Chapter 05:

boundary layer

boundary layer

Introduction

The concept of the boundary layer was introduced for the first time by a German engineer, Prandtl, in 1904. According to Prandtl’s theory, when a real fluid flows over a fixed solid wall, the flow is divided into two regions.
i) A thin layer in the vicinity of the solid wall, where viscous forces and rotation cannot be neglected.
ii) An outer region where viscous forces are very small and can be neglected; the flow behavior there is similar to the free-stream flow upstream.

2.2 Définitions et caractéristiques de la couche limite

The flow of a viscous fluid over a solid wall forms a region in which the velocity increases from zero at the wall and approaches the free-stream velocity. This region is called the boundary layer. Figure 2.1 shows the development of a boundary layer along one side of a long flat plate aligned with the flow direction. The velocity gradient produces a significant shear stress at the wall, τ_0) or τ_w ,(as illustrated in Figure 2.1

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0}$$

The velocity gradient in a turbulent boundary layer is larger than that in a laminar boundary layer.

An entrance region exists in which the boundary layer develops and $dp/dx \neq cst$, while the pressure remains constant. A second region is the fully developed region, in which:

•The boundary layer fills the entire flow domain.

•The velocity profiles, the pressure gradient and the wall shear stress are constant, i.e. they do not depend on x.

•The flow is either laminar or turbulent over the entire length, meaning that the transition phase is not taken into account.

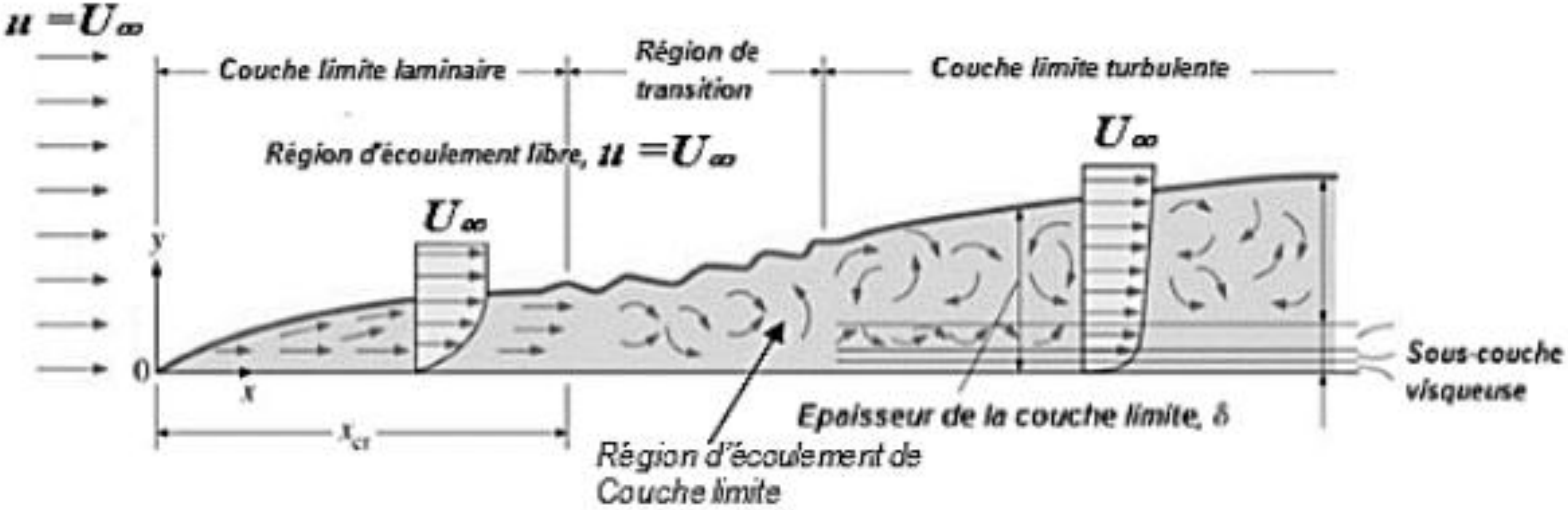

Figure 2.1 Schéma d'écoulement de couche limite sur une plaque plane

- **<u>Two regions are distinguished</u>**: a region close to the wall where viscous forces are much larger than inertial forces and where the velocity gradient is not zero (**called boundary layer**) . The second region, also called ***the free stream***, **is far from the wall**, where inertial forces are much larger than viscous forces and the velocity gradient is zero.

