

## Solution of series N° 5

### Exercise 1

Variable	Type	Subtype / Comment
Place of residence	Qualitative	Nominal
Sex	Qualitative	Nominal
Blood group	Qualitative	Nominal
Number of white blood cells	Quantitative	Discrete
Size (height)	Quantitative	Continuous
Age	Quantitative	Continuous (often recorded as discrete)
Number of languages spoken	Quantitative	Discrete
Level of obesity	Qualitative	Ordinal
Eye color	Qualitative	Nominal

### Exercise 2

#### ❶ Population, Character, and Nature

- **Population:** 88 employees of the company.
- **Character studied:** Number of sports sessions per month ( $x_i$ ).
- **Nature:** Quantitative discrete variable.

#### ❷ Table Completion

$x_i =$ Number of sessions per month	$n_i$	$n_i^{c\uparrow}$	$f_i$	$f_i^{c\uparrow}$
8	7	7	0.079	0.079
12	20	27	0.227	0.306
16	23	50	0.261	0.567
20	19	69	0.216	0.783
24	14	83	0.159	0.942
28	5	88	0.057	1.000
<b>Total</b>	88		1.000	

### ③ Mode, Mean, and Median

- **Mode:** 16 sessions (highest frequency)

- **Mean:**

$$\bar{x} = \frac{\sum n_i x_i}{N} = \frac{1520}{88} \approx 17.27$$

- **Median:**  $N/2 = 44^{th}$  observation  $\Rightarrow$  falls in  $x = 16$  class  $\Rightarrow$  Median  $\approx 16$

### ④ Quartiles and Interquartile Range (IQR)

- Q1 position:  $N/4 = 22 \Rightarrow Q1 = 12$
- Median (Q2) = 16
- Q3 position:  $3N/4 = 66 \Rightarrow Q3 = 20$
- $I_Q = Q3 - Q1 = 20 - 16 = 4$

### ⑤ Range, Variance, Standard Deviation, and Coefficient of Variation

- Range:  $28 - 8 = 20$

- Variance:

$$\sigma^2 = \frac{\sum n_i (x_i - \bar{x})^2}{N} = \frac{2546.1}{88} \approx 28.93$$

- Standard deviation:  $\sigma \approx 5.38$

- Coefficient of variation:

$$CV = \frac{\sigma}{\bar{x}} \times 100 \approx \frac{5.38}{17.27} \times 100 \approx 31.2\%$$

## Exercise 3

### ① Population, character and nature

- **Population:** The 70 presumed healthy men.
- **Character studied:** Haemoglobin level in the blood (g/L).
- **Nature:** Quantitative continuous variable.

### ② Completed table

Classes	$n_i$	$n_i^{c\uparrow}$	$f_i$	$f_i^{c\uparrow}$
[105, 115[	0	0	0.000	0.000
[115, 125[	0	0	0.000	0.000
[125, 135[	3	3	0.043	0.043
[135, 145[	4	7	0.057	0.100
[145, 155[	18	25	0.257	0.357
[155, 165[	19	44	0.271	0.628
[165, 175[	12	56	0.171	0.800
[175, 185[	14	70	0.200	1.000

#### ④ Mode, mean and median

- **Mode:**

The modal class is the class with the highest frequency:

$$[155, 165[$$

For grouped data, the mode is given by:

$$\text{Mode} = L_i + \frac{d_1}{d_1 + d_2} \cdot a$$

where:

- $L_i = 155$  is the lower bound of the modal class,
- $d_1 = n_i - n_{i-1} = 19 - 18 = 1$
- $d_2 = n_i - n_{i+1} = 19 - 12 = 7$
- $a = 165 - 155 = 10$  is the class width.

Substituting the values:

$$\text{Mode} = 155 + \frac{19 - 18}{(19 - 18) + (19 - 12)} \cdot 10 = 155 + \frac{1}{8} \cdot 10$$

$$\text{Mode} \approx 156.25 \text{ g/L}$$

- **Mean:** For grouped data, the mean is calculated using class center.

Class	class center $c_i$	Frequency $n_i$	$n_i c_i$
[105, 115[	110	0	0
[115, 125[	120	0	0
[125, 135[	130	3	390
[135, 145[	140	4	560
[145, 155[	150	18	2700
[155, 165[	160	19	3040
[165, 175[	170	12	2040
[175, 185[	180	14	2520
Total			11250

The mean is given by:

$$\bar{x} = \frac{\sum n_i c_i}{N} = \frac{11250}{70} \approx \boxed{160.7 \text{ g/L}}$$

- **Median:**

Median position:

$$\frac{N}{2} = 35$$

Median class: [155, 165[

$$\text{Median} = L_i + \frac{\frac{N}{2} - \sum_{i=1}^{<\text{Me}} n_i}{n_{\text{Me}}} \cdot a = 155 + \frac{35 - 25}{19} \cdot 10 \approx 160.3 \text{ g/L}$$

### ⑤ Range, variance, standard deviation and coefficient of variation

- **Range:**

$$R = 185 - 105 = 80 \text{ g/L}$$

- **Variance:** the variance is computed using the class centres (midpoints).

Class	Centre $x_i$	$n_i$	$(x_i - \bar{x})^2$	$n_i(x_i - \bar{x})^2$
[125, 135[	130	3	941.49	2824.47
[135, 145[	140	4	428.49	1713.96
[145, 155[	150	18	114.49	2060.82
[155, 165[	160	19	0.49	9.31
[165, 175[	170	12	86.49	1037.88
[175, 185[	180	14	376.49	5270.86
Total				12917.30

The variance is given by:

$$\sigma^2 = \frac{\sum n_i(x_i - \bar{x})^2}{N} = \frac{12917.30}{70} \approx \boxed{184.53}$$

- **Standard deviation:**

$$\sigma = \sqrt{184.53} \approx 13.58 \text{ g/L}$$

- **Coefficient of variation:**

$$CV = \frac{\sigma}{\bar{x}} \times 100 = \frac{13.58}{160.7} \times 100 \approx 8,45\%$$