes the quality of the result, is defined as:

$$\varepsilon_r = \frac{|m - m_0|}{|m_0|}$$

and is generally expressed as a percentage.

The term *error* is used only when a reference value is available and considered to be "true". In most practical cases, the true value is unknown; therefore, we speak of **uncertainty** rather than error.

A complete measurement result is expressed as:

$$m \pm \Delta m$$

where Δm is the **absolute uncertainty**, defined as half the width of the confidence interval.

The absolute uncertainty always results from an estimation and depends both on the measurement means and on the experimenter's judgment.

The **relative uncertainty** is defined as:

$$\frac{\Delta m}{m}$$

1.3 Propagation of Uncertainties

Measured quantities are often used to compute derived results. The propagation of uncertainties must therefore be evaluated.

- **Addition:**

$$s = a + b \qquad \Delta s = \Delta a + \Delta b$$

- **Subtraction:**

$$d = a - b \qquad \Delta d = \Delta a + \Delta b$$

- **Multiplication:**

$$p = a \cdot b \qquad \frac{\Delta p}{p} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

- **Division:**

$$q = \frac{a}{b} \qquad \frac{\Delta q}{q} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

The purpose of error analysis is to estimate measurement errors and evaluate their consequences on results. Absolute error alone is insufficient to characterize measurement quality; relative error (or relative uncertainty) provides a more meaningful assessment.

2 Random and Systematic Errors

2.1 Random Error

Repeatability conditions are satisfied when the same operator or program performs N measurements under identical conditions.

Under repeatability conditions, the best estimator of the measurand is the arithmetic mean:

$$\overline{m} = \frac{1}{N} \sum_{i=1}^{N} m_i$$

The deviation of an individual measurement m_i from the mean is called the **random error**.

2.2 Systematic Error

The **systematic error** is defined as the difference between the mean of an infinite number of measurements (performed under repeatability conditions) and the true value of the measurand.

Since the true value is unknown and infinite measurements cannot be performed, the systematic error cannot be known exactly; only an estimate is possible.

Random error varies unpredictably between measurements, whereas systematic error remains constant (though unknown).

2.3 Trueness and Precision

Measurement error generally has two components:

$$ER = ER_a + ER_S$$

where ER_a is the random error and ER_S is the systematic error.

The estimate of systematic error is called **measurement bias**.

- **Precision** (or repeatability) describes the closeness of agreement between repeated measurements.
- **Trueness** describes the closeness of agreement between the mean measurement result and the true value.

3 Estimation of Experimental Uncertainty and Presentation of Results

3.1 Type A Evaluation of Standard Uncertainty

Type A evaluation is based on statistical analysis of repeated observations. Assuming n independent measurements m_k:

- Best estimate (mean value):

$$\overline{m} = \frac{1}{n} \sum_{k=1}^{n} m_k$$

- Experimental standard deviation:

$$S_n = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(m_k - \overline{m})^2}$$

- Standard uncertainty of the mean:

$$u = \frac{S_n}{\sqrt{n}}$$

3.2 Type B Evaluation of Standard Uncertainty

Type B evaluation is based on scientific judgment using all available information, such as:

- previous measurements,
- experience with instruments and materials,
- manufacturer specifications,
- calibration certificates,
- reference data from handbooks.

3.3 Presentation of Measurement Results

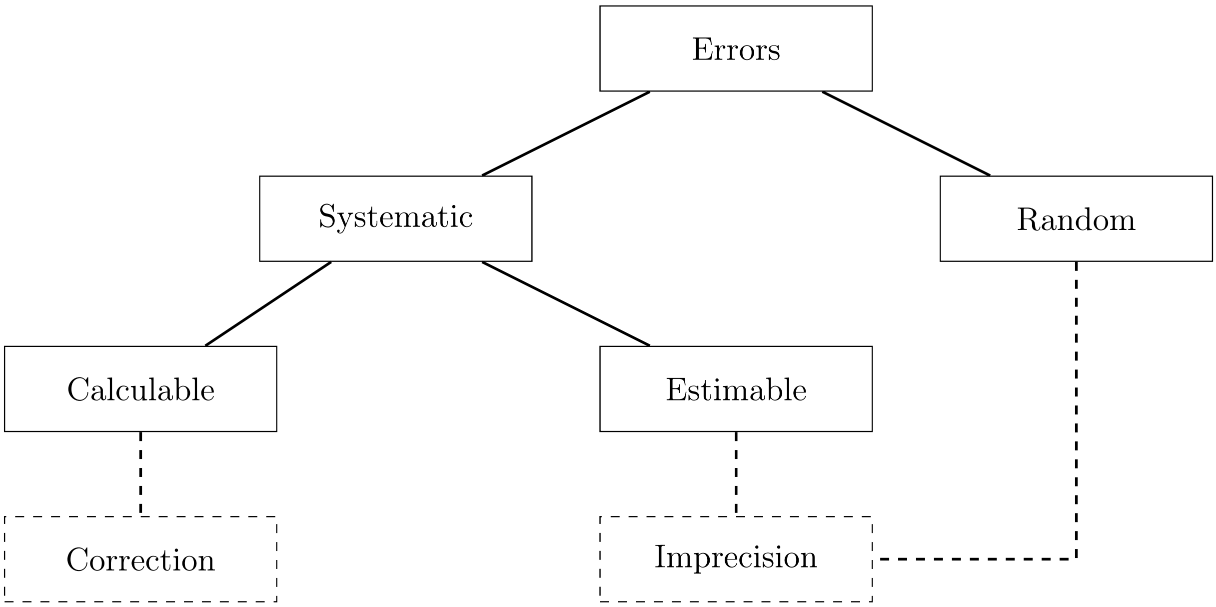

Figure 1: Classification of measurement errors.

A measurement result is generally expressed as:

$$X = X_{\text{corrected}} \pm U$$

where U is the expanded uncertainty.

3.4 Indirect Measurements

For a function of several variables:

$$y = f(x_1, x_2, \ldots, x_n)$$

The uncertainty is approximated using the differential method:

$$\Delta y = \sum_{i=1}^{n} \frac{\partial y}{\partial x_i} \Delta x_i$$

The combined uncertainty is:

$$U_y = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} U_{x_i} \right)^2}$$

For composite functions:

$$F(y) = f(x_1, x_2, \ldots, x_n)$$

$$U_y = \frac{1}{F'(y)} \sqrt{\sum_{i=1}^{n} \left(\frac{\partial F}{\partial x_i} U_{x_i} \right)^2}$$