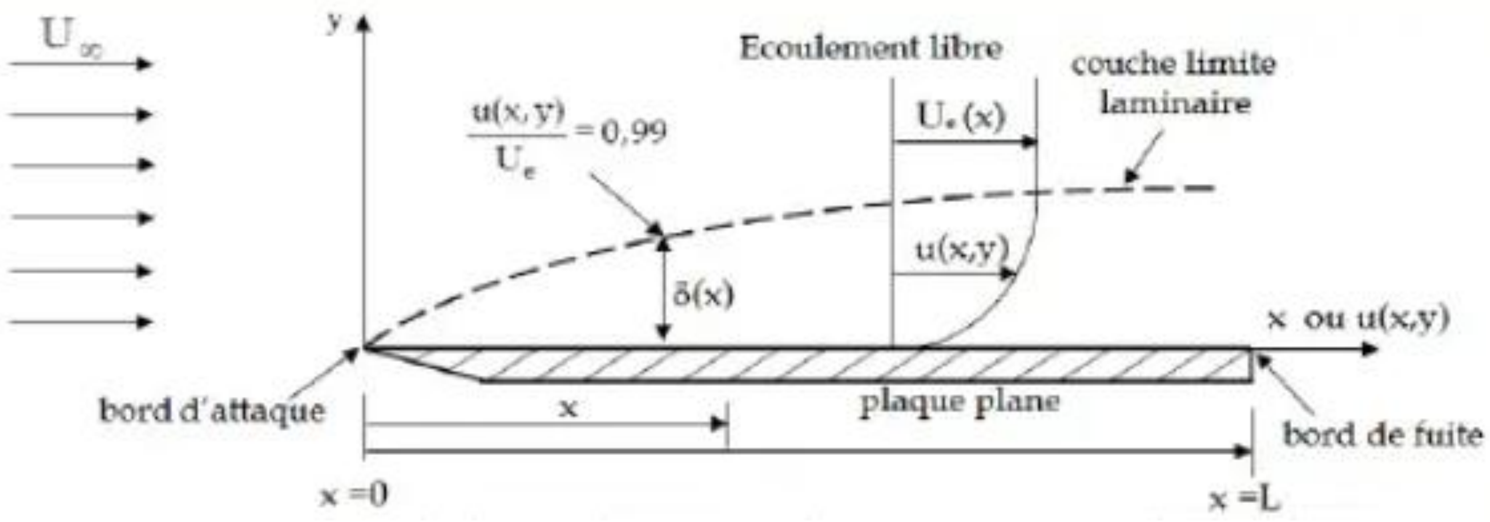

Figure: laminar dynamic boundary layer over a flat plate

Main characteristics:

•It has a finite thickness that depends on the viscosity and the velocity of the fluid.

•Inside the boundary layer, there is a rapid variation of velocity (a strong velocity gradient).

•One often distinguishes between a laminar boundary layer and a turbulent boundary layer, depending on the nature of the flow.

Consider a flow over a flat plate, as illustrated in Figure 2.1. The flow over the plate can be divided into two regions:

i) $0 \leq y \leq \delta$: boundary-layer flow (a region close to the wall) : in which the effect of viscous forces is important.

Because of the no-slip condition at the wall, the first layer of fluid experiences a deceleration. This slowed layer causes an additional delay for the adjacent layer, thus developing a thin region in which the flow velocity increases from zero at the solid wall and approaches the free-stream velocity.

Because of the presence of a velocity gradient within the boundary-layer region $du/dy \neq 0$, the fluid particles at the top begin to deform, having a higher velocity than those located below. This force causes the fluid particle to rotate when it enters the boundary-layer region **(see Figure 2.2)**. Consequently, this fluid layer is also called rotational flow.

ii) $y > \delta$: external flow region (**far from the wall**): external flow region outside the boundary layer, where the viscous force is very small and can be neglected. There is no velocity gradient in this zone and the fluid particle does not rotate when it enters the region outside the boundary layer; therefore, the flow is also called irrotational flow.

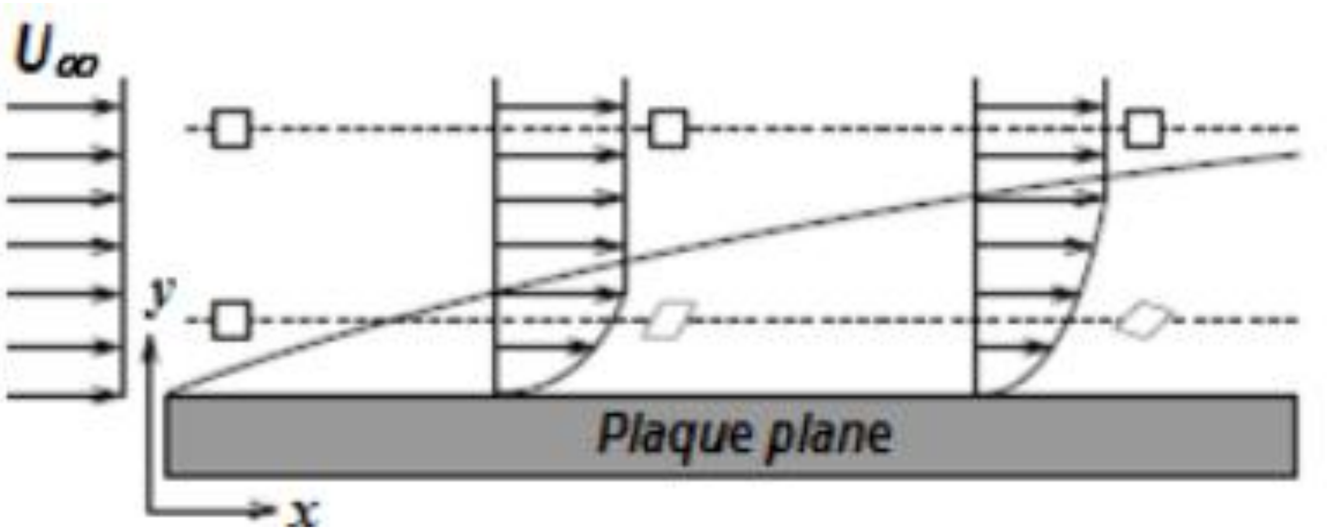

Figure 2.2 Rotation of fluid particles in the boundary-layer region.

- As shown in the figure, the boundary-layer conditions are that the fluid sticks to the solid wall:

$$\text{at} \quad y = 0 \qquad u = v = 0 \qquad (2.1)$$

- Outside the boundary layer, the fluid velocity is equal to the free-stream velocity, that is:

$$0 \leq y \leq \delta \qquad du/\mathrm{d}y \neq 0 \text{ (there is a speed gradient)}$$

$$\text{at} \quad y = \delta \qquad u(x, y)/U_\infty = 0.99 \qquad (2.2)$$

The following boundary condition is also valid for boundary-layer flow:

$$\text{when} \quad y > \delta \qquad du/\mathrm{d}y = 0 \qquad (2.3)$$

2.2.1 Boundary-layer thickness

Three types of boundary-layer thickness are distinguished,

namely:

•Conventional boundary-layer thickness, $\boldsymbol{\delta}$

•Displacement thickness of the boundary layer, $\boldsymbol{\delta}^*$

•Momentum thickness of the boundary layer, $\boldsymbol{\theta}$

2.2.2 Conventional boundary-layer thickness, δ

The thickness of the boundary layer is defined as the vertical distance between the solid wall and the point where the flow velocity reaches **99%** $(\boldsymbol{u} = \mathbf{0.99}\ \boldsymbol{U}_\infty)$ of the free-stream velocity **(Fig. 2.3)**.

- Conventional thickness: $\boldsymbol{\delta}$ such that $\boldsymbol{u}(\boldsymbol{\delta}) = \mathbf{0.99}\ \boldsymbol{U}_\infty$.
- The ordinate $\boldsymbol{y} = \boldsymbol{\delta}$ such that $\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y})/\boldsymbol{U}_\infty = \mathbf{0.99}$.

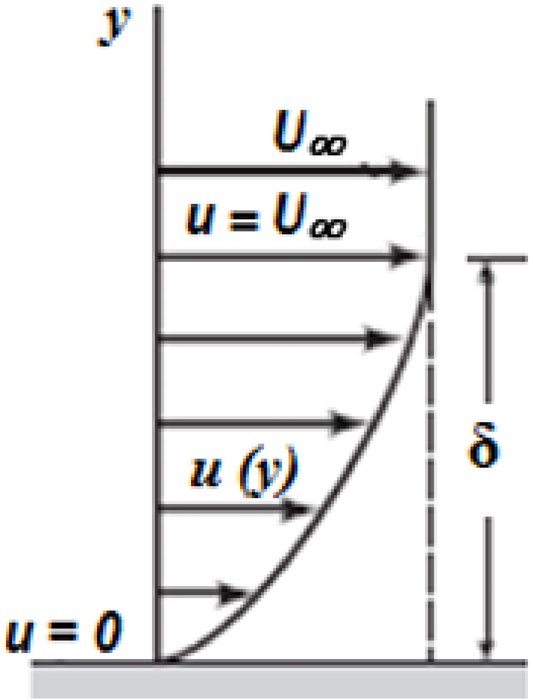

Figure 2.3 Conventional boundary-layer thickness.

2.2.3 Displacement thickness of the boundary layer, δ^*

The displacement thickness represents the vertical distance by which the solid wall has to be shifted upward so that the real fluid has the same mass flow rate as the ideal (inviscid) fluid.

The displacement thickness $\boldsymbol{\delta}^*$ measures the overall effect of viscous slowing on the mass flow rate, as if the wall were "thickened" and pushed the outer flow outward by a distance $\boldsymbol{\delta}^*$.

- A two-dimensional steady flow is considered along a plate, with an external velocity U_∞ ideal, inviscid fluid) and a real flow with profile $\boldsymbol{u}(\boldsymbol{y})$ in the boundary layer.

The displacement thickness is then the vertical distance $\boldsymbol{\delta}^*$ such that, if the real profile were replaced by a uniform flow U_∞ above a fictitious plate located at $\boldsymbol{y} = \boldsymbol{\delta}^*$,the same mass flow rate would be obtained as in the real flow above the real plate at $\boldsymbol{y} = \boldsymbol{0}$.

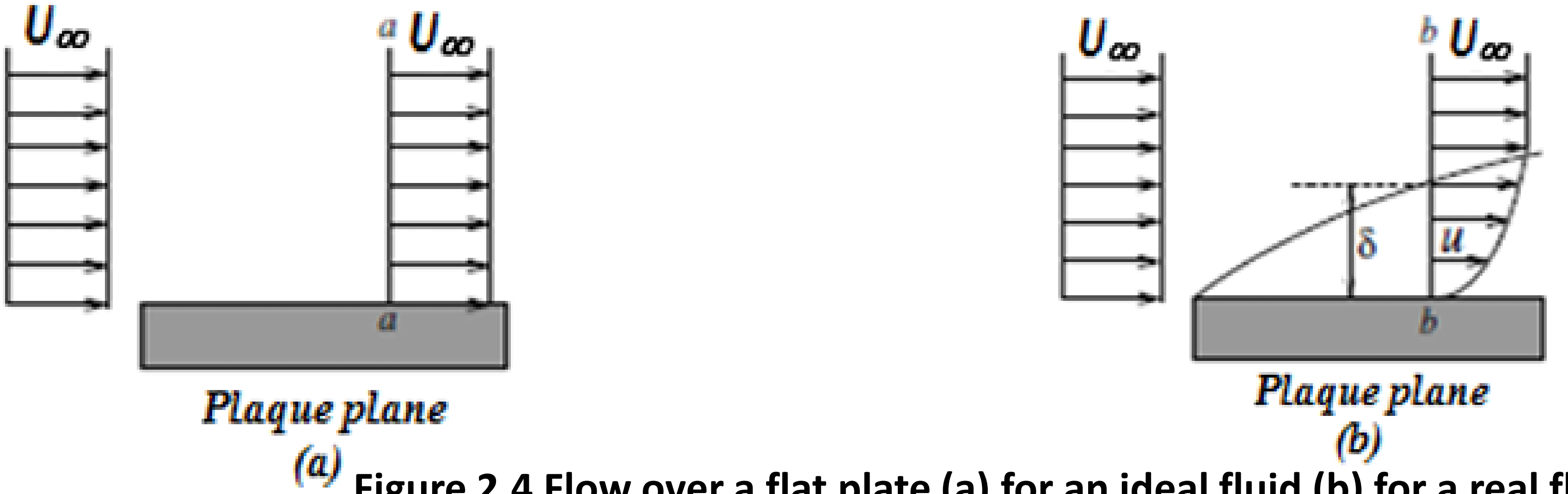

Figure 2.4 Flow over a flat plate (a) for an ideal fluid (b) for a real fluid.

Consider two types of fluid flow over a fixed horizontal flat plate with a flow velocity U_∞ ,as illustrated in Figure 2.4.

In the absence of viscosity, in the case of an ideal fluid **(Figure 2.4(a))**, a uniform velocity profile is developed above the solid wall. However, in the case of a viscous (real) fluid with no slip at the wall, a velocity gradient develops in the boundary-layer region, as shown in **Figure 2.4(b).**

The displacement thickness represents the vertical distance by which the solid wall must be shifted upward so that the ideal fluid has the same mass flow rate as the real fluid.

Here, $\boldsymbol{\delta}^*$ is the displacement thickness of the boundary layer.

$\boldsymbol{\delta*}$: is the **"apparent distance"** by which the outer flow is shifted because the fluid layers near the wall are slowed down and carry less mass than in an ideal fluid.

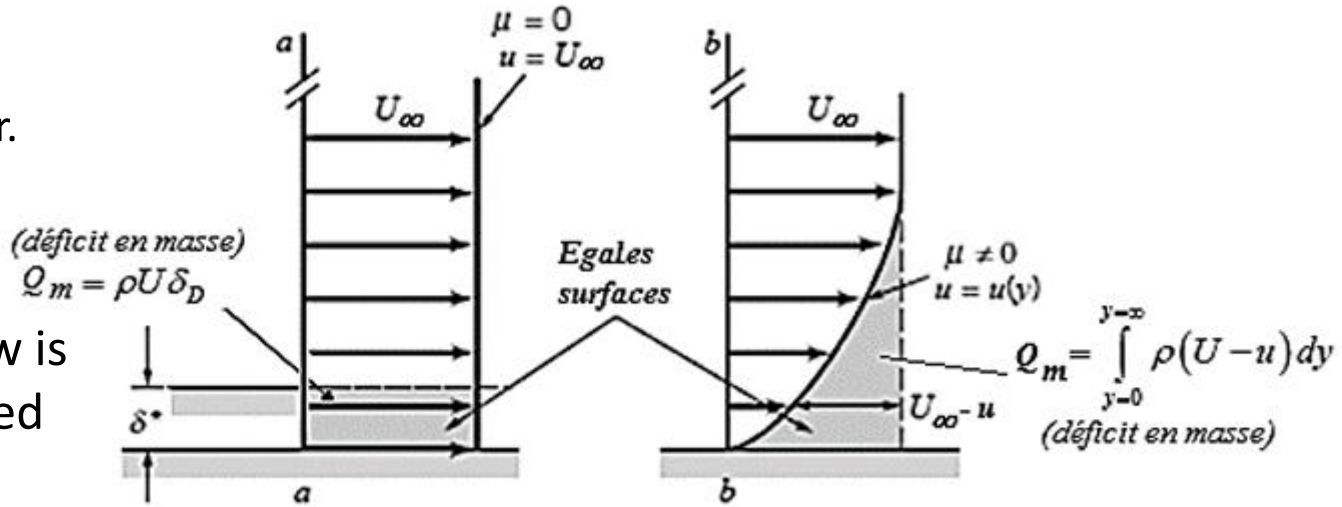

Figure 2.5 Displacement thickness of the boundary layer

Mathematical expression

In Cartesian coordinates, for an incompressible fluid of constant density, it can be shown that

$$\delta^*(x) = \int_0^\infty \left(1 - \frac{u(x, y)}{U_\infty}\right) dy$$

2.2.4 Momentum thickness of the boundary layer, $\boldsymbol{\theta}$

The "**momentum thickness**" $\boldsymbol{\theta}$ measures the loss of momentum flux in the boundary layer compared with a perfect (inviscid) flow having the same mass flow rate.

$\boldsymbol{\theta}$ **:** is the displacement thickness of the boundary layer.

Physical definition

- In an ideal fluid, the momentum flux per unit width is $\boldsymbol{\rho U_\infty^2}$ everywhere, whereas in the real flow the profile $\boldsymbol{u(y)}$ reduces this flux near the wall.
- $\boldsymbol{\theta}$ is defined as the vertical distance such that, if the real profile were replaced by a uniform flow of speed $\boldsymbol{U_\infty}$ above a fictitious plate located at $\boldsymbol{y = \theta}$, the momentum flux would equal the real flux despite the deficit created by the boundary layer.
- The momentum thickness represents the vertical distance by which the solid wall must be shifted upward so that the ideal fluid has the same momentum as the real fluid

Mathematical expression

For a two-dimensional incompressible flow, one obtains $\boldsymbol{\theta(x)} = \int_{0}^{\infty} \frac{u(\boldsymbol{x},\boldsymbol{y})}{U_\infty}\left(\mathbf{1} - \frac{u(\boldsymbol{x},\boldsymbol{y})}{U_\infty}\right) \boldsymbol{dy}$

which weights the deficit $(\mathbf{1} - \boldsymbol{u}/\boldsymbol{U}_\infty)$ by the local velocity u, making it a direct indicator of the loss of momentum due to viscosity.

In practice, $\boldsymbol{\theta}$ is strongly linked to the plate's skin-friction drag and appears in integral formulations of the von Kármán type for the development of the boundary layer.

- The three different boundary-layer thicknesses are shown and compared in Figure 2.7 below.

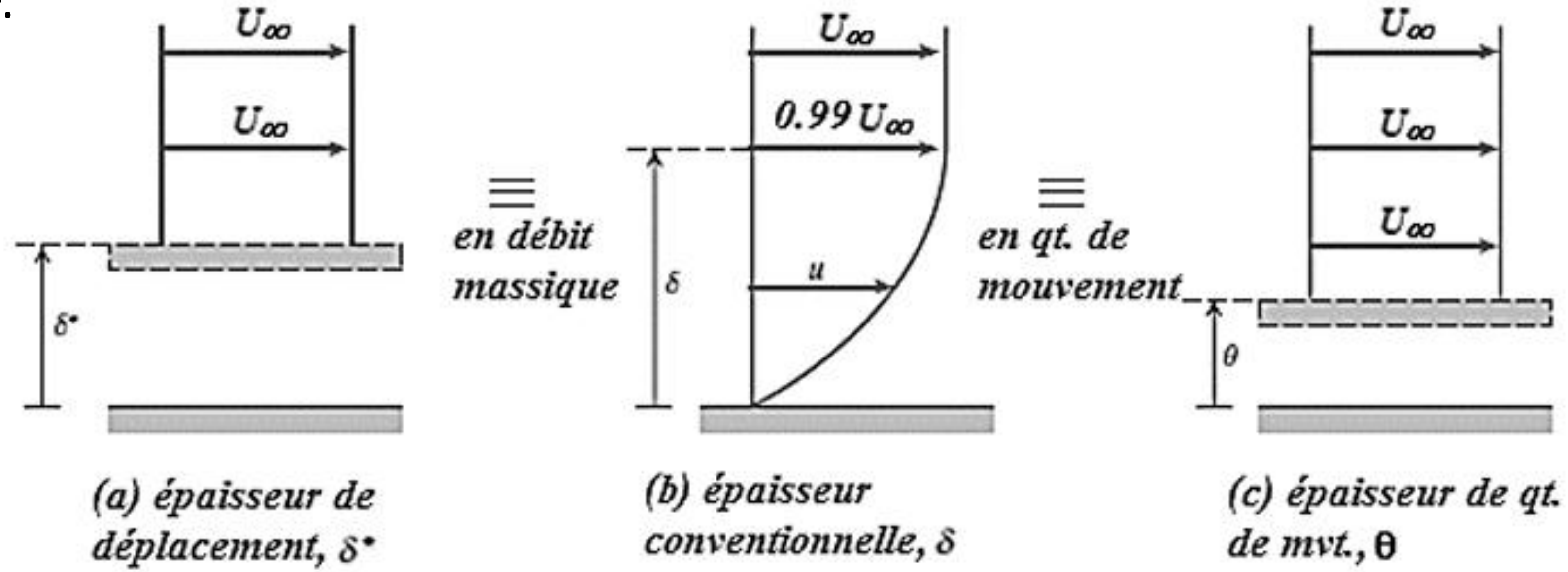

Figure 2.7 Boundary layer thicknesses.

The three thicknesses based on velocity-deficit integrals, $\boldsymbol{\delta}$**,** $\boldsymbol{\delta}^*$ **and** $\boldsymbol{\theta}$, are more directly connected to mass-flow and momentum losses in the boundary layer.

A dynamic boundary layer is turbulent when the Reynolds number $\boldsymbol{Re_x}$ exceeds the critical Reynolds number $\boldsymbol{Re_c} \approx \mathbf{5 \times 10^5}$.

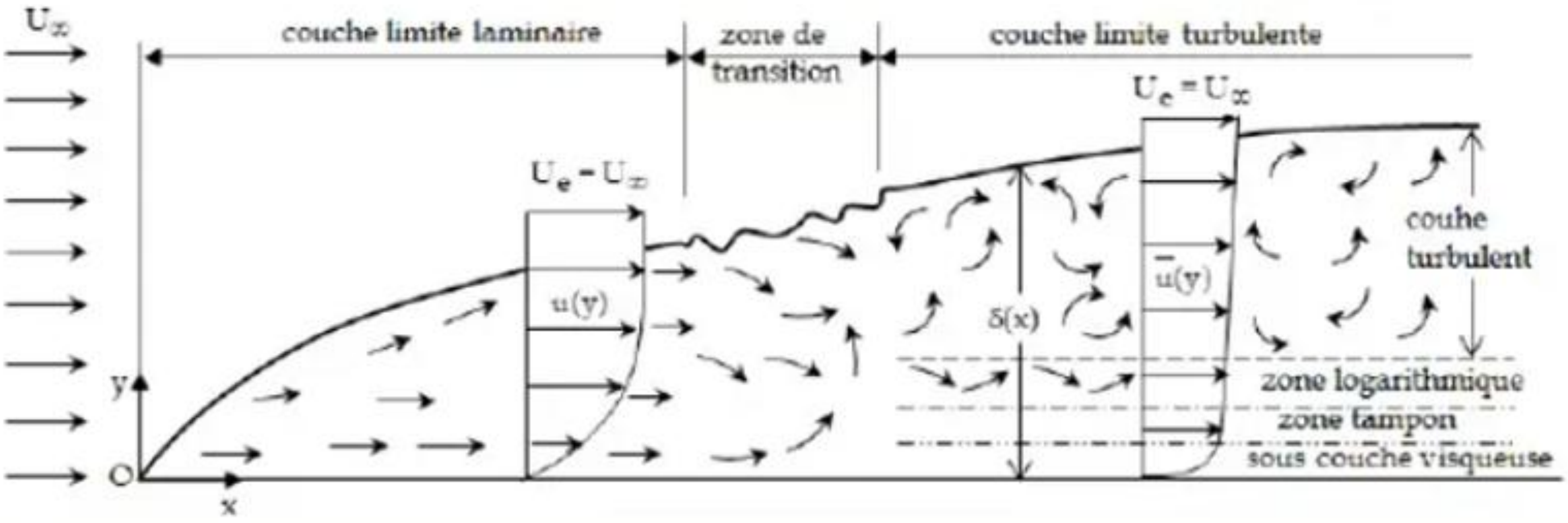

Figure 8.4: Development of the turbulent momentum boundary layer over a flat plate.

The transition from laminar to turbulent flow is accompanied by a sudden and significant increase in boundary-layer hickness and wall shear stress, and the velocity profile becomes much flatter.

Role of the Reynolds number

The local Reynolds number for a flow over a flat plate is defined as: $\boldsymbol{Re_x = \frac{U_\infty x}{\nu}}$,

where $\boldsymbol{\nu}$ is the kinematic viscosity.

•**Small $\boldsymbol{Re_x}$** → laminar boundary layer.

• **Increasing $\boldsymbol{Re_x}$** → instability, transition.

• **High $\boldsymbol{Re_x}$** → predominantly turbulent boundary layer.

In practice, the transition for an idealized flat plate often occurs around $\boldsymbol{Re_x \sim 5 \times 10^5}$,but this depends on the roughness of the plate and the level of turbulence in the upstream flow.

Scientific analysis of laminar, transitional and turbulent regions

a) Laminar boundary layer

Near the leading edge, the local Reynolds number

$$\boldsymbol{Re_x = \frac{U_\infty x}{\nu}}$$

is small. The flow is laminar and can be described by the Blasius solution.

•**Boundary-layer thickness:**

$$\boldsymbol{\delta(x) \approx \frac{5x}{\sqrt{Re_x}}}.$$

•Velocity profile is monotonic and smooth. This law is a fundamental result, confirmed experimentally.

b) Transition region

When $\boldsymbol{Re_x}$ increases, the boundary layer becomes unstable:

•Appearance of Tollmien–Schlichting waves.

•Growth of three-dimensional disturbances.

•Local alternation of laminar and turbulent structures.

This region is **intrinsically unsteady**, which justifies its qualitative representation in the figure.

c) Turbulent boundary layer

Beyond a certain Re_x ,the flow becomes fully turbulent:

$$\delta_{\mathrm{turb}}(x) \approx 0{,}37 \frac{x}{Re_x^{1/5}}$$

•Intense mixing.

•Boundary-layer thickness larger than in the laminar case.

•Presence of a viscous sublayer near the wall.

The structure shown in the figure (vortices, viscous sublayer) is consistent with modern models.

Under standard conditions for flow over a flat plate, with the same upstream conditions and at the same distance x from the leading edge, **the turbulent boundary layer is generally thicker than the laminar boundary layer**.

$$\delta_{turbulent} > \delta_{\mathrm{laminaire}}$$

This is explained by the **intense turbulent mixing**, which transports momentum more efficiently toward the outer part of the fluid, thereby increasing the thickness of the region where the velocity is influenced by the wall. The turbulence increases the transverse transport of momentum, which extends the area affected by the wall.

👉 Properties:

- Faster growth of $\boldsymbol{\delta}$.
- Intense turbulent mixing.
- “Thicker” boundary layer